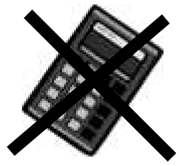
MathsMadeEasy

GCSE Mathematics



Non-Calculator
Higher Tier
Free Practice Set 1

1 hour 45 minutes

ANSWERS

Grade Boundaries

A*	A	B	C	D	E
88	71	57	43	22	13

Authors Note

Every possible effort has been made to ensure that everything in this paper is accurate and the author cannot accept responsibility for any errors.

1. a) Estímate 14.1×99

Estimate 99 as 100 and 14.1 as 14.

Then we have 14 x 100 = 1400 (add two zeros to the end when x 100)

1400 .... **(1)**

b) Estimate : $\dfrac{14.9 \times 40.1}{9.7 \times 3.1}$

If we ROUND each number to the nearest whole number we get:
14.9 × 40.1 is 15 × 40 and 9.7 × 3.1 is 10 × 3

So we have $\dfrac{15^5 \times 40}{10 \times 3^1}$ which cancels nicely to $\dfrac{5 \times 4}{1 \times 1} = 20$

20 .... **(2)**

2. The formula $v = u + at$ gives the final velocity of an object as it accelerates.

a Find the value of v when:

i $u = 20$, $a = 5$ and $t = 9$

Replace the letters with the values given:
So $v = u + at$ becomes $v = 20 + 5 \times 9 = 20 + 45 = 65$

65 .... **(2)**

ii if $v = 35$ **,** $a = 4$ and $t = 5$ find u

Again replace the letters with the values given and solve the equation
So $v = u + at$ becomes $35 = u + 4 \times 5$ or $35 = u + 20$

Solve $35 = u + 20$ by taking 20 from both sides
$35 - 20 = u + 20 - 20$
$15 = u$

15 .... **(2)**

3. David, Jane and Matthew shared out £1000 between them in the ratio 4 : 6 : 10

How much did they each get.

Add the ratios together: 4 + 6 + 10 = 20
So we are splitting the money into 20 parts.

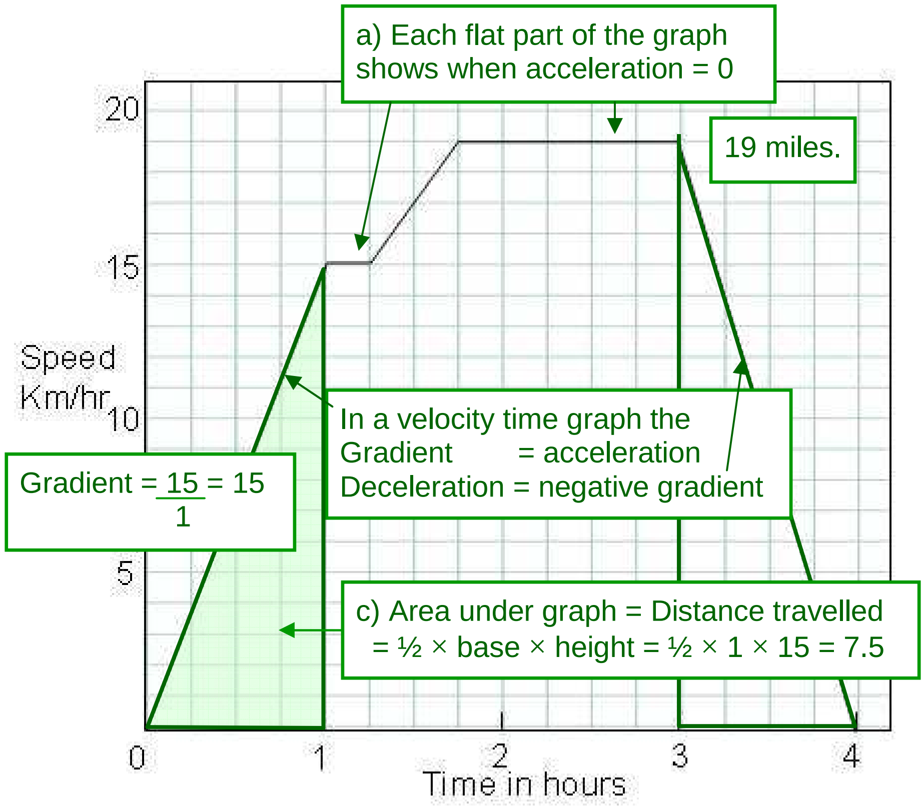
Divide 1000 by 20 to find the value of one part 1000 ÷ 20 = 50

David gets 4/20ths or 4 parts	= 4 × 50 = 200;
Jane gets 6/20ths or 6 parts	= 6 × 50 =300;
Matthew gets 10/20ths or ten parts or half	= 500

£ 200 ✓ and £ 300 ✓ and £ 500 ✓

(2)

4. The graph below is a **velocity-time** graph.



a) How many times was the acceleration zero?

There are two flat sections when the acceleration was zero.

........2........ **(1)**

b) What is the acceleration during the first part of the graph?

Acceleration is the gradient = $\frac{\text{change in Y}}{\text{change in X}}$ = $\frac{\text{15 km per hour}}{\text{hour}}$

........15........ kmph^2 **(1)**

c) What was the distance travelled in the first hour

........7.5........ km **(1)**

d) What was the deceleration shown by the graph?

........19........ kmph^2 **(1)**

Deceleration is the gradient = $\frac{\text{change in Y}}{\text{change in X}}$ = $\frac{\text{–19 km per hour}}{\text{hour}}$

5. What is.

a) $3\frac{1}{2} + 2\frac{3}{5}$ Write your answer as a mixed number.

Add the 3 and the 2 = 5 and put aside for now

Use this simple trick to add the fractions:

$\frac{1}{2} + \frac{3}{5}$ multiply as shown by the arrows. Red arrow gives base number

We get $\frac{1}{2} + \frac{3}{5} = \frac{5}{10} + \frac{6}{10} = \frac{11}{10}$

This is $1\frac{1}{10}$. Add back the 5 to give $6\frac{1}{10}$

What we have done is convert both fractions to the same denominator

$6\frac{1}{10}$ ✓

(2)

b) $3\frac{1}{2} \times 2\frac{3}{5}$ Write your answer as a mixed number.

Convert $3\frac{1}{2}$ to a fraction = $\frac{7}{2}$ and $2\frac{3}{5} = \frac{13}{5}$

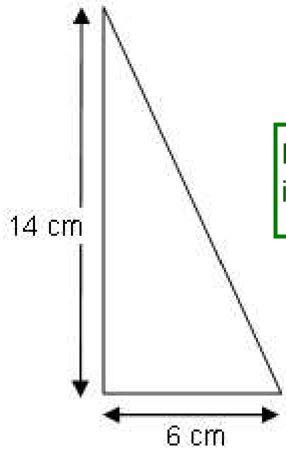
Now just multiply out top and bottom

$\frac{7}{2} \times \frac{13}{5} = \frac{91}{10} = 9\frac{1}{10}$

$9\frac{1}{10}$ ✓

(2)

6. **a)** Calculate the area of this triangle.



Remember area of triangle is: $\frac{1}{2} \times$ base $\times$ height

area of triangle is: $\frac{1}{2} \times 14 \times 6 = 6 \times 7 = 42$ cm^2

42 ....cm^2 **(2)**

b) i Calculate the area of a circle with *diameter* 20 cm.

Area of circle is πr^2 where r is the radius
Diameter is twice the radius, so radius =10 cm
Area = $\pi \times r^2 = 3.142 \times 10 \times 10 = 3.142 \times 100 = 314.2$

REMEMBER – when we multiply by 100 we move the decimal point two places to the right.

314.2 ....cm^2 **(2)**

ii This circle is the cross-section of a cylinder of height 10 cm.
Calculate the volume of the cylinder.

Volume of cylinder = cross sectional area $\times$ height
= 314.2 $\times$ 10 = 3142

REMEMBER – when we multiply by 10 we move the decimal point one place to the right.

3142 ....cm^3 **(2)**

7. Seven students had a Maths and English test. Here are the scores out of 10.

Student	Maths	English
David	7	7
Jane	8	5
Laura	8	6
Stuart	6	8
Matthew	5	6
Pete	6	7
Vicky	8	3

a) What is the Range of the English scores

The range is the difference from the largest to the smallest.
Largest – smallest = 8 – 3 = 5

…………**5**………… **(1)**

b) What is the Median score for Maths

The median is the middle value when they are in order of size.
5 6 6 7 8 8 8
The middle is the 4^{th} value = 7

…………**7**………… **(1)**

c) What is the mean score for English?

The mean is all the values added and divided by the number of values

$$= \frac{7 + 5 + 6 + 8 + 6 + 7 + 3}{7} = \frac{42}{7} = 6$$

…………**6**………… **(2)**

8. **a)** Simplify:

$6a \times 2a$

Multiplying $6 \times 2 = 12$; Multiply $a \times a$ we write as a^2. So we have $12a^2$

$12a^2$ ✓

(1)

b) Solve
$4t - 6 = 14$

Get rid of the 6 on the left side by adding 6 to both sides

$4t - 6 + 6 = 14 + 6$ so $4t = 20$

Divide both sides by 4 so we only have t on the left. $t = 20 \div 4 = 5$

t = **5** ✓

(1)

c) Expand and simplify:

$5(x + y) + 3(4x - 2y)$

Expand means multiply out the brackets:

$5(x + y) = 5x + 5y$ and $3(4x - 2y) = 12x - 6y$

Simplify by putting same types together

$5x + 5y + 12x - 6y = 17x - y$

$17x - y$ ✓

(2)

d) Solve
$7(x + 2) = 5x + 21$

Expand the left side first: $7x + 14 = 5x + 21$

(–14 from both sides) $7x = 5x + 7$

(– 5x from both sides) $2x = 7$

(÷ 2) $x = 7 \div 2 = 3\frac{1}{2}$

x = **3 ½** ✓

(2)

e) Factorise $y^2 - 49$

This is the difference of two squares. Make sure you learn how they work:

$y^2 - 4 = (y - 2)(y + 2)$
$y^2 - 9 = (y - 3)(y + 3)$
$y^2 - 16 = (y - 4)(y + 4)$

$(y - 7)(y + 7)$ ✓

(1)

f) Simplify $\dfrac{2x^2 + 7x - 4}{x^2 + 5x + 4}$

Factorise both top and bottom then see if any bracketed terms cancel

Let's do the bottom first: $x^2 + 5x + 4$

1. Start with: $(x \quad A)(x \quad B)$

2. To find A and B look at the quadratic

3. A and B are two numbers which

make + 5 by adding or subtracting

and multiply to make 4

$x^2 + 5x + 4 = 0$

4 = 4 x 1 and
5 = 4 + 1

$\therefore x^2 + 5x + 4 = (x + 4)(x + 1)$

Now do the top: $2x^2 + 7x - 4$.
This is more complicated because the x^2 term is greater than 1.
It can be done by trial and error or by the method below:

Replace the $+7x$ term with two terms which make $+7x$

$2x^2 + 7x - 4$

We need two numbers which

make + 7 by adding or subtracting

and multiply to make –8 which we get from 2 × – 4

– 8 = 8 × – 1 and 7 = 8 – 1

We *rewrite the equation* replacing + 7 with + 8 and – 1
So $2x^2 + 7x - 4$

$2x^2 - 1x + 8x - 4$

Now *factorise* the two pairs of terms
$2x^2 - 1x + 8x - 4$.

$x(2x - 1)$ $+4(2x - 1)$

Notice that these both contain (2x – 1)
Which means we can take this out as a factor.

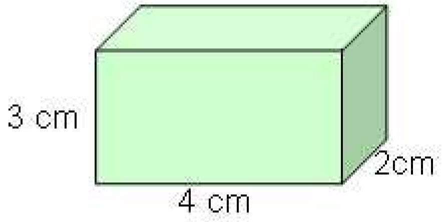
$\therefore x(2x - 1) + 4(2x - 1) = (2x - 1)(x + 4)$

$\therefore \dfrac{2x^2 + 7x - 4}{x^2 + 5x + 4x} = \dfrac{(2x - 1)(x + 4)}{(x + 4)(x + 1)}$ then cancel $x + 4$ terms to get $\dfrac{2x - 1}{x + 1}$

.... $\dfrac{2x - 1}{x + 1}$ ....

(5)

9. **a)** Work out the total surface area of this cuboid..



Surface area = front + back + top + bottom + left + right
$= 3 \times 4 + 3 \times 4 + 4 \times 2 + 4 \times 2 + 3 \times 2 + 3 \times 2$
$= 12 + 12 + 8 + 8 + 6 + 6$
$= 52\ cm^2$

........**52**.....cm^2

(2)

b) The cuboid has a density of 7 grams per cm^3
What is the mass of the cuboid

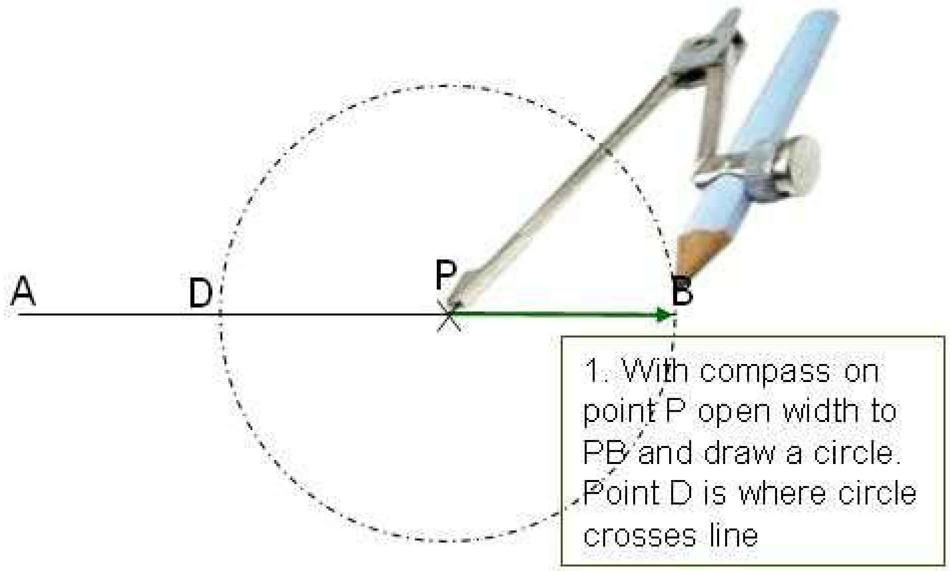
Density = mass ÷ volume ∴ mass (g) = density ($g\ cm^{-3}$) × volume (cm^3)

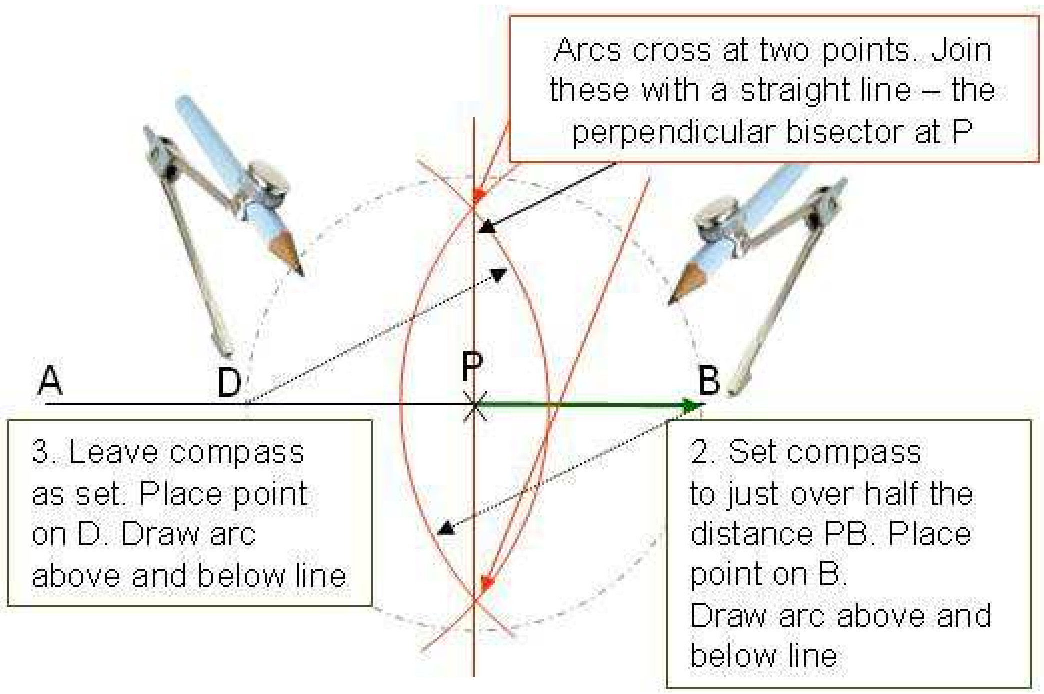
The volume of the cuboid = length × width × height
$= 4 \times 3 \times 2 = 24\ cm^3$
Mass = $7 \times 24 = 168$ g

............**168**..grams

(2)

10. A horizontal line AB is drawn below with point P is on the line
Using a compass and pencil construct a perpendicular line that passes through P**.**



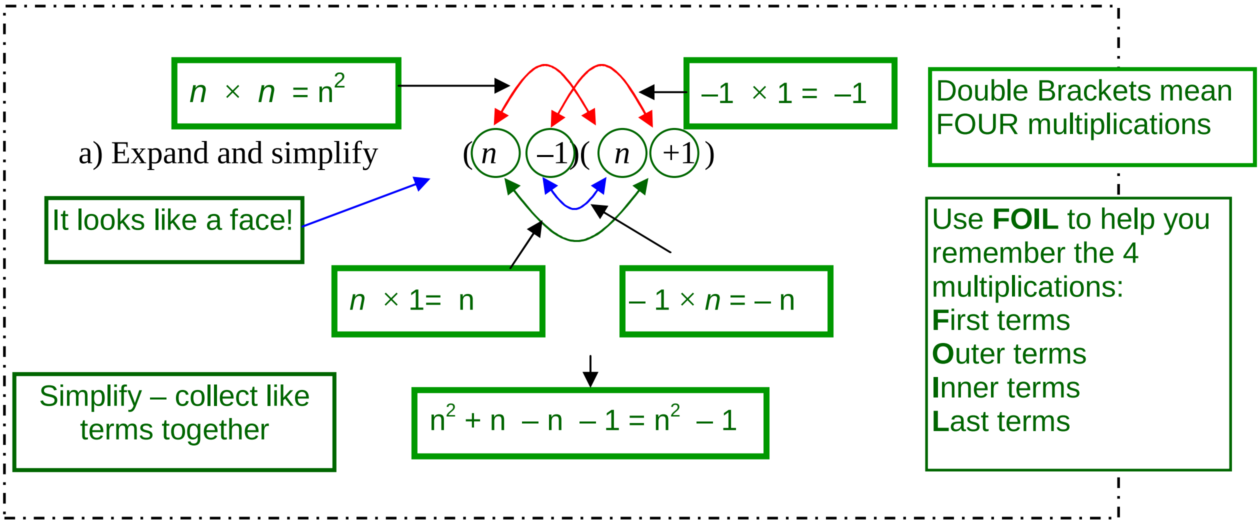


(2)

11. If we take a number and square it, the answer is also the product of the two numbers either side of it plus one. i.e. $5^2 = 4 \times 6 + 1 = 25$; $6^2 = 5 \times 7 + 1 = 36$;

Prove algebraically that this works for all numbers

Using algebra.
Let the number in the middle = n. So our numbers either side are: (n – 1) (n) (n + 1)
Work out the product as: (n – 1) x (n + 1) or (n – 1) (n + 1)
First expand and simplify (n – 1) (n + 1)



When we add 1 to $n^2 - 1$ we get n^2 This is the same as the middle number n squared and it proves our theorem for all numbers

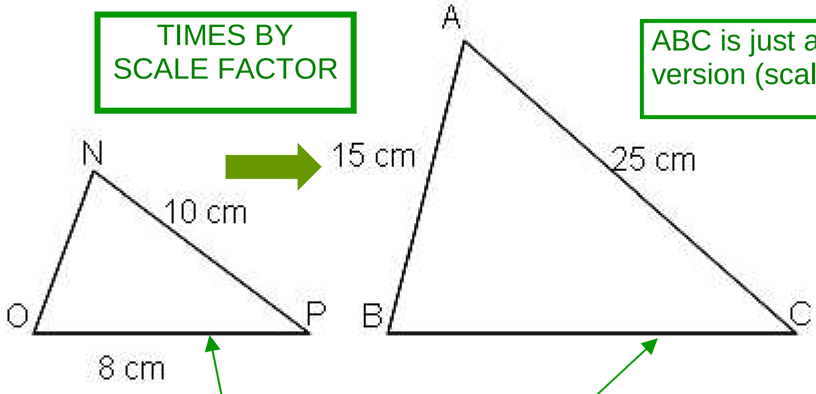
……………..……………

(2)

12.

TIMES BY SCALE FACTOR

ABC is just a bigger version (scaled up) of NOP



Diagrams NOT drawn accurately

To find the scale factor use sides from each triangle that are the similar sides.

The two triangles NOP and ABC are mathematically similar.
Angle N = angle A
Angle P = angle C
OP = 8 cm; NP = 10 cm
AC = 25 cm; AB = 15 cm

If two shapes are similar, one is an enlargement of the other with the same angles

a) What is the length of BC

When we are getting BIGGER, work out the scale factor this way

Scale factor $= \frac{\text{Big side}}{\text{Small side}} = \frac{25}{10}$

Side BC can be found from side OP using the scale Factor

$BC = OP \times \frac{25}{10} = 8 \times \frac{25^{5}}{10^{2}} = \frac{40}{2}$

20 ✓ ....cm
(2)

b) What is the length of NO?

When we are getting Smaller work out the scale factor this way

Scale factor $= \frac{\text{Small side}}{\text{Big side}} = \frac{10}{25}$

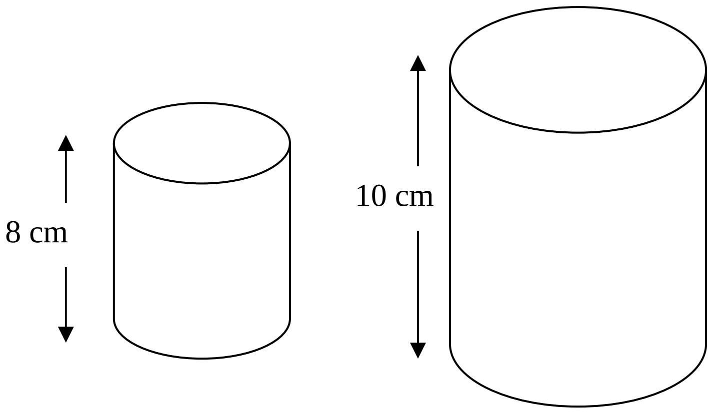
Side NO can be found from side AB using the Scale Factor

$NO = AB \times \frac{10}{25} = 15 \times \frac{10^{2}}{25^{5}} = \frac{30}{5}$

This is exactly like dividing by the scale factor

.... **6** ✓ ..cm
(1)

c) Two mathematically similar cylinders are shown



Diagrams NOT drawn accurately

The volume of the smaller cylinder is 64 cm^3
Calculate the volume of the larger cylinder.

Since they are mathematically similar we can work out the scale factor for length

Scale factor (for length) = $\frac{\text{Big side}}{\text{small side}} = \frac{10}{8}$

The scale factor for volume = (scale factor for length) ^{3} = $\left(\frac{10}{8}\right)^3$

The volume of the large cylinder can be found using the volume of the small cylinder × scale factor for volume

Vol = $64 \times \frac{10}{8} \times \frac{10}{8} \times \frac{10}{8} = \frac{1000}{8} = 125$

.........**125**...cm^3
(3)

13. A survey of 80 children was made to see how long they spent playing computer games in a week

The table below shows how long in hours the children spent.

Time (t hours)	Frequency
$5 \leq t < 10$	10
$10 \leq t < 15$	16
$15 \leq t < 20$	30
$20 \leq t < 25$	21
$25 \leq t < 30$	3

a) Complete the cumulative frequ

Cumulative means add find a new total as you go along by adding on each new number

(1)

Notice that we start from 5 each time

Here we want time between 5 hours and less than 20 hours

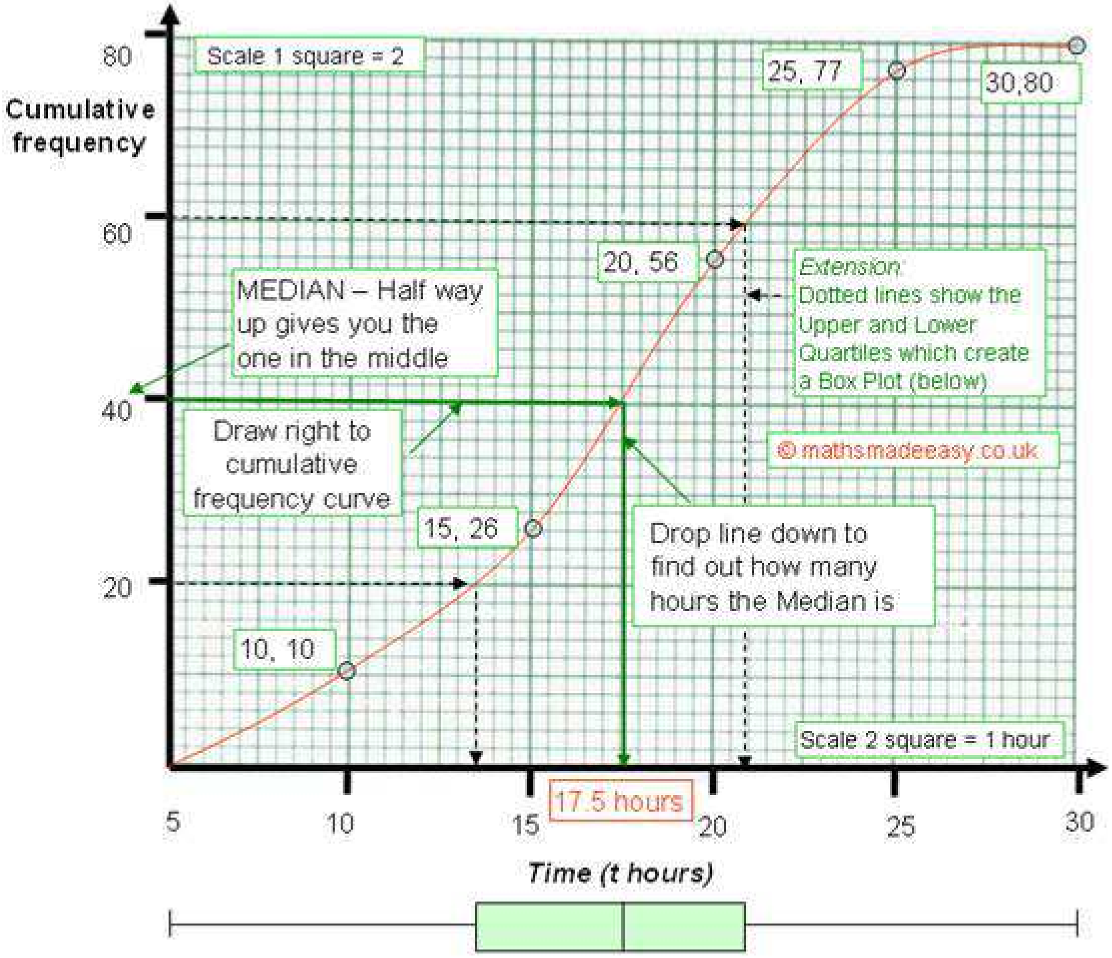
Time (t hours)	Cumulative Frequency
$5 \leq t < 10$	10
$5 \leq t < 15$	**26** ✓
$5 \leq t < 20$	**56** ✓
$5 \leq t < 25$	**77** ✓
$5 \leq t < 30$	**80** ✓

26 = 10 + 16

56 = 26 + 30

b) Using your completed table draw a cumulative frequency graph on the grid **(2)**

When plotting points use the **upper number** in each time interval
e.g. plot 10, 10 ; 15, 26; 20, 56; 25, 78 and 30, 80 for the last point



c) Using the completed graph estimate the median time
Remember to state the units in your answer

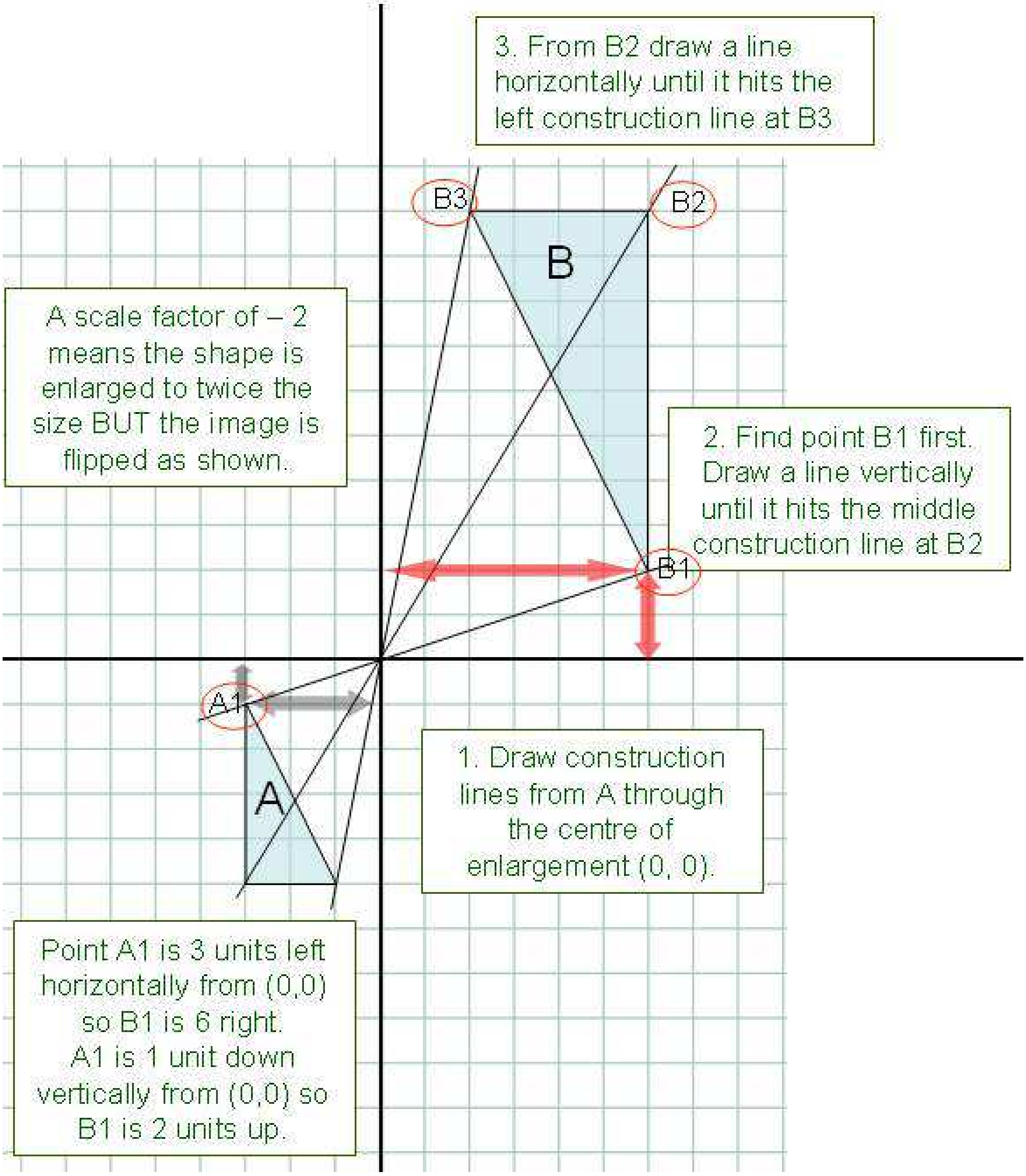
17.5 hours

17 to 18 hours are also OK

(2)

14. Enlarge triangle **A** by scale factor –2 , centre *O*. Label the new triangle B.

(3)

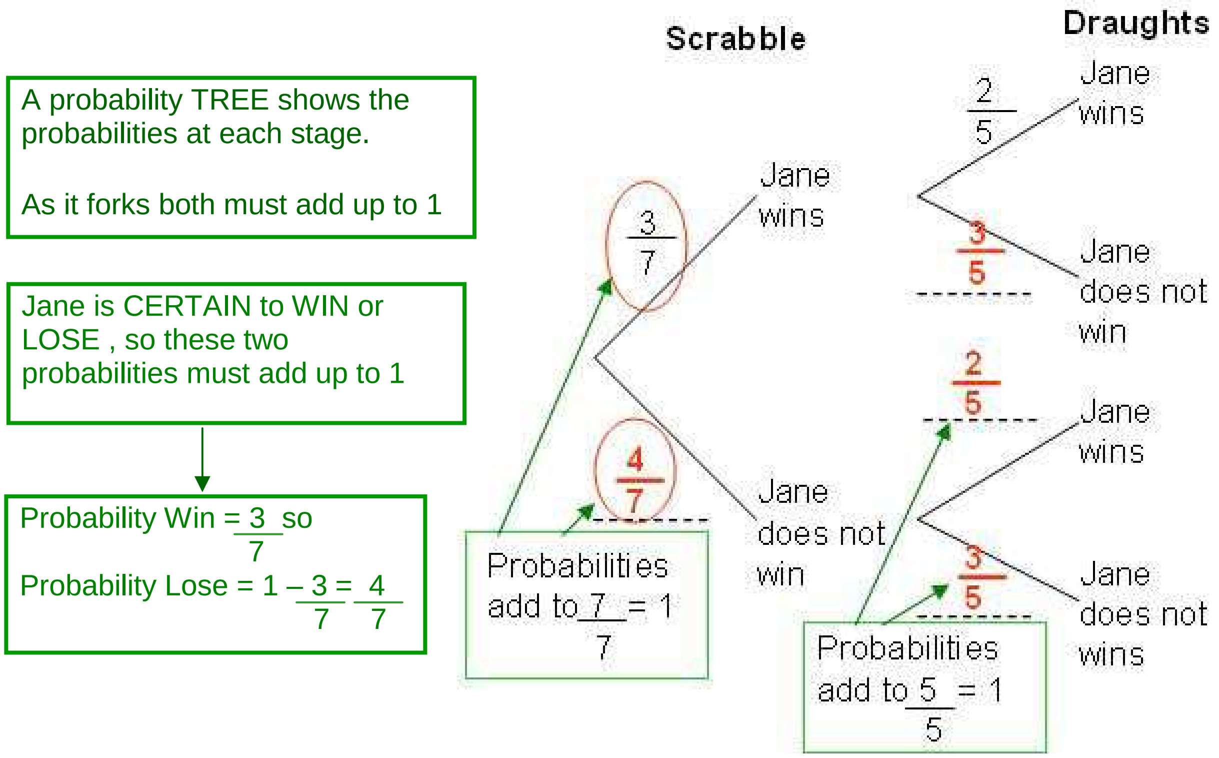


15. Jane played a game of scrabble and then a game of draughts.

The probability that she will win the game of scrabble is $\frac{3}{7}$

The probability that she will win the game of draughts is $\frac{2}{5}$

a) Draw a probability tree to show this information



(2)

b) Work out the probability that Jane will win exactly one game

If Jane wins one game she must lose the other one. There are two paths on the probability tree. We work out each path and add them. So we ADD probability (win-lose) and probability (lose-win)

probability (win-lose) = $\frac{3}{7} \times \frac{3}{5} = \frac{9}{35}$

probability (lose-win) = $\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$

probability (win-lose) + probability (lose-win) = $\frac{9}{35} + \frac{8}{35} = \frac{17}{35}$

$\frac{17}{35}$ ✓

……………………………

(2)

Jane played a game of scrabble and a game of draughts in several competitions.
She won at both scrabble and draughts in ten of these competitions.

c) Estimate in how many competitions Jane did not win either game.

If Jane wins both games there is only one path on the probability tree.

probability (win-win) = $\frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$

This equates to 10 competitions

The probability (lose-lose) = $\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$

Since the probability (lose-lose) *is twice* probability (win -win) then probability (lose-lose) equates to 2 × 10 competitions

20 ✓

(2)

16. Make x the subject of the formula $3(2x - y) = ax - 4$

Expand the left side

$3(2x - y) = ax - 4$

$6x - 3y = ax - 4$

Get *x*'s on left – subtract *ax* from both sides

$6x - 3y - ax = ax - ax - 4 = -4$

Get *y*'s on right – add 3*y* to both sides

$6x - 3y - ax + 3y = + 3y - 4$

$6x - ax = 3y - 4$

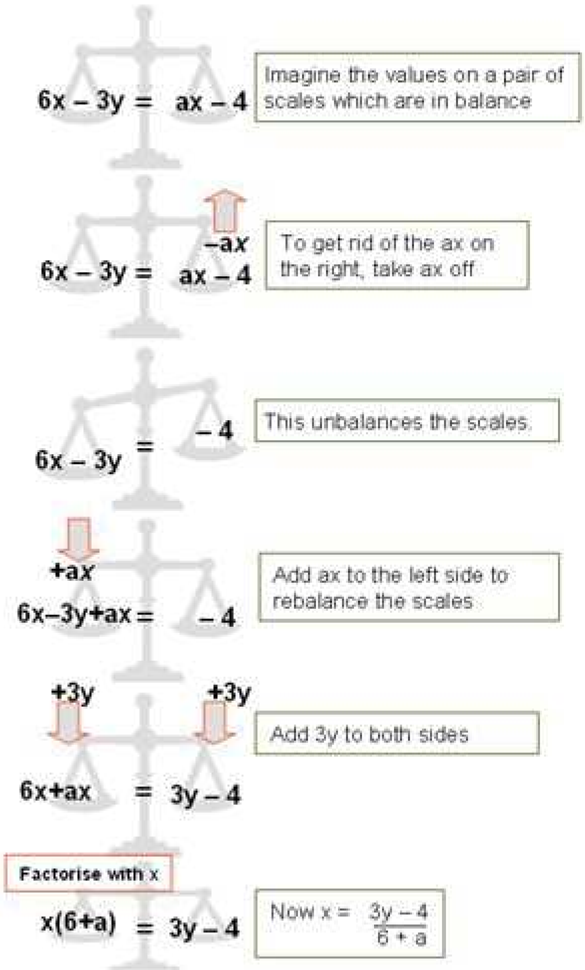
Factorise left with *x* as the factor

$x(6 - a) = 3y - 4$

Divide both sides by (6 – a) to get *x*

$$\frac{x(6 - a)}{6 - a} = \frac{3y - 4}{6 - a}$$

$$x = \frac{3y - 4}{6 - a}$$



$$\frac{3y - 4}{6 - a}$$

(3)

17. Prove that the recurring decimal $0.\dot{3}\dot{6} = \frac{4}{11}$

Recurring decimals have a pattern of digits which repeat forever e.g. $0.\dot{3}$ means 0. 333333333 …

Count the number of digits in the pattern that are repeating. The dot above a digit tells you that it is repeating. So $0.\dot{3}\dot{6}$ is 0.36363636 and has two repeating digits

Multiplying your recurring decimal by either 10, 100, 1000 etc.
Select the one which has the same number of zeros as the repeating pattern
So multiply $0.\dot{3}\dot{6}$ with two repeating digits by 100 which has 2 zeros and has 2 repeating digits

$0.\dot{3}\dot{6}$ x 100 = 36.363636

Subtract the original : 36.363636 – 0.363636 = 36
This is like multiplying our original by 99 (100 – 1)
So 0.363636 x 99 = 36

Rearranging 0.363636 × 99 = 36 we get $0.363636 = \frac{36}{99} = \frac{4}{11}$

So $0.\dot{3}\dot{6} \times 100 - 0.\dot{3}\dot{6} \times 1 = 36$
or $0.\dot{3}\dot{6} \times 99 = 36 \rightarrow 0.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$

……………. **(3)**

18. Given that $x^2 - 4x + 11 = (x - a)^2 + b$ find a and b

To get our answer in the form $(x - a)^2 - b$ means we have to ***complete the square.***

$x^2 - 4x \rightarrow (x - 2)^2$

Work on the first two terms
Look at the number for the x term and halve it. Put this number inside the squared bracket as shown.

$(x - 2)^2 = x^2 - 4x + 4$

$x^2 - 4x = (x - 2)^2 - 4$

There will be an extra value created by squaring the number in the bracket.
To make both sides of the equation equal we have to subtract it.

$x^2 - 4x + 11 = (x - 2)^2 - 4 + 11$
$= (x - 2)^2 + 7$

Finally we have to put the third term back and then simplify the equation

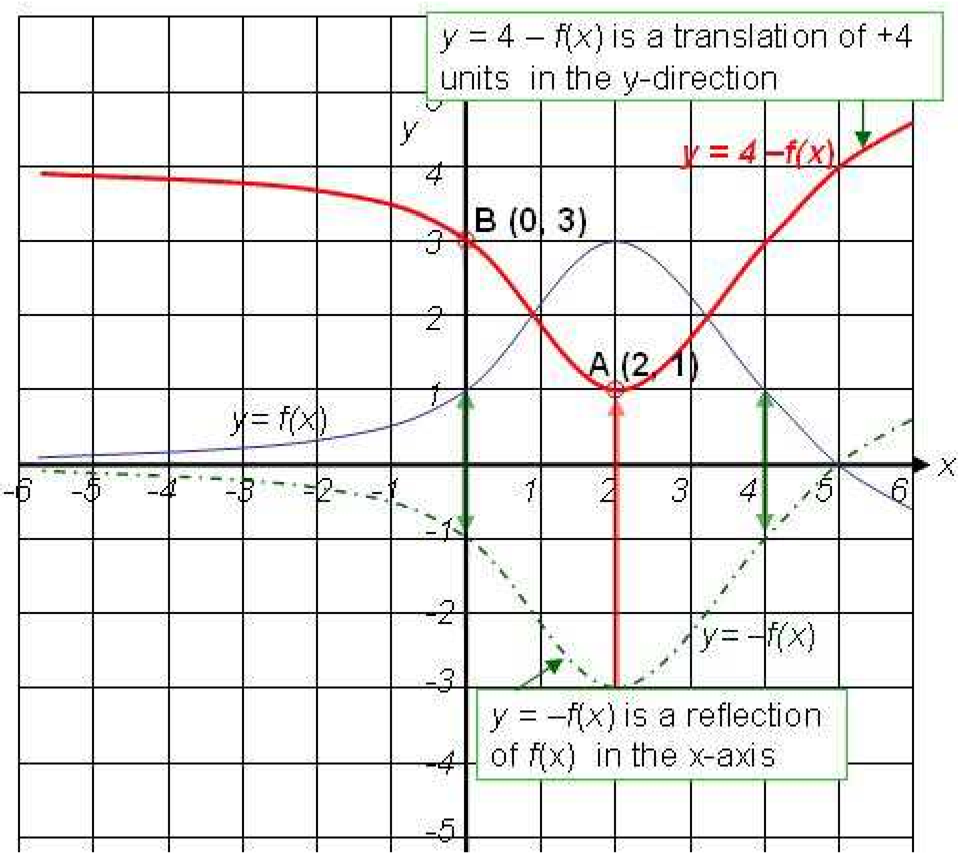
$x^2 - 4x + 11 = (x - 2)^2 + 7$
∴ a is 2 and b is 7

a= ..**2**.. b=...**7**.....
(3)

19. The diagram shows a sketch of y = f(x).

a) Sketch the graph of y = 4 – f(x) on the grid showing the co-ordinates of points A and B.

(3)



20. x is inversely proportional to the square of y.
$x = 3$ when $y = 4$

a) Express x in terms of y

We write x is inversely proportional to the square of y as: $x \propto \frac{1}{y^2}$

This can be written as an equation using a constant k so, $x = \frac{k}{y^2}$

Work out constant k using the value of $x = 3$ when $y = 4$

So $3 = \frac{k}{16}$ → $k = 3 \times 16 = 48$

Substituting k back in the equation $x = \frac{48}{y^2}$

$x = \frac{48}{y^2}$

(2)

b) Work out the value of x when $y = 6$

$x = \frac{48}{y^2}$, when $y = 6$, $x = \frac{48}{6^2} = \frac{48}{36} = \frac{4}{3}$

$x = \frac{4}{3}$

(1)

c) Work out the value of positive value of y when $x = 12$

$x = \frac{48}{y^2}$, when $x = 12$, $12 = \frac{48}{y^2}$

Rearranging above

$12 = \frac{48}{y^2} \rightarrow 12y^2 = 48 \rightarrow y^2 = \frac{48}{12} = 4 \rightarrow y = \sqrt{4} = \pm 2$

$y = 2$

(2)

21. (a) Find the value of

i) 64^0

Remember: any number to the power 0 is 1

1 ✓ .... **(1)**

ii) $64^{\frac{1}{2}}$

½ means square root

$64^{\frac{1}{2}} = \sqrt{64} = \pm 8$

± 8 ✓ .... **(1)**

iii) $64^{-2/3}$

minus (–) means reciprocal or 1 over the number

$^{1/3}$ means cubed root $\sqrt[3]{\ }$ $^{2/3}$ means cubed root squared $(\sqrt[3]{\ })^2$

$$64^{-2/3} = \frac{1}{(\sqrt[3]{64})^2} = \frac{1}{(4)^2} = \frac{1}{16}$$

$\frac{1}{16}$ ✓ ..... **(2)**

iv) $3\sqrt{n} = 9^{3/2}$

Find the value of *n*.

$9^{3/2} = (\sqrt{9})^3 = (\pm 3)^3 = \pm 27$

Substituting 27 for $9^{3/2}$ we get $3\sqrt{n} = \pm 27$

÷ both sides by 3 → $\frac{3\sqrt{n}}{3} = \frac{27^9}{3}$ → $\sqrt{n} = +9$ so $n = 9^2$ and n= 81

n =81 ✓ .... **(2)**

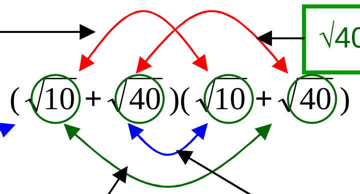
e) Prove that $(\sqrt{10} + \sqrt{40})^2 = 90$

This is a SURDs question. SURDs are irrational numbers – usually square roots

Multiply out

$\sqrt{10} \times \sqrt{10} = 10$

$\sqrt{40} \times \sqrt{40} = 40$



It looks like a face!

$\sqrt{10} \times \sqrt{40} = \sqrt{400} = \pm 20$

$\sqrt{10} \times \sqrt{40} = \sqrt{400} = \pm 20$

(2)

Simplify – collect like terms together

$10 + 40 + 20 + 40 = 90$

Alternatively a better proof is $(\sqrt{10} + \sqrt{40})^2 = (\sqrt{10} + 2\sqrt{10})^2 = (3\sqrt{10})^2 = 9 \times 10 = 90$

22. **a)** Factorise $3x^2 + 26x + 16$

This quadratic equation has an x^2 term greater than 1.
It can be done by trial and error or by the method below:

Replace the $+26x$ term with two terms which make $+26x$

$3x^2 + 26x + 16$

$48 = 2 \times 24$ and $26 = 24 + 2$

We need two numbers which → make + 26 by adding or subtracting

and multiply to make +48 which we get from 24×2

We *rewrite the equation* replacing + 7 with + 8 and – 1
So $3x^2 + 26x + 16$

$3x^2 + 24x + 2x + 16$

Notice that we put $24x$ near the $3x^2$ and $2x$ near the 16
It makes it easier to factorise later

Now *factorise* the two pairs of terms
$3x^2 + 24x + 2x + 16$

$3x(x + 8)$ $+2(x + 8)$

Notice that these both contain $(x + 8)$
Which means we can take this out as a factor.

$\therefore\ 3x(x + 8) + 2(x + 8) = \mathbf{(3x + 2)(x + 8)}$

$(3x + 2)(x + 8)$ ✓

.... **(2)**

b) Solve $3x^2 + 26x + 16 = 0$

$\therefore\ 3x^2 + 26x + 16 = (3x + 2)(x + 8) = 0$

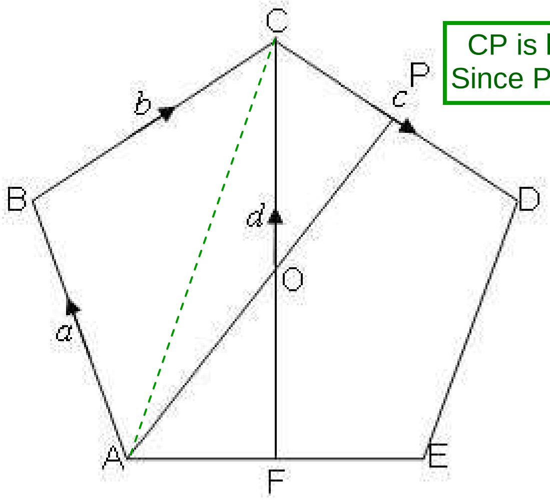
So $(3x + 2) = 0\ \ \therefore 3x = -2\ \ \ x = -\frac{2}{3}$

Or $(x + 8) = 0\ \ \therefore x = -8$

$x = -\frac{2}{3}$ or -8 ✓

(1)

23.



CP is half CD = c/2
Since P is the midpoint

ABCDE is a regular pentagon

$\overrightarrow{AB} = a$ $\overrightarrow{BC} = b$ $\overrightarrow{CD} = c$ $\overrightarrow{FC} = d$

P is the mid point of CD and F is the mid point of AE

a) Find the vector $\overrightarrow{AC}$

AC is AB + BC = a + b

a + b ✓ ....

(1)

b) Find the vector $\overrightarrow{AP}$

AP is AB + BC +CP = a + b + ½ c

a + b + ½c ✓ ....

(2)

O is the mid point of AP and FC

c) Find the vector $\overrightarrow{AF}$

AF is AO + OF = ½(a + b + ½ c) – ½ d

AO is half AP in (b)
OF is half of FC but in the opposite direction

½ (a + b + ½c) – ½d
or
½a + ½b + ¼c – ½d ✓

(2)

TOTAL FOR PAPER: 100 MARKS
END