

GCSE Mathematics
 Non Calculator
 Higher Tier
 Free Practice Set 5
 1 hour 45 minutes



ANSWERS

Marks shown in brackets for each question (2)

A*	A	B	C	D	E
88	75	60	45	25	15

Legend used in answers

Green Box - Working out

5b means five times b
 $b = -3$ so $5 \times -3 = -15$

Red Box and ✓ - Answer

48 % ✓

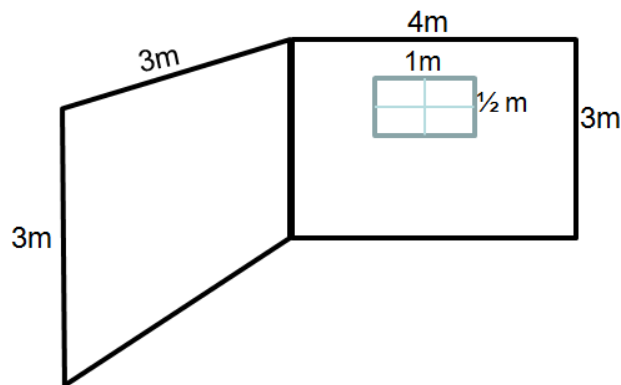
Authors Note

Every possible effort has been made to ensure that everything in this paper is accurate and the author cannot accept responsibility for any errors.

Apart from any fair dealing for the purposes of research or private study as permitted under the Copyright, Designs and Patents Act 1988, this paper may only be reproduced, stored or transmitted in any form or by any means with the prior permission in writing of the author, or in the case of reprographic reproduction in accordance with the terms and licence by the CLA. Enquiries concerning reproduction outside these terms should be sent to the author.

The right of David Weeks to be identified as the author of this work has been asserted by him in accordance with the Copyright, Designs and Patents Act 1988.

1. Two walls of a bathroom need tiling. The walls are shown below.



not drawn accurately

One wall is 3 metres by 3 metres.

The other wall is 3 metres by 4 metres with a window which is 1 metre by $\frac{1}{2}$ metre.

Work out the area that needs tiling.

$$\begin{aligned} \text{Side wall: } & 3 \times 3 = 9\text{m}^2 & \text{back wall: } & 4 \times 3 = 12\text{m}^2 & \text{window} & = 1 \times \frac{1}{2} = \frac{1}{2} \text{m}^2 \\ \text{Total area} & = 9 + 12 - \frac{1}{2} = 20\frac{1}{2} \text{m}^2 \end{aligned}$$

20 $\frac{1}{2}$ ✓

..... m²

Tiles are squares measuring 25cm by 25 cm. They cost £3.00 each
Calculate the costs of tiling the bathroom

(3)

$$\begin{aligned} \text{Tiling area} & = 9 + 12 - \frac{1}{2} = 20\frac{1}{2} \text{m}^2 \\ \text{Each tile is } & \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \text{m}^2 \\ \text{How many } \frac{1}{16} & \text{ go into } 20\frac{1}{2} = 320 + 8 = 328 \text{ tiles} \\ \text{Cost} & = £3 \times 328 = £984 \end{aligned}$$

984 ✓

£....

(3)

2. a) Expand $y(y - 5)$

Multiply everything inside brackets by y outside

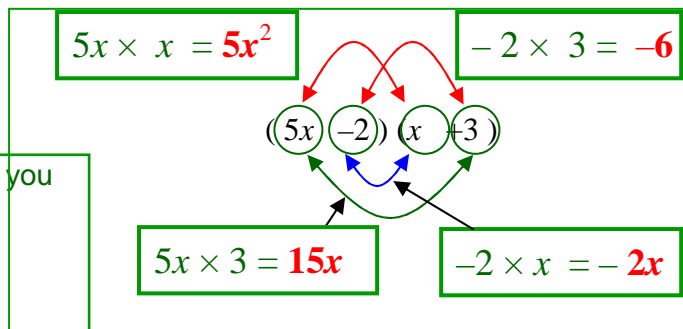
$$y^2 - 5y$$

(2)

b) Expand and simplify $(5x - 2)(x + 3)$

Double Brackets mean FOUR multiplications

Use **FOIL** to help you remember the 4 multiplications:
First terms
Outer terms
Inner terms
Last terms



Simplify – collect like terms together

$$15x - 2x + 5x^2 - 6 = 5x^2 + 13x - 6$$

$$5x^2 + 13x - 6$$

(2)

c) Simplify $10a^2 - 6b - 3a^2 + 3b - b$

$$7a^2 - 4b$$

(1)

d) Factorise $9x^2 - 4$

This is a difference of two squares

$$(3x - 2)(3x + 2)$$

(2)

e) Solve $21a^2 + 5a = 4$

Rearrange to make quadratic: $21a^2 + 5a - 4 = 0$

$$21a^2 + 5a - 4 = 0$$

Multiply 1st and last numbers to give $21 \times -4 = -84$

1. Find two numbers which multiply to make -84

and also makes the middle a value $+5$ by adding or subtracting

$$\begin{aligned} -84 &= 12 \times -7 \\ +5 &= 12 - 7 \end{aligned}$$

$$21a^2 + 5a - 4 = 0$$

2. Rewrite $+5a$ as $(-7a + 12a)$ in the equation

$$21a^2 + 5a - 4 = 21a^2 - 7a + 12a - 4$$

Notice :
we put $12a$ next to the -4 and $-7a$ next to the $21a^2$
We use it for factorisation in the next step

3. Factorise each pair of terms:
 $21a^2 - 7a + 12a - 4$
 $\rightarrow 7a(3a - 1) + 4(3a - 1)$

4. Simplify: we have $(3a - 1)$ in both terms so we can take out as a factor
 $7a(3a - 1) + 4(3a - 1) = (7a + 4)(3a - 1)$

A test that we are right is that both factors are the same

$$(7a + 4)(3a - 1)$$

$$\text{So } a = -4/7 \text{ or } 1/3$$

3. Given that

$$543 \times 21 = 11403$$

find the value of

Count how many places the decimal point has moved

a) 54.3×2.1

114.03

(1)

b) 0.543×0.21

0.11403

(1)

c) $1140.3 \div 5.43$

210

(1)

4. For a qualification $\frac{2}{5}$ of the marks were given for coursework.

The *rest* of the marks were given for a written paper of which $\frac{3}{8}$ were given for a mental test.

Total marks were out of 120.

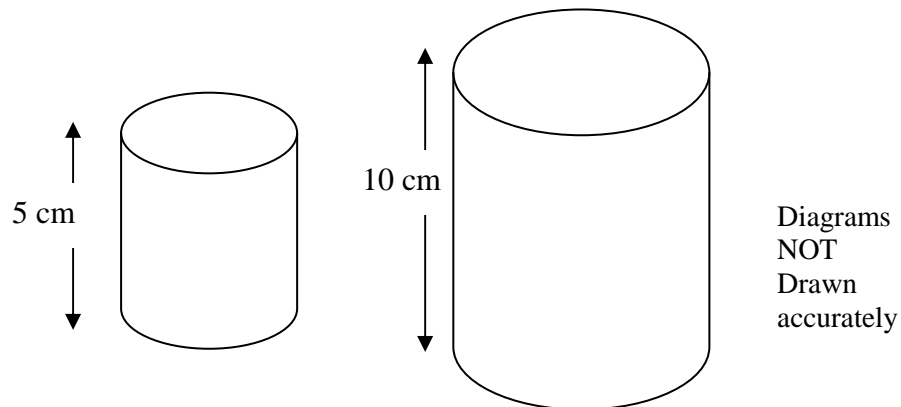
How many marks were given for the mental test.

$$\frac{3}{5} \text{ given for written paper}$$
$$\frac{3}{5} \times \frac{3}{8} = \frac{9}{40} \quad \frac{9}{40} \times 120 = 27$$

27

(3)

5. Two mathematically similar cylinders are shown



The volume of the smaller cylinder is 50 cm^3
Calculate the volume of the larger cylinder.

$$\text{Scaling factor for length} = 2$$
$$\text{Scaling factor for volume is } 2^3$$
$$\text{Volume of larger cylinder} = 50 \times 8 = 400$$

400 .cm³

(3)

6. Cyril had a pack of playing cards.

Each card in the pack is green or blue with a circle, square, triangle or rectangle symbol. The table below shows the probability of picking different cards.

Symbol	Circle	Square	Triangle	Rectangle
Colour	Green	Green	Blue	Blue
Probability of picking a card	$2x$	x	$3x$	$5x$

Cyril picked **two** cards at random without replacement

a) What is the probability of picking a blue card *and* a green card.

Draw a tree diagram

There are $11x$ cards and $8x$ are blue

Blue = $\frac{7}{10}$

Blue = $\frac{8}{11}$

Green = $\frac{3}{10}$

Green = $\frac{3}{11}$

Blue = $\frac{8}{10}$

Green = $\frac{2}{10}$

$$BG = \frac{8}{11} \times \frac{3}{10} = \frac{24}{110} \quad GB = \frac{8}{11} \times \frac{3}{10} = \frac{24}{110}$$

.....

$\frac{48}{110}$

(3)

Cyril put the cards back, then picked **two** cards at random without replacement

b) What is the probability of **not** picking a card with the square symbol

There is 1 square card out of 11 cards

SQ = $\frac{0}{10}$

SQ = $\frac{1}{11}$

NSQ = $\frac{10}{10}$

SQ = $\frac{1}{10}$

NSQ = $\frac{10}{11}$

NSQ = $\frac{9}{10}$

$$NSQ-NSQ = \frac{10}{11} \times \frac{9}{10} = \frac{90}{110}$$

.....

$\frac{90}{110}$

(3)

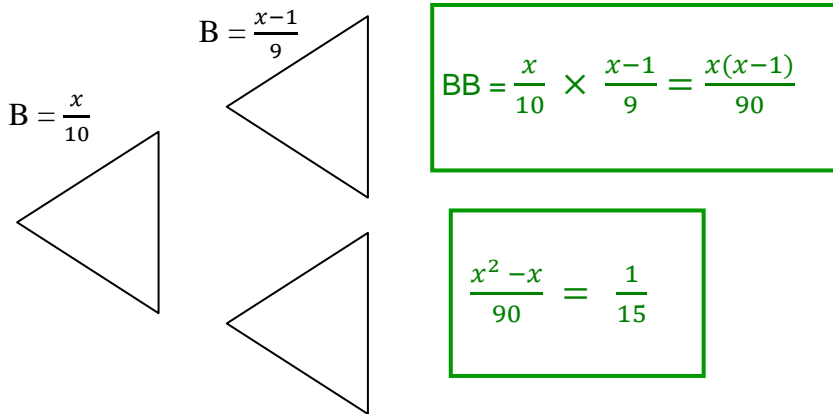
A bag contained some blue and green balls.
Two balls are picked at random without replacement

The probability of picking a blue ball on the first selection is $\frac{x}{10}$

The probability of picking *two blue* balls is $\frac{1}{15}$

How many blue balls were in the bag

Draw a tree diagram



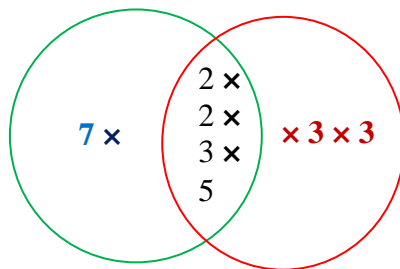
Multiply out $15x^2 - 15x - 90 = 0 \rightarrow x^2 - x - 6 = 0$

Factorise $\rightarrow (x - 3)(x + 2) = 0 \rightarrow x = 3$

3

(4)

7. What is the LCM of 420 and 540



$420 = 2 \times 2 \times 3 \times 5 \times 7$ and $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$ Prime factors

Put common factors into centre of Venn diagram. Then multiply everything

$7 \times 2 \times 2 \times 3 \times 5 \times 3 \times 3$

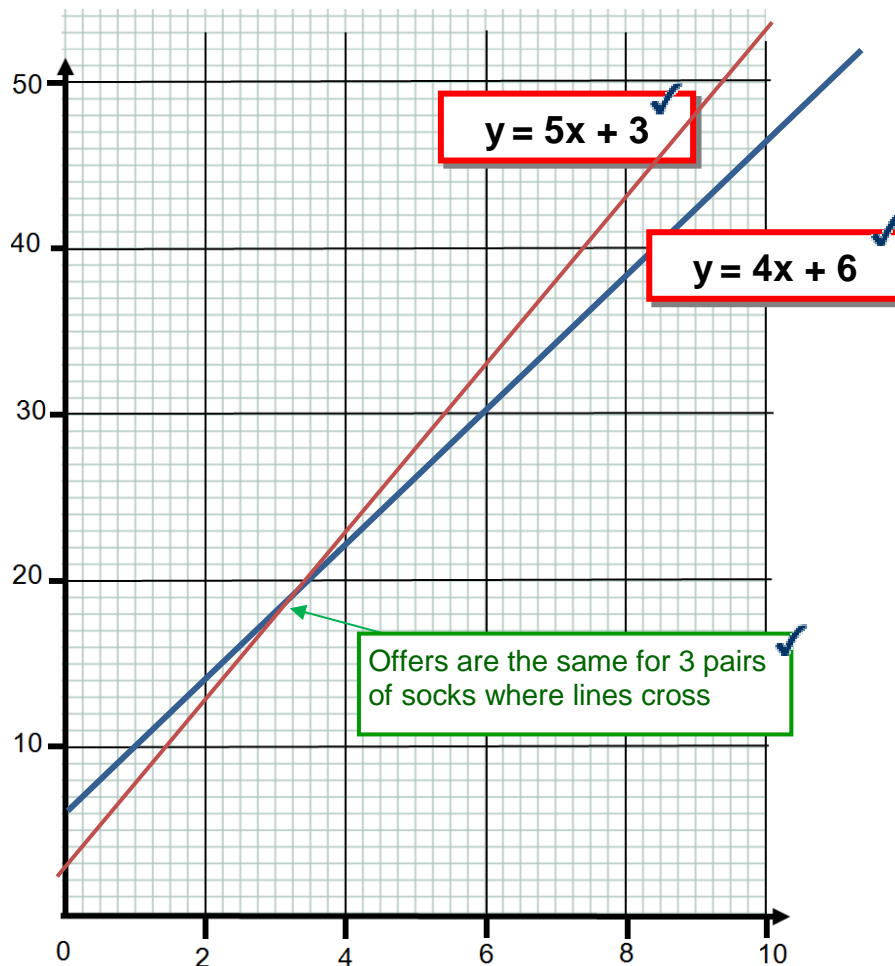
3780

(2)

8. Bill had an internet business selling socks. He starts by charging £5.00 for each pair plus postage of £3 for each order.

a) On the graph below plot the equation

$$y = 5x + 3$$



(2)

- b) Bill reduced the price for a pair of socks to £4, but increased postage to £6. On the same graph plot the equation showing this information.

Use the graph to compare both offers and work out for how many pairs of socks the two offers charge the same.

Plot the equation $y = 4x + 6$ for new offer

The point where two graphs cross shows where the two offers are the same

The two offers are the same for 3 pairs of socks

(2)

9. Matt's looked at his yearly income statement below and noticed some values were missing.

Yearly income report: Matt M. Easy	
Employee No: 123456 Tax Code: 747L	
	Gross pay £ 2 3 0 0 0 . 0 0
a)	Taxable Pay £ 15525.00
b)	Income Tax £ 3105.00
	NI £ 1 8 9 2 . 6 4
c)	Total Deductions £ 4997.64
d)	Net Pay £ 18002.36

His taxable pay is worked out using the tax code 747L.
This means, he is doesn't have to pay tax on £7475 of his gross pay.

Use the following formula to complete Matt's income statement

- a) Taxable Pay = Gross Pay – £7475.
Work out Matt's Taxable Pay and enter it above. (1)

$$23000 - 7475 =$$

- b) Matt's Income tax is 20% of his Taxable Pay.
Work out Matt's Income tax and enter it above. (1)

$$15525 \times 0.2 =$$

- c) Matt's Total Deductions = Income Tax + NI (national Insurance)
Work out Matt's Total Deductions and enter it above. (1)

$$3105 + 1892.64 =$$

- d) Net Pay = Gross pay – Total deductions.
Work out Matt's Net Pay and enter it above. (1)

10. Peter made some bricklaying mortars using the proportions:

	Think of $\frac{1}{4}$ as 1 share
1 part cement	Cement has 4 shares
$\frac{1}{4}$ part lime	Lime has 1 share
3 parts sand	Sand has 12 shares

He made 34 kg of mortar.
Work out the proportions for each

$$12 + 4 + 1 = 17 \text{ shares}$$

$$34 \div 17 = 2 \text{ kg for each share}$$

Cement ...	8	g
Lime ..	2	g
Sand ..	24	g

(3)

11. The formula

$$v^2 = u^2 + 2as$$

gives the velocity v of an object dropped from a height.

u is the starting velocity of an object

v is the final velocity of an object

a is the acceleration due to gravity. $a = 9.8$ on Earth.

s is the height dropped in metres.

- a) An object is dropped from rest and reaches a final velocity of 10m/s
Calculate the height dropped
Give your answer to 1 decimal place.

$$u = 0 \text{ as it starts from rest; } a = 9.8; v = 10$$

$$10^2 = 0 + 2 \times 9.8 \times s \rightarrow 100 = 19.6 s$$
$$\text{So } s = 100 \div 19.6 = 5.1$$

$s = \dots\dots\dots$ **5.1** m (2)

Another object has an initial velocity of 20 m/s and falls 25m

$$u = 20\text{m/s}$$

$$s = 25\text{m}$$

- b) Calculate the final velocity v
Give your answer to 1 decimal place.

$$v^2 = 20^2 + 2 \times 9.8 \times 25 \rightarrow v^2 = 400 + 490$$

$$v = \sqrt{890} = 29.83$$

$$v = \dots \boxed{29.8} \text{ m/s} \quad (2)$$

12. Chantelle wants to buy *two pairs* of trainers.
Three shops sell the trainers she wants as shown below.

DW Shoes	CB Sports	Joggers
Trainers	Trainers	Trainers
Normal price £40	Normal price £38	Normal price £43
$\frac{1}{5}$ th off	15% off	Buy one pair get 2 nd pair Half price

Calculate which shop is the cheapest for *two pairs* of trainers.
Show all your working

DW	JC	Joggers
$\frac{1}{5}$ th of £40 = £8	10% of £38 = £3.80	1 st pair costs £43
So 1 pair costs £40-8 = £32	5% of £38 = £1.90 £5.70	Half of £43 is £21.50
Two pairs cost £64	So 1 pair costs £38 - £5.70 = £32.30	Two pairs cost £64.50

Cheapest Shop is.....

**DW shoes is cheapest –
two pairs of trainers cost £64** (4)

13. a) Make y the subject of the formula

$$y + 3x = \frac{2y}{x}$$

Multiply both sides by x $yx + 3x^2 = 2y$

Take $2y$ from both sides $yx + 3x^2 - 2y = 0$

Take $3x^2$ from both sides $yx - 2y = -3x^2$

Factorise left side $y(x - 2) = -3x^2$

Divide by $(x - 2)$ Either answer correct

$$\frac{-3x^2}{x - 2}$$

$$\frac{3x^2}{2 - x}$$

(2)

b) Simplify $\frac{6x^3y^5}{3x^2y^6}$

Cancel 6 and 2 and x and y powers.
Remember when dividing powers subtract bottom from top

$$\frac{2x}{y}$$

(2)

c) Simplify $\frac{2x^2 + 7x - 15}{x^2 + x - 20}$

Factorise top and bottom then see if you can cancel

It factorises to

$$\frac{(2x - 3)(x + 5)}{(x + 5)(x - 4)}$$

$$\frac{2x - 3}{x - 4}$$

(4)

d) Solve the simultaneous equations

$$x^2 + y^2 = 25$$

$$y = x - 1$$

Substitute $y = x - 1$ in the 1st equation

$$x^2 + (x - 1)^2 = 25$$

$$x^2 + x^2 - 2x + 1 = 25$$

$$2x^2 - 2x - 24 = 0 \rightarrow x^2 - x - 12 = 0$$

Factorise $(x + 3)(x - 4) = 0$

$$x = 4 \text{ or } -3$$

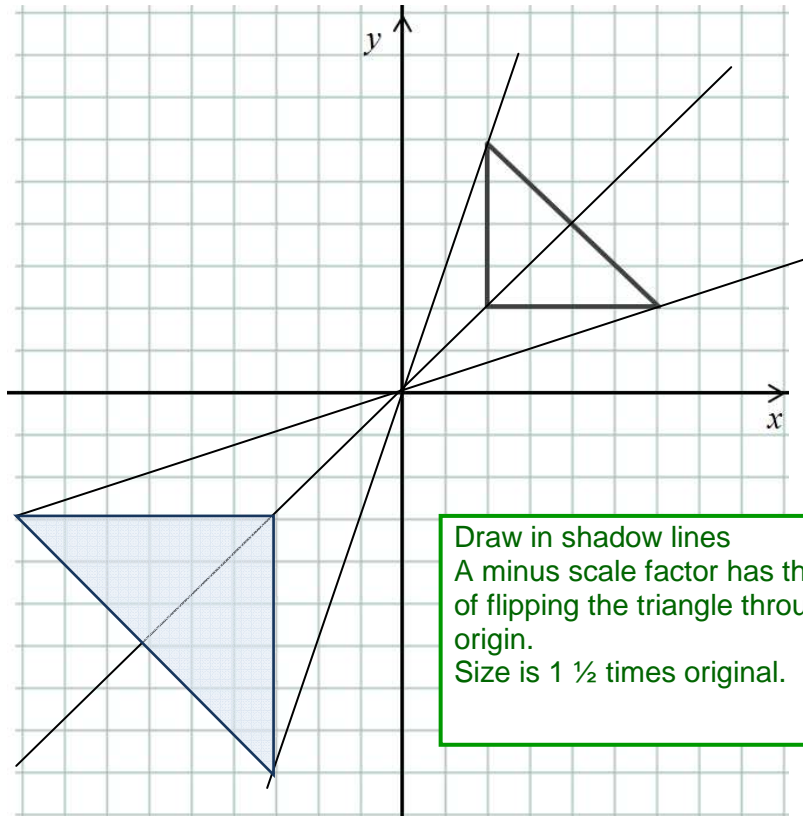
SUBSTITUTE these values into the 2nd equation

$$y = 4 - 1 = 3 \text{ and } y = -3 - 1 = -4$$

x **4 and -3** ✓ (3)

y..... **3 and -4** ✓

14. Enlarge triangle A by scale factor of $-1\frac{1}{2}$, centre O .



(3)

15. The sale price for a coat was £108.
The normal price was reduced by 20%.

Work out the normal price for the coat.

Don't work out 20% of 108 and add it on.

$$\text{Normal price} = 108 \div 0.8 = 135$$

£... **135** (3)

16. Given that $x^2 - 8x - 4 = (x - a)^2 + b$ find a and b

This is a complete the square question.

$$x^2 - 8x \rightarrow (x - 4)^2$$

Work on the first two terms first.
Look at the number for the x term and **halve** it. Put this number inside the squared bracket as shown.

$$x^2 - 8x = (x - 4)^2 - 16$$

There will be an extra value created by the squaring the number in the bracket. To make both sides of the equation equal we have to subtract it.

$$\text{Subtract } 4^2 = 16$$

$$x^2 - 8x - 4 = (x - 4)^2 - 16 - 4 \\ = (x - 4)^2 - 20$$

Finally we have to put the third term back and then simplify the equation

a = **4** b = **-20** (3)

Hence solve $x^2 - 8x - 4 = 0$

Give your answer in the form $c \pm d\sqrt{5}$

Using above	$(x - 4)^2 - 20 = 0$
	$(x - 4)^2 = 20$
Square root both sides	$x - 4 = \pm\sqrt{20}$
Subtract 5 both sides	$x = \pm\sqrt{20} + 4$
	$x = 4 \pm 2\sqrt{5}$

$4 \pm 2\sqrt{5}$ (3)

17. The amount of energy (E) released when matter is converted to energy is proportional to mass of that object (m).

When $E = 9 \times 10^{13}$ Joules, $m = 1 \times 10^{-3}$ kg

- a) Find a formula for E in terms of m giving your answer in standard form

Since E is proportional to m we write this as: $E \propto m$
 We can replace the \propto sign with $= k$ where k is a constant. So $E = km$

We know that when $E = 9 \times 10^{13}$, $m = 1 \times 10^{-3}$ and can use this to find k :

$$9 \times 10^{13} = k \times 1 \times 10^{-3}$$

$$\text{so } k = \frac{9 \times 10^{13}}{1 \times 10^{-3}} = 9 \times 10^{16}$$

We can rewrite the formula as: $E = 9 \times 10^{16} m$

E = $9 \times 10^{16} m$

(3)

- b) Calculate the mass, in kg when $E = 1.8 \times 10^{15}$ Joules
 Give your answer in standard form

We have: $E = 9 \times 10^{16} m$ so when $E = 1.8 \times 10^{15}$

$$1.8 \times 10^{15} = 9 \times 10^{16} m$$

$$m = \frac{1.8 \times 10^{15}}{9 \times 10^{16}} = \frac{1.8 \times 10^{-1}}{9} = 0.2 \times 10^{-1} = 2 \times 10^{-2}$$

Since we increase 0.2 by 10 we have to reduce the power from -1 to -2

E = 2×10^{-2} kg

(2)

- c) Calculate the Energy, in joules when $m = 0.00000015$ kg
 Give your answer in standard form

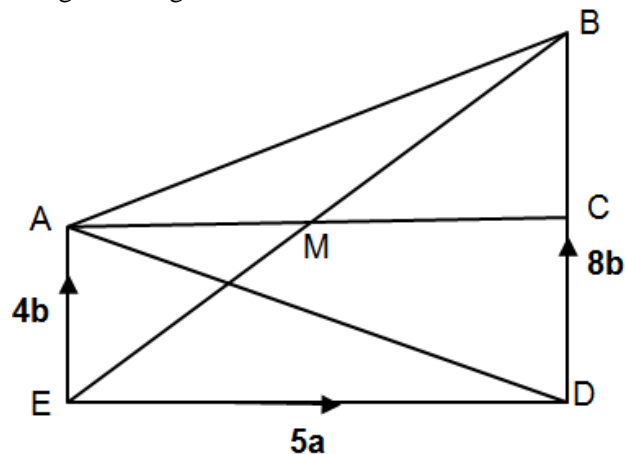
$$0.00000015 = 1.5 \times 10^{-7}$$

$$\text{We have: } E = 9 \times 10^{16} \times 1.5 \times 10^{-7} = 13.5 \times 10^9 = 1.35 \times 10^{10}$$

m = 1.35×10^{10} Joules.

(1)

18. ACDE is a rectangle with $\overrightarrow{AE} = 4b$, $\overrightarrow{ED} = 5a$ and $\overrightarrow{DB} = 8b$
M is the midpoint of AC
ABC is a right angled triangle



Write each of the following vectors in terms of a and b

a) \overrightarrow{BA}

$$\begin{aligned}\overrightarrow{BA} &= \overrightarrow{BD} + \overrightarrow{DE} + \overrightarrow{EA} \\ \overrightarrow{BA} &= -8b - 5a + 4b = -4b - 5a\end{aligned}$$

$$\boxed{-4b - 5a}$$

(1)

b) \overrightarrow{BM}

$$\begin{aligned}\overrightarrow{BM} &= \overrightarrow{BA} + \overrightarrow{AM} \\ \overrightarrow{BM} &= -4b - 5a + \frac{5}{2}a = -4b - \frac{5}{2}a\end{aligned}$$

$$\boxed{-4b - 5/2a}$$

(1)

- c) Show that M is the midpoint of the line BE

Show that \overrightarrow{ME} is in the same length as \overrightarrow{BM}

$$\begin{aligned}\overrightarrow{ME} &= \overrightarrow{MA} + \overrightarrow{AE} \\ \overrightarrow{ME} &= -\frac{5}{2}a - 4b\end{aligned}$$

As both vectors have the same magnitude and direction M must be the midpoint.

(2)

19. A survey of 100 trainee teachers was made to see how long they spent revising for their QTS numeracy test

The table below shows how long in hours the trainee teachers spent.

Time (t hours)	Frequency
$0 \leq t < 4$	8
$4 \leq t < 8$	23
$8 \leq t < 12$	37
$12 \leq t < 16$	25
$16 \leq t < 20$	7

- a) Complete the cumulative frequency table

Time (t hours)	Cumulative Frequency
$0 \leq t < 4$	8
$0 \leq t < 8$	$8+23 = 31$ ✓
$0 \leq t < 12$	$8+23+37 = 68$
$0 \leq t < 16$	$8+23+37+25 = 93$
$0 \leq t < 20$	$8+23+37+25+7 = 100$

(1)

- b) Using your completed table draw a cumulative frequency graph on the grid

Plot highest value in each time interval

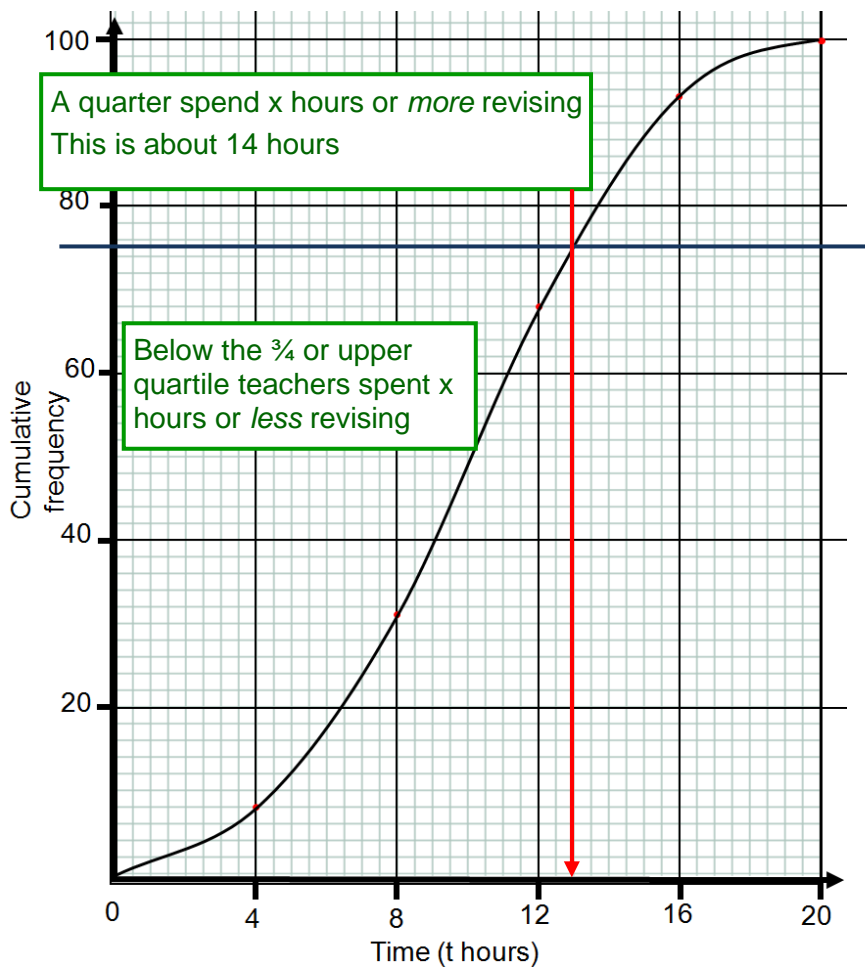
(2)

- c) One quarter of the trainee teachers *spent x hours or more* revising. Using the cumulative frequency graph estimate the value of x.

13-14

 ✓

(1)



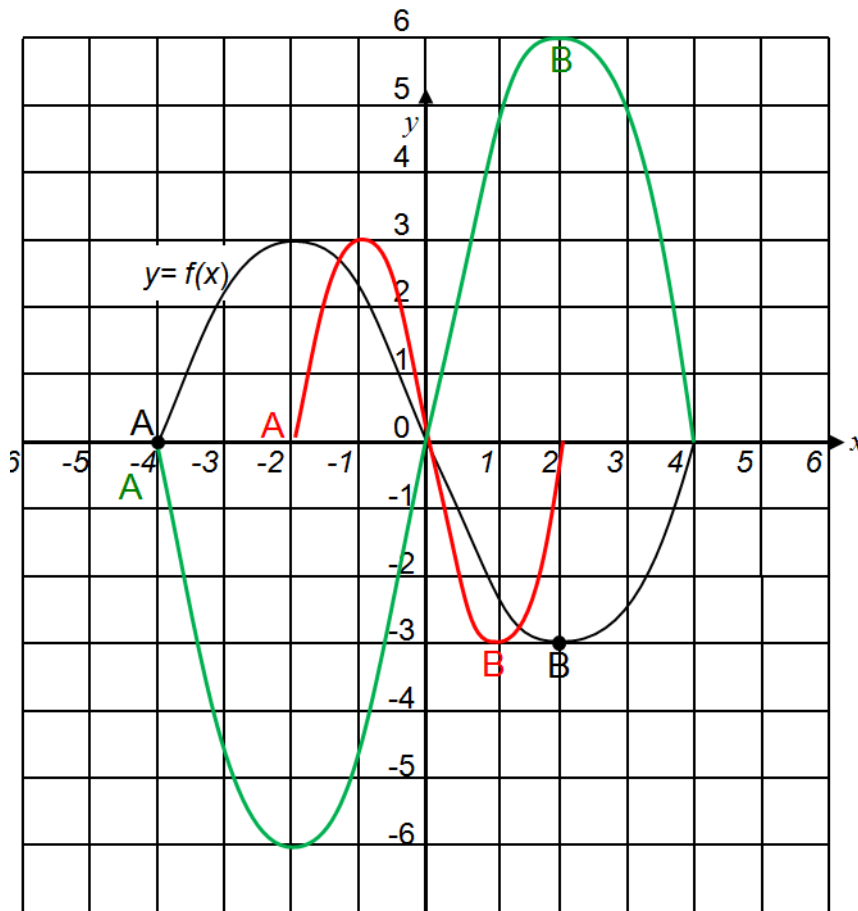
20. Prove that the recurring decimal $4.2\dot{7} = \frac{77}{18}$

The dot above the 7 means that number is repeated

$$\begin{aligned}
 4.277777 \text{ etc} \times 100 &= 427.77777 \text{ etc} \\
 4.277777 &\times 10 = 42.777777 \\
 4.277777 &\times 90 = 427.77777 - 42.777777 = 385 \\
 4.277777 &= \frac{385}{90} = \frac{77}{18}
 \end{aligned}$$

(3)

21. The diagram shows a sketch of $y = f(x)$.



(2)

- a) Sketch the graph of $y = f(2x)$ on the grid showing the co-ordinates of points A and B.

$f(2x)$ means scale factor *divided* by 2 in x-direction so graph squashed

A (**-2, 0**)
 B (**1, -3**)

- b) Sketch the graph of $y = -2f(x)$ on the grid showing the co-ordinates of points A and B.

$-2f(x)$ means scale factor multiplied by 2 in y-direction and graph also reflected in x-axis- so graph flipped vertically

A (**-4, 0**)
 B (**2, 6**)

(2)