

GCSE Mathematics

Calculator

Higher Tier

Free Practice Set 3

1 hour 45 minutes



ANSWERS

Marks shown in brackets for each question (2)

A*	A	B	C	D	E
88	75	60	45	25	15

Legend used in answers

Green Box - Working out

5b means five times b
 $b = -3$ so $5 \times -3 = -15$

Red Box and ✓ - Answer

48 % ✓

Authors Note

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1. a) Draw a prime factor tree for 168 and 40

A prime factor tree breaks a number down into its prime number factors.

2 is the first prime number

Start with your number at the top. See if it can be halved. This halves into 2 and 84

84 can be halved again = 2×42

42 can be halved again = 2×21

21 = 3×7 both prime numbers

$168 = 2 \times 2 \times 2 \times 3 \times 7$
 These are all prime numbers

$40 = 2 \times 2 \times 2 \times 5$

(2)

- b) Using your prime factor tree or otherwise work out the Lowest Common Multiple for 168 and 40

The lowest common multiple is the value which both 40 and 168 will divide into

Look at the prime numbers that we got for 168 and 40 and mark the those that **do not** appear in both. Cross multiply these with the top of the opposite prime factor tree

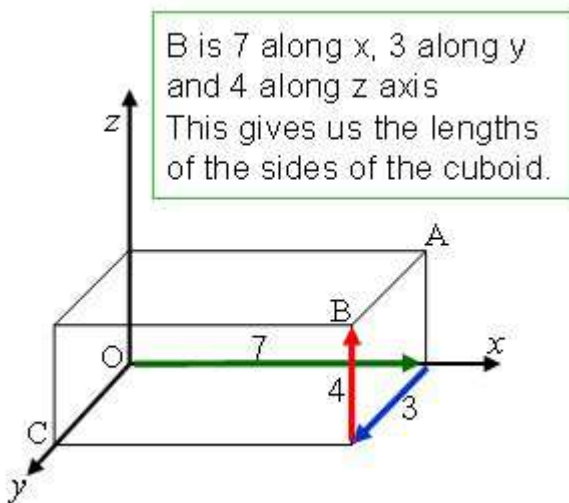
$168 = 2 \times 2 \times 2 \times 3 \times 7$
 $40 = 2 \times 2 \times 2 \times 5$

$21 \times 40 = 840$
 $5 \times 168 = 840$

840

(2)

2. A cuboid lines on the co-ordinate axes.



Distances in cm
Not drawn accurately

When you have co-ordinates of (x, y) they are on a graph. If you add a z co-ordinate it makes it into a 3-D space. Each corner of the cuboid is given by the co-ordinate (x, y, z)

The point B has co-ordinates (7, 3, 4)

- a) What are the co-ordinates of the point A

B is 5 along the x-axis, 3 long the y-axis and 2 along the z-axis

A is the same distance as B along the x-axis and z-axis but has no distance along the y-axis

7, 0, 4

(1)

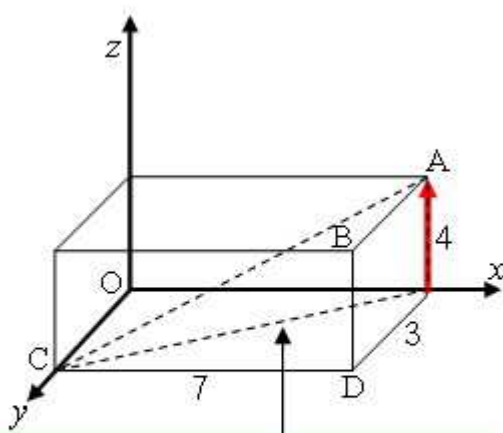
- b) What are the co-ordinates of the point C

A is zero along the x-axis 3 along the and y-axis and zero along the z-axis

0, 3, 0

(1)

- c) What is the distance between A and C correct to 1 decimal place



To work out AC which is diagonally across the cuboid we need to draw a triangle as shown dotted. The base length of this triangle = $\sqrt{58} = 7.616\text{cm}$

Use Pythagoras with
 $AC^2 = \text{base length}^2 + 4^2$
 $= 58 + 16 = 74$

$AC = \sqrt{74} = 8.6\text{cm}$

Work out this diagonal side using Pythagoras.
 $= \sqrt{7^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58}$

8.6

cm
(3)

3. Jane paid £10.56 for five mp3 downloads and two CDs

If six mp3 downloads cost £7.68 how much is it for one CD.

You can work out the cost of 1 mp3 download

If 6 mp3 downloads cost £7.68 then 1 mp3 download costs $£7.68 \div 6 = £1.28$

$$7 \ . \ 6 \ 8 \ \div \ 6 \ =$$

So the cost of 5 mp3 downloads is cost of 1 mp3 download $\times 5 = £6.40$

$$1 \ . \ 2 \ 8 \ \times \ 5 \ =$$

5 mp3 downloads and 2 CDs cost £10.56.

To work out the cost of 2 CDs take the cost of the 5mp3 downloads away from this $£10.56 - £6.40 = £4.16$

$$1 \ 0 \ . \ 5 \ 6 \ - \ 6 \ . \ 4 \ =$$

Finally to work out the cost of 1 CDs divide by 2

$£4.16 \div 2 = £2.08$

$$4 \ . \ 1 \ 6 \ \div \ 2 \ =$$

2.08 ✓

(3)

4. a) Kath gives her three grandchildren, Laura, Matthew and Nathan £1000 in the ratio of 1:2:5.
How much does Nathan get?

Add up the ratios first: $1 + 2 + 5 = 8$

1000 ÷ 8 gives what one ratio is worth **1 0 0 0 ÷ 8 =** £125

Multiply this by Nathan's ration **1 2 5 × 5 =** £625

625

£.....
(2)

- b) Kath invests the rest of her money, in the Bank.
She gets 3.6% compound interest per year on £4000.
How much will she have after 2 years.

Interest for year 1 = $4000 \times 3.6 \div 100 = \text{£}144$

REMEMBER: Compound interest means that we get interest on the interest, so it is different every year

4 0 0 0 × 3 . 6 ÷ 1 0 0 =

4 0 0 0 × 3 . 6 shift % = 144

Add the interest to the £4000: $\text{£}4000 + \text{£}144 = \text{£}4144$

Find the % button on your calculator to make it simpler

Interest for year 2 = $4144 \times 3.6 \div 100 = \text{£}149.18$

4 1 4 4 × 3 . 6 ÷ shift % = 149.18

Total after year 2 = $4144 + 149.18 = \text{£}4293.18$

4293.18

£.....
(3)

Alternatively use

4 0 0 0 × 1 . 0 3 6 x^y 2 =

Initial amount

Percentage change is 3.6%. as a decimal it is 0.036

Use x^y button for power

2 years

5. There are 21 coloured buttons in a bag

6 buttons are blue
10 buttons are green
4 buttons are red.
1 black button

$$\text{Total number buttons is } 6 + 10 + 4 + 1 = 21$$

If you take two buttons at random from the bag *without replacement*

What is the probability that you pick

a) Two red buttons

$$\text{Probability (1}^{\text{st}} \text{ red)} = \frac{\text{number of red buttons}}{\text{total number buttons}} = \frac{4}{21}$$

$$\text{Probability (2}^{\text{nd}} \text{ red)} = \frac{3}{20}$$

REMEMBER: we have already taken one red out so have 3 left out of a total of 20

$$\text{Probability (two reds)} = \frac{4}{21} \times \frac{3}{20} = \frac{1}{35}$$

$$\frac{1}{35}$$

(2)

b) A blue button and a green button

We can select this in two ways: either a blue button followed by a green button OR a green button followed by a blue button. We work out the probability for each event and then add them

$$\text{Probability (blue - green)} = \frac{6}{21} \times \frac{10}{20} = \frac{6}{42} = \frac{1}{7}$$

$$\text{Probability (green - blue)} = \frac{10}{21} \times \frac{6}{20} = \frac{1}{7}$$

$$\text{Probability (both events)} = \frac{1}{7} + \frac{1}{7} = \frac{2}{7}$$

$$\frac{2}{7}$$

(2)

6. The equation

$$x^3 - 7x = 4$$

Trial and improvement means trying out different values for the letters in an equation to see how close you get to the given answer

Has a solution between 2 and 3.

Using trial and improvement find the solution to 1 decimal place.

Show all your working.

For this equation make a table with columns for the x , x^3 , $-7x$ and answer

Use the power button to get x^3

4×3^3 gives $4^3 = 64$

x^y

x	x^3	$-7x$	$=$	
2	8	-14	-6	Too low

2 x^y 3
 - 7 x 2
 =

3	27	-21	6	Too high
---	----	-----	---	----------

3 x^y 3
 - 7 x 3
 =

6 is closer to 4 than -6 so x is nearer 3 than 2. Try $x = 2.7$ next

2.7	19.68	-18.9	0.78	Too low
-----	-------	-------	------	---------

2 . 7 x^y 3
 - 7 x 2 . 7
 =

This is too low so try a higher value for x . Go up by one decimal point $x = 2.8$

2.8	21.95	19.6	2.35	Too low
-----	-------	------	------	---------

2 . 8 x^y 3
 - 7 x 2 . 8
 =

This is too low so try $x = 2.9$

2.9	24.39	20.3	4.09	OK
-----	-------	------	------	----

2 . 9 x^y 3
 - 7 x 2 . 9
 =

$x =$ 2.9 (4)

7. a) Factorise $2y^2 + 4y$

Find a factor that goes into $2y^2$ and $4y$
 y is in both y^2 and y so it is a factor
 2 goes into 2 and 4 so it is a factor

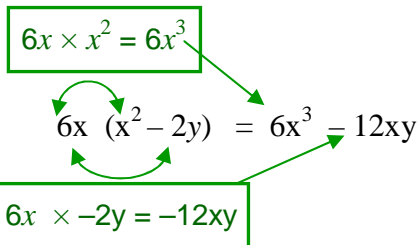
Rewrite $2y^2 + 4y$ as $2 \times y \times y + 2 \times 2 \times y$
 Put factor $2y$ outside bracket: $2 \times y (y + 2)$
 or $2y(y + 2)$

$2y(y + 2)$

(2)

b) Expand $6x(x^2 - 2y)$

Expand means multiplying everything inside the brackets by what's outside.
 So the bracket replaces a multiplication sign.



$6x^3 - 12xy$

(1)

c) Factorise $y^2 - 4y - 21$

1. Start with: $(x \quad A)(x \quad B)$

2. To find A and B look at the quadratic

3. A and B are two numbers which

$y^2 - 4y - 21 = 0$

make -4 by adding or subtracting

and multiply to make -21

The two values are 3 and -7
 Because:
 $-21 = +3 \times -7$
 $-4 = +3 - 7$

Rewriting the quadratic using 3 and -7 we get:
 $y^2 - 4y - 21 = (y + 3)(y - 7) = 0$

To solve it either $(y + 3) = 0$
 or $(y - 7) = 0$
 so $y = -3$ or $y = 7$

$y = 7$ or $y = -3$

(2)

d) $c = 1.2(a - b^2)$

Find the value of c when $a = 3$ and $b = 4$

using $a = 3$ and $b = 4$	$c = 1.2 \times (a - b \times b)$
	$c = 1.2 \times (3 - 4 \times 4)$

1 . 2 x (3 - 4 x 4) =

$c = \dots\dots\dots$ **- 15.6** ✓
(2)

e) Factorise fully $4y^2 - 4$

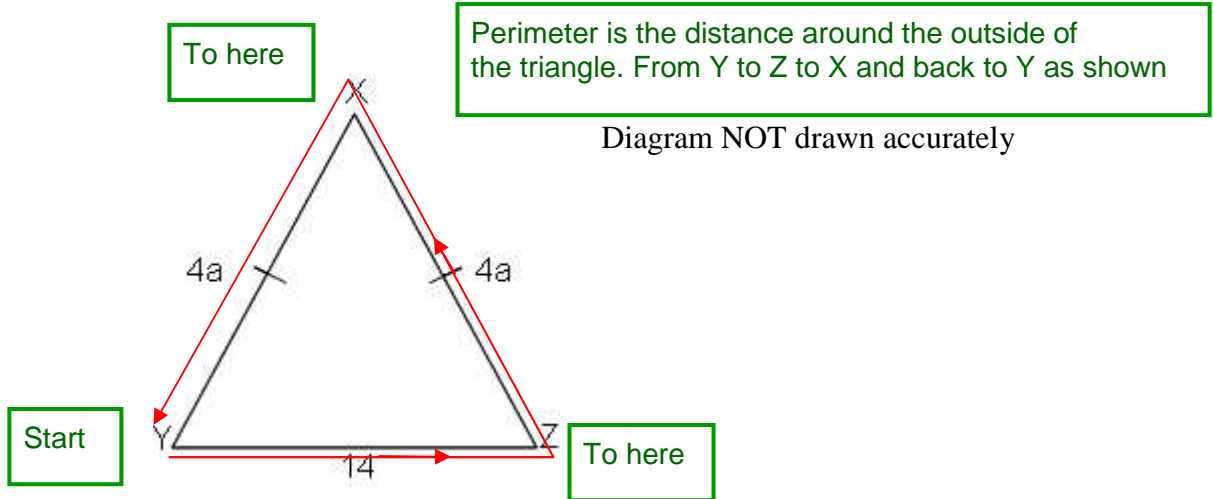
This is a difference of two squares written as $(a + b)(a - b) = a^2 - b^2$
 Common ones:
 $x^2 - 4 = (x - 2)(x + 2)$
 $x^2 - 9 = (x - 3)(x + 3)$
 The + and - signs make sure that the x terms disappear.

Rewrite $4y^2 - 4 = (2y - 2)(2y + 2)$

$(2y - 2)(2y + 2)$ ✓

(2)

8.



In the diagram, above shows an isosceles triangle XYZ with measurements in centimetres.
 $XY = 4a$
 $XZ = 4a$
 $YZ = 14$

a) Find an expression in terms of a , for the **Perimeter** of the triangle in its simplest form

Perimeter = $YZ + ZX + XY$

→

$YZ + ZX + XY = 4a + 4a + 14 = 8a + 14$

Simplify $4a + 4a + 14 = 8a + 14$

$8a + 14$

.....(2)

b) If the perimeter of the triangle is a^2 , calculate a , correct to 1 decimal point.

Perimeter = a^2 so $8a + 14 = a^2$
 Rearranging gives $a^2 - 8a - 14 = 0$
 We have a quadratic equation.
 Solve using the quadratic formula

CLUE: An answer to 1 decimal place shows that we will not be able to factorise to solve the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, b = -8, c = -14$

$$x = \frac{+8 \pm \sqrt{64 - 4 \times 1 \times -14}}{2} = \frac{+8 \pm \sqrt{64 + 56}}{2} = \frac{+8 \pm \sqrt{120}}{2} = \frac{+8 \pm 10.95}{2}$$

$x = \frac{18.95}{2} \text{ or } \frac{-2.95}{2}$

We cannot have a negative perimeter

$+8 \text{ because } -b = -(-8)$

9.5

$a =$ (3)

9. Laura paid £78 for an mp3 player in a sale with 35% off the original price. How much was the mp3 player originally?

Sale
35% off original price

It's **NOT** 35% of £78 !!!! because £78 was NOT the original price

The original price is reduced by 35% so, £78 is 65% or 0.65 of the original price.

If we say the original price is O and the sale price is S we can make an equation:

$$O \times 0.65 = S$$

$$O = S \div 0.65$$

and $O = 78 \div 0.65 = 120$

Sale price 65% = 0.65

$$\overbrace{78} \div \overbrace{.65} =$$

£ 120 (3)

10. The first four terms in a sequence are:

a)

-3 1 5 9

Write an expression for the nth term of the sequence

Find the DIFFERENCE: between each number the gap is always 4

This is really like the Four Times Table:

But the sequence starts at -3 not 4

How can you change the 4 times table to make it like our sequence?
Subtract 7 ($4 - 7 = -3$)

We write the 4 times table as $4 \times n$ where n can be 1 or 2 or 3 etc
 We can also write this as $4n$

Subtracting 7 gives $4n - 7$

$4n - 7$

(1)

b)

3 6 11 18

Write an expression for the nth term of the sequence

Find the DIFFERENCE: between each number the gap increases

Find the 2nd DIFFERENCE:
it is always 2

This means we have an n^2 type sequence like:

1 4 9 16

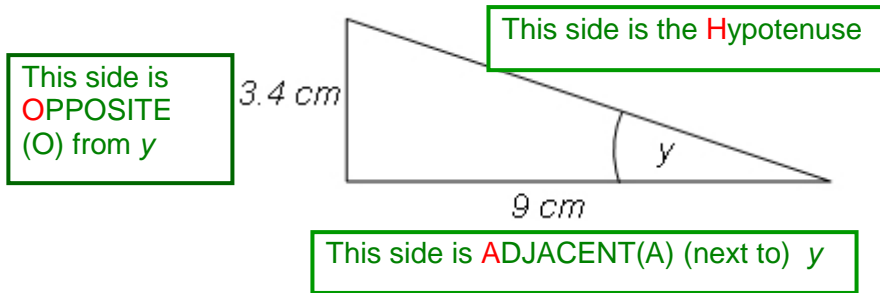
How can you change the n^2 sequence to make it like our sequence?
Add 2 ($1 + 2 = 3$)

We write this as $n^2 + 2$

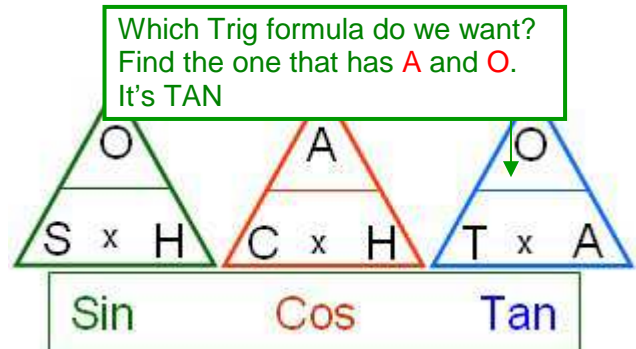
$n^2 + 2$

(2)

11.



What is the size of angle y in this triangle?
Give your answer to 3 significant figures.



Cover up T because we want to find this and the formula is $T = \frac{O}{A}$

REMEMBER:
Use Tan^{-1} on your calculator

$$\text{Tan}^{-1} y = \frac{3.4}{9} = 20.69$$

3 significant figures
round up to 20.7

3 . 4 ÷ 9 = shift Tan⁻¹ =

$y = \dots \boxed{20.7} \dots^\circ$

(3)

12. a) Simplify 2^0

Remember: any number to the power 0 is 1

1

(1)

b) Simplify 2^{-3}

minus (-) means reciprocal or 1 over the number

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$\frac{1}{8}$

(1)

c) Simplify $(27)^{\frac{2}{3}}$

$^{1/3}$ means cubed root $^3\sqrt{\quad}$ $^{2/3}$ means cubed root squared $(^3\sqrt{\quad})^2$

$$27^{2/3} = (^3\sqrt{27})^2 = (3)^2 = 9$$

9

(2)

When powers are multiplied ADD the powers

d) Simplify $2^{\frac{1}{2}} \times 2^{\frac{3}{2}}$

$$2^{\frac{4}{2}} = 2^2 = 4$$

4

(1)

13. Solve the simultaneous equations

$$\begin{aligned} 6x - 4y &= 21 \\ 12x + 12y &= 12 \end{aligned}$$

Often we can just add or subtract equations to eliminate **x** or **y**. But this time it's harder- first we have to multiply one equation so x or y is the same as the other equation

x 3 → $6x - 4y = 21$

It becomes → $18x - 12y = 63$

Multiply by 3 so 4y becomes 12y then we can add the equations to eliminate it

Now add the two equations to eliminate y

$$\begin{array}{r} 18x - 12y = 63 \\ + 12x + 12y = 12 \\ \hline 30x = 75 \end{array}$$

So $x = 75 \div 30 = 2 \frac{1}{2}$

SUBSTITUTE $x = 2 \frac{1}{2}$ into one of the equations

$$\begin{aligned} 6x - 4y &= 21 \\ 6 \times 2 \frac{1}{2} - 4y &= 21 \\ 15 - 4y &= 21 \\ (-15) - 4y &= 6 \\ y &= -1 \frac{1}{2} \end{aligned}$$

2 1/2 ✓

x.....

·

-1 1/2 ✓

y.....

·

(2)

14. Express $\frac{5}{3 + \sqrt{8}}$ in the form $a + b\sqrt{2}$

This is a SURDs question.
We need to make the denominator rational.
To do this multiply both top and bottom by $2 + \sqrt{3}$. (sign change from original)

$$\frac{5}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} = \frac{15 - 5\sqrt{8}}{9 + 3\sqrt{8} - 3\sqrt{8} - \sqrt{8}\sqrt{8}} = \frac{15 - 5\sqrt{8}}{9 - 8} = 15 - 5\sqrt{8} = 15 - 10\sqrt{2}$$

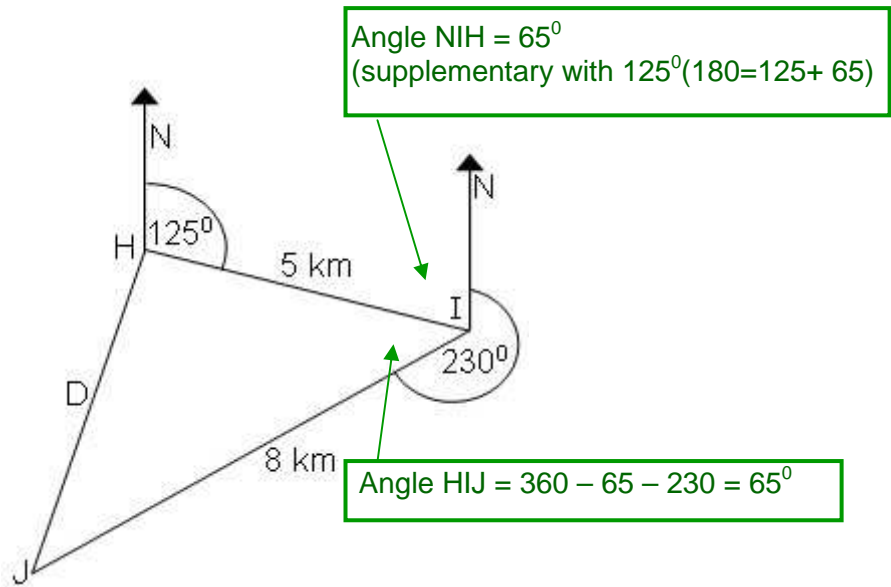
Notice that these parts are opposite so cancel out.

$5\sqrt{8} = 5\sqrt{4 \times 2} = 5 \times 2\sqrt{2}$

15 - 10√2 ✓

(3)

15. A man starts at home H and runs for 5 km on a bearing of 125° to I
 He then runs for 8 km on a bearing of 230° for 8km to J
 He then runs home in a total time of 1 hour and nine minutes.



- a) Work out how far away (D) from home he is, correct to one decimal place

In triangle HIJ we have 2 sides and an angle and need to find side D – in other words, 3 sides and an angle so use the **Cosine Rule**:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Assume side D is 'a' and so angle A = 65° b = 8 and c = 5

Substituting:

$$a^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 65$$

$$a^2 = 64 + 25 - 80 \times \cos 65$$

$$a^2 = 89 - 33.809$$

$$a = \sqrt{55.19} = 7.4$$

✓ (8 x² + 5 x² - 2 x 8 x 5 x Cos 65) =

7.4 ✓
0
 (3)

- b) Calculate his average speed in km per hour to one decimal place.

Speed = total distance ÷ time

Time taken = 1 hour 9 minutes.

To convert minutes to hours divide by 60 $9 \div 60 = 0.15$ hours

$$= (5 + 8 + 7.42) \text{ km} \div 1.15 \text{ hours}$$

$$= 20.42 \div 1.15 \text{ hours}$$

$$= 17.756 \text{ or } 17.8 \text{ km/hr}$$



17.8 ✓
km/hr
 (2)

(5 + 8 + 7 . 4 2) ÷ 1 . 1 5 =

- c) Calculate the area of the triangle HIJ
Give your answer correct to three significant figures.

We can't use the normal formula for the area of a Δ since we don't have a right angled Δ
Use: **area of $\Delta = \frac{1}{2} a b \sin C$**

The two sides a and b are either side of the angle.
 $C = 65$, $a = 5$ and $b = 8$

Substitute into formula: Area = $\frac{1}{2} \times 5 \times 8 \times \sin 65$
= 20×0.4226
= 8.452

0 . 5 x 5 x 8 x Sin 6 5 =

8.452 to 3 significant figures is 8.45

8.45 ✓

.....km²
(2)

16. Cyril walks 50 metres in a time of 35.6 seconds

The distance of 50 metres was measured to the nearest metre.
The time of 35.6 seconds was measured to the nearest tenth of a second.

- a) What is the upper bound for the distance of 50 metres

Upper and Lower Bounds are related to accuracy. Once you know the degree of accuracy to which a measurement has been rounded, you can then find the Upper and Lower bounds.

The Upper Bound is the biggest possible value the measurement can have before it is rounded down.

The Lower Bound is the smallest possible value the measurement can have before it was rounded up

Note: Measured value = V
Take the level of accuracy A
Divide by two: A/2
The Bounds = $\pm A/2$
Lower Bound = $V + A/2$
Upper Bound = $V - A/2$

We want 50m to the nearest metre.
So Accuracy = 1m and Bounds will be ± 0.5 m

Upper bound = $50 + 0.5 = 50.5$ m.

50.5 ✓

.....m
(1)

- b) What is the lower bound for the time of 35.6 seconds

We want 35.6 seconds to the nearest second
So Accuracy = $1/10$ s = 0.1 and Bounds will be ± 0.05 s

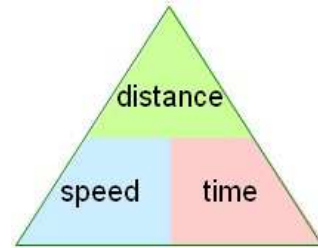
Lower bound = $35.6 - 0.05 = 35.55$ s

35.55 ✓

.....seconds
(1)

- c) Calculate the upper bound for Cyril's average speed
Show all the figures on your calculator display

Speed = Distance \div Time
The maximum Speed is when we have
an **upper bound for Distance** and
a **lower bound for Time**



Upper bound for distance = 50.5 m.
Lower bound for time = 35.55s.

$$\text{Speed} = \frac{\text{Upper bound distance}}{\text{Lower bound time}} = \frac{50.5}{35.55}$$

$$= 1.4205344$$

5 0 . 5 \div 3 5 . 5 5 =

1.4205344 ✓

.....metres per second
(2)

- d) Calculate the lower bound for Cyril's average speed
Show all the figures on your calculator display

Speed = Distance \div Time
The minimum Speed is when we have
an **lower bound for Distance** and
a **upper bound for Time**

Lower bound for distance = 49.5 m.
Upper bound for time = 35.65s.

$$\text{Speed} = \frac{\text{lower bound distance}}{\text{upper bound time}} = \frac{49.5}{35.65}$$

$$= 1.388499$$

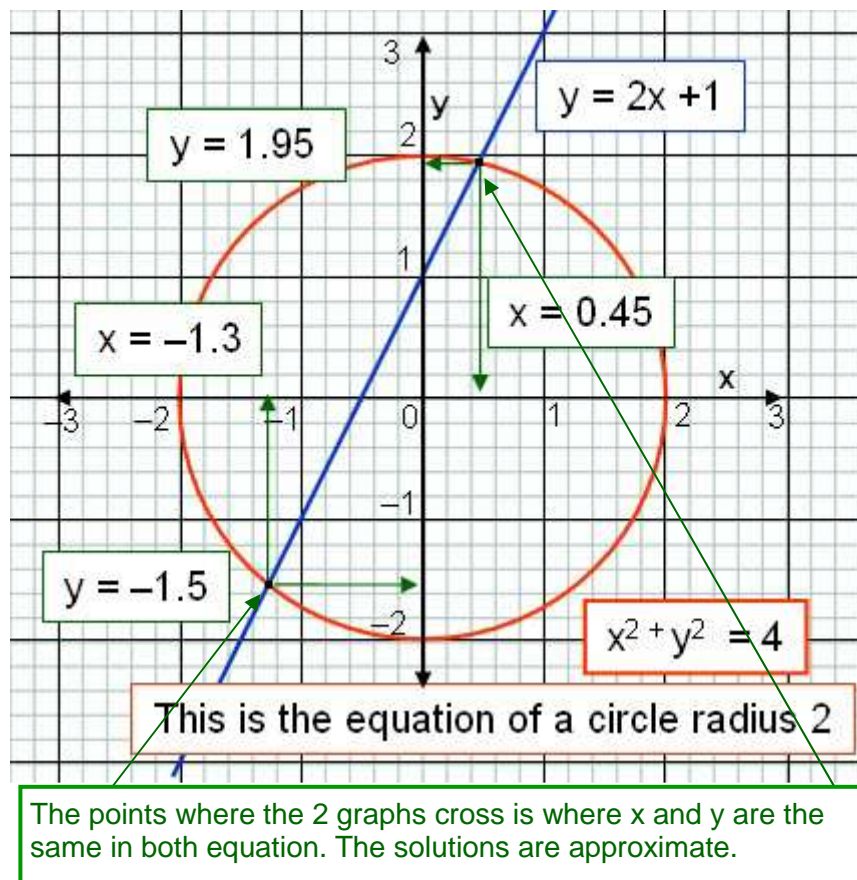
4 9 . 5 \div 3 5 . 6 5 =

1.388499 ✓

.....metres per second
(2)

17. Draw the graphs for these simultaneous equations and use them to find the solutions

$$\begin{aligned} x^2 + y^2 &= 4 \\ y &= 2x + 1 \end{aligned}$$



x... **-1.3** ✓ and ... **0.45** ✓ ...
y.... **-1.5** ✓ and **1.95** ✓
(3)

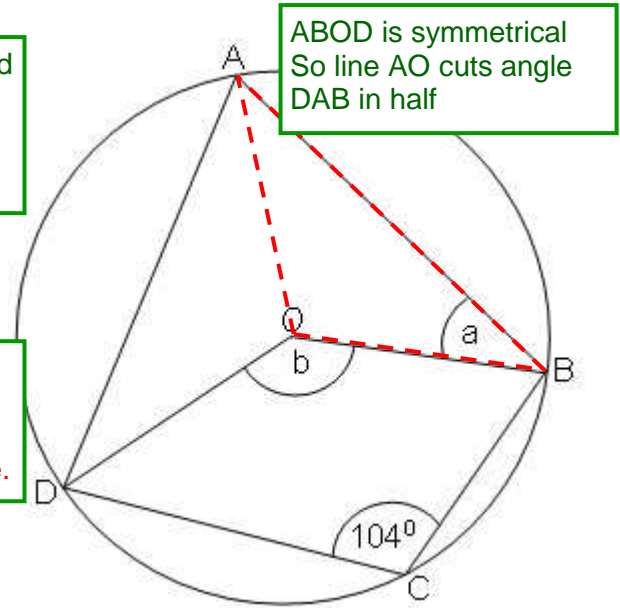
18. In the diagram O is the centre of the circle and $DCB = 104^\circ$
The shape ABOD is symmetrical

ABCD is a **cyclic quadrilateral** – a four sided shape where each corner touches the circumference of a circle
Opposite angles add up to 180°

$$DAB = 180 - 104 = 76^\circ$$

DOB is an angle at the centre of the circle
DAB is an angle at the circumference.
They both start at the same sector.
Angle at centre = 2 x angle at the circumference.

$$\begin{aligned} \text{DOB} &= 2 \times \text{DAB} \\ \text{DOB (b)} &= 2 \times 76 = 152^\circ \end{aligned}$$



- a) Work out the value of angle b

152 ✓
.....0
(2)

- b) Explain how you got your answer

DAB = 76° - cyclic quadrilateral, sum opposite angles = 180
B = 2 x DAB – angle at centre = 2x angle at circumference

(1)

- c) Work out the value of angle a

Since ABOD is symmetrical we can draw a line through the shape

Reflex angle $\text{DOB} = 360 - b = 360 - 152 = 204$

The line AO cuts reflex angle DOB in half $\text{AOB} = 102$

The line AO also cuts angle DAB in half $\text{OAB} = 38$

$$\text{ABO(a)} = 180 - 102 - 38 = 40$$

40 ✓
.....0
(2)

19. a) Solve the equation

$$\frac{x}{3} + \frac{2x}{4} = 5$$

multiply everything by 12 (3 x 4) will get rid of all bottom terms (denominators)

$$12 \times \frac{x}{3} + 12 \times \frac{2x}{4} = 12 \times 5$$

So

$$4x + 6x = 60$$
$$10x = 60$$

x = 6 ✓

(2)

b) Solve the equation

$$\frac{2x}{x-3} + \frac{4}{x+1} = 1$$

Get rid of everything at the bottom by multiplying through by these terms

multiply both sides by $(x-3)(x+1)$

$$\frac{\cancel{(x-3)}(x+1) \times 2x}{\cancel{x-3}} + \frac{\cancel{(x-3)}(x+1) \times 4}{\cancel{x+1}} = (x-3)(x+1) \times 1$$

The terms at the bottom cancel out

So $(x+1) \times 2x + (x-3) \times 4 = (x-3)(x+1)$
 or $2x^2 + 2x + 4x - 12 = (x-3)(x+1)$
 or $2x^2 + 6x - 12 = (x-3)(x+1)$

Expand the right hand side

$x \times x = x^2$

$-3 \times 1 = -3$

Double Brackets mean FOUR multiplications

$(x-3)(x+1)$

$x \times 1 = +x$

$-3 \times x = -3x$

Simplify – collect like terms together

$+x - 3x + x^2 - 3 = x^2 - 2x - 3$

Rewriting the equation : $2x^2 + 6x - 12 = x^2 - 2x - 3$

Rearrange the equation

	$2x^2 + 6x - 12$	$=$	$x^2 - 2x - 3$
($-x^2$)	$x^2 + 6x - 12$	$=$	$-2x - 3$
($+2x$)	$x^2 + 8x - 12$	$=$	-3
($+3$)	$x^2 + 8x - 9$	$=$	0

Solve this Quadratics equation by factorising.

1. Start with:
 $(x \quad A)(x \quad B)$

2. To find A and B look at the quadratic

3. A and B are two numbers which

make +8 by adding or subtracting

and multiply to make -9

$x^2 + 8x - 9 = 0$

$-9 = 9 \times -1$

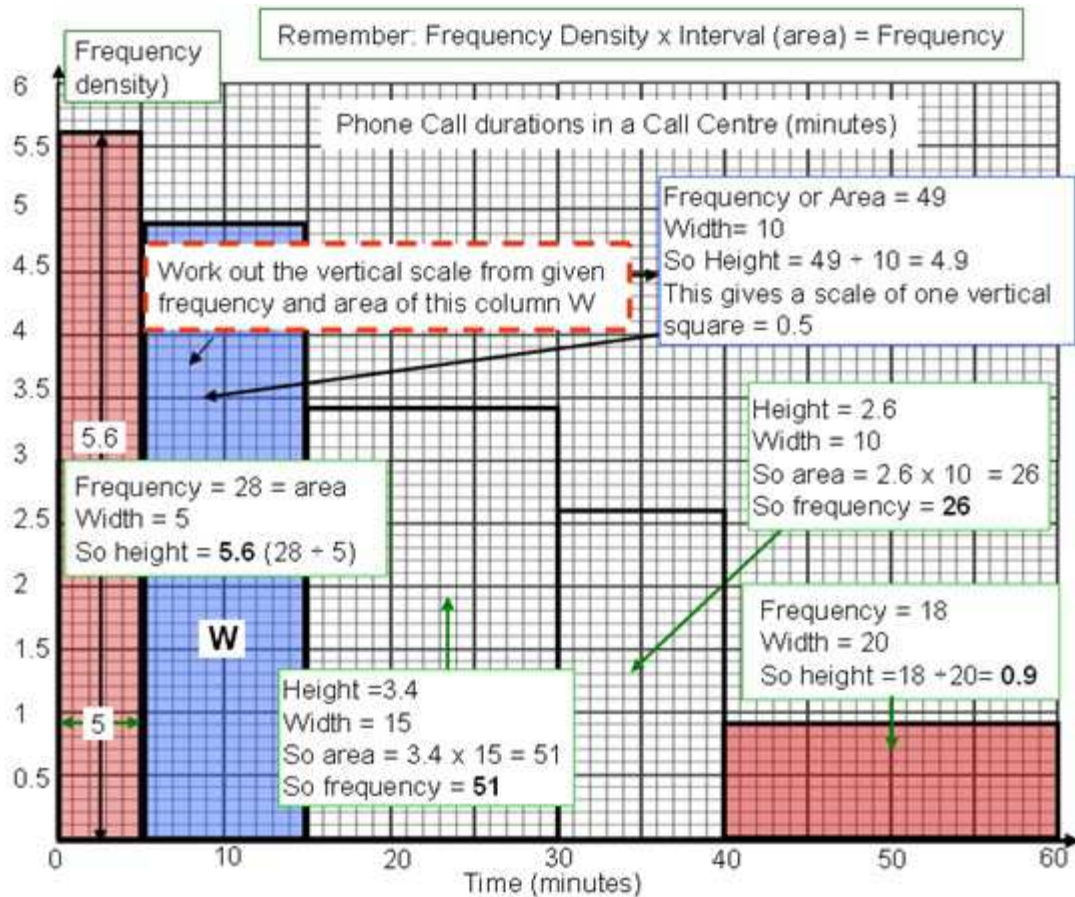
$+8 = 9 - 1$

$x^2 + 8x - 9 = (x+9)(x-1) = 0$

$x = -9 \text{ or } x = 1$

If $(x+9)(x-1) = 0$ so $x = -9$ or $x = 1$

20. The table and histogram show information about the time of phone calls in a Call centre.



- a) Use the histogram to complete the table

(2)

Time (t minutes)	Frequency
$0 < t \leq 5$	28
$5 < t \leq 15$	49
$15 < t \leq 30$	51 ✓
$30 < t \leq 40$	26 ✓
$45 < t \leq 60$	18

- b) Use the table to complete the histogram

(2)

21 a) Factorise $4x^2 - 23x + 15$

$$4 \times 15 = 60$$

1. Find two numbers which multiply to make 60

and also makes the *middle x value* - 23 by adding or subtracting

$$60 = -20 \times -3$$

$$-23 = -20 + -3$$

$$4x^2 - 23x + 15$$

2. Rewrite $-23x$ as $(-20x - 3x)$ in the equation

$$4x^2 - 23x + 15 = 4x^2 - 20x - 3x + 15$$

Notice:
we put $-3x$ next to the 15
and $-20x$ next to the $4x^2$
We use it for factorisation in the next step

3. Factorise each pair of terms:

$$4x^2 - 20x - 3x + 15$$

$$\rightarrow 4x(x - 5) - 3(x - 5)$$

$\swarrow \quad \searrow \quad - \times - = +$

4. Simplify: we have $(x - 5)$ in both terms so we can take it out as a factor

$$4x(x - 5) - 3(x - 5) = (4x - 3)(x - 5)$$

A test that we are right is that both factors are the same

$$(4x - 3)(x - 5)$$

(3)

b) Hence or otherwise solve $4x^2 - 23x + 15 = 0$

$$4x^2 - 23x + 15 = (4x - 3)(x - 5) = 0$$

So rearrange

$$4x - 3 = 0 \quad \text{and} \quad x - 5 = 0$$

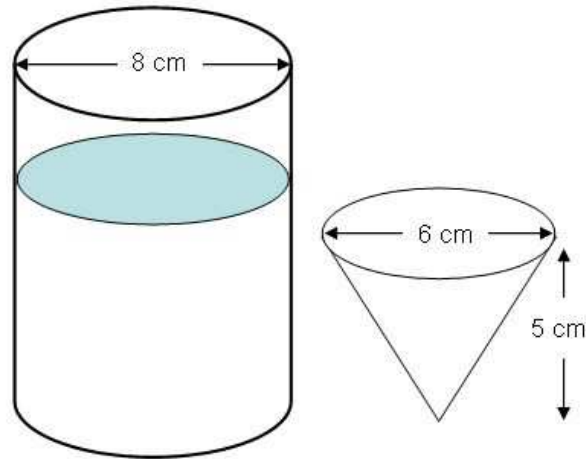
$$4x = 3 \quad \text{and} \quad x = 5$$

$$x = \frac{3}{4} \quad \text{and} \quad x = 5$$

$$x = \dots \boxed{5} \text{ or } x = \dots \boxed{\frac{3}{4}} \dots$$

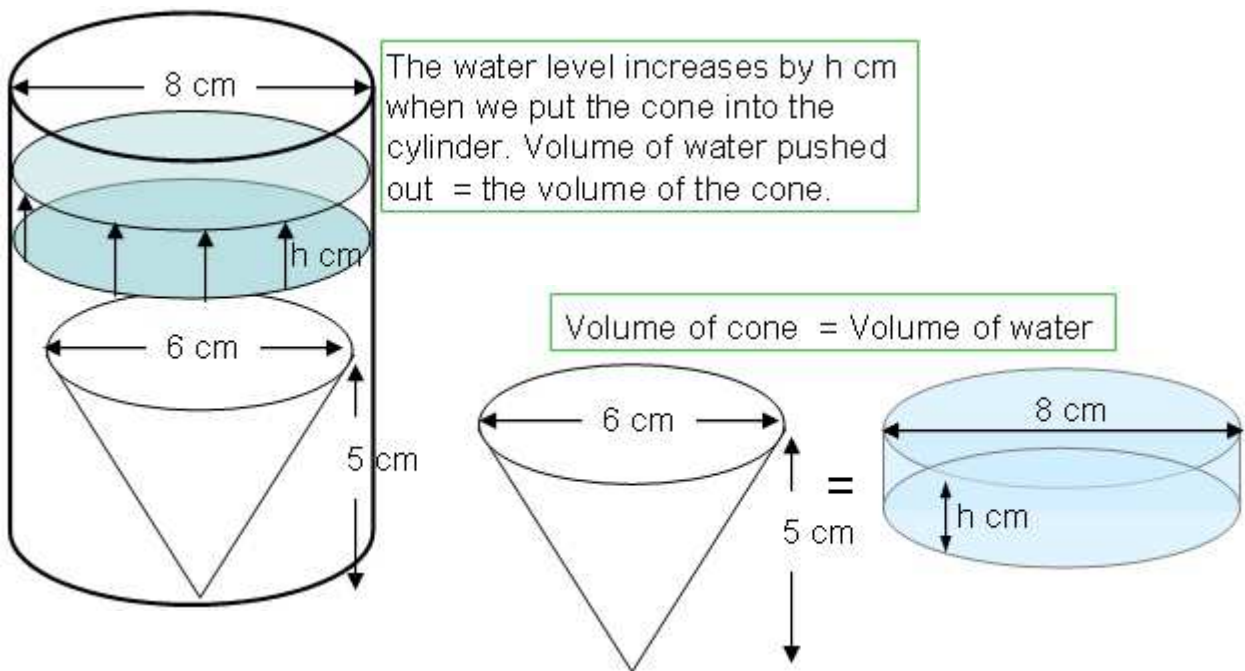
(1)

22.



A cone with base diameter of 6cm and height 5cm is dropped into a cylinder of diameter 8 cm part filled with water.

- a) Calculate the increase in the height of the water in the cylinder.
Leave your answer as a fraction in its simplest form.



<p>Volume water pushed out (cylinder) =</p> $\pi r^2 h$ $\pi \times 4^2 \times h$ $\pi \times 16 \times h$ <p>Rearrange h</p>	<p>Volume cone</p> $= \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi \times 3^2 \times 5$ $= \pi \times 3 \times 5$ $\frac{15 \pi}{16 \pi} = \frac{15 \text{ cm}}{16}$	<p>Radius cylinder = 4cm Radius cone = 3 cm</p>	$\frac{15}{16}$	<p>cm (3)</p>
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The π 's cancel top and bottom

- b) The density of water is 1 gram per cm^3 .
 Calculate the mass of the water displaced by the cone.
 Leave your answer in terms of π

The units given for density give you a clue for it's formula :

Density of water is in $\frac{\text{grams}}{\text{cm}^3}$ or $\frac{\text{mass}}{\text{volume}}$

Rearrange mass = density \times volume

$1 \times 15\pi \text{ gms}$

15π ✓

.....gm
(2)

23. David recorded how many maths questions students downloaded from his website.
 He recorded downloads for each quarter from January 2007 to June 2008.

Year	Months	Number of downloads
2007	Jan – Mar	172
	Apr – Jun	446
	Jul – Sept	58
	Oct – Dec	124
2008	Jan – Mar	195
	Apr – Jun	479

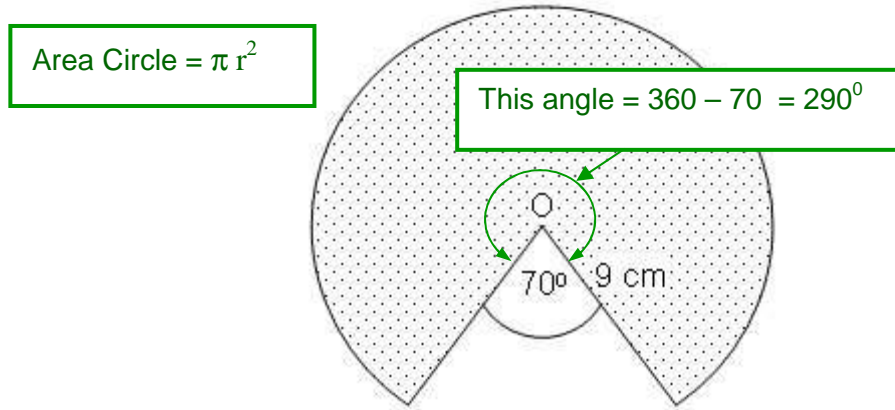
- a) What was the means number of downloads in 2007?
-
(1)

- b) Calculate the three-point moving averages for this information.
-
(2)

c) Can you explain why the downloads changed during the year

(1)

24.



A circular pizza with a radius of 9 cm has a sector of angle 70° removed. Calculate the area of the remaining sector to 3 significant figures.

$$\begin{aligned} \text{Area of the full pizza} &= \pi r^2 = \pi \times 81 \\ \text{Fraction of pizza shown} &= \frac{290}{360} \text{ because } 360^\circ \text{ is 1 full rotation} \\ \text{Area of pizza shown} &= \frac{290 \times \pi \times 81}{360} \end{aligned}$$

$$2 \quad 9 \quad 0 \quad \times \quad \pi \quad \times \quad 8 \quad 1 \quad \div \quad 3 \quad . \quad 6 \quad 0 \quad =$$

205 ✓
.....cm²
(2)

TOTAL FOR PAPER: 100 MARKS
END