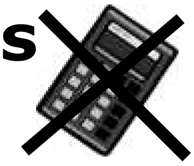
MathsMadeEasy

GCSE Mathematics



Non-Calculator
Higher Tier
Mock 1, paper 1
ANSWERS

1 hour 45 minutes

Legend used in answers

Blue dotted boxes – instructions or key points

Start with a column or row that has only one number missing

Green Box - Working out

5b means five times b
b = -3 so 5 x -3 = -15

Red Box and ✓ - Answer

48 % ✓

24

Marks shown in brackets for each question (2)

Authors Note

Answer ALL questions.
Write your answers in the spaces provided.
Do NOT use a Calculator

You must write down all stages in your working.

1. $4y^2 = 256$

a) Find a value for *y*

$y = \boxed{8}$ ✓……

(2)

1. Divide both sides by 4 to get rid of the 4 on left

$\frac{\cancel{4}y^2}{\cancel{4}} = \frac{\cancel{256}^{64}}{\cancel{4}}$

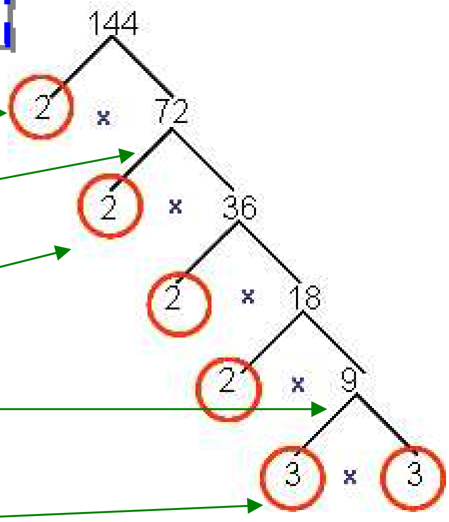
$y^2 = 64$

$\sqrt{y^2} = \sqrt{64}$

$y = 8$

b) Express 144 as a product of its prime factors

Make a Prime factor **TREE** – put your number at the top and constantly divide by prime numbers as shown.



The first prime numbers to try is 2

144 ÷ 2 = 72

72 also divides by 2

9 won't divide by 2 so try next prime, 3

Prime numbers at the end so we have finished
Multiply all the prime numbers circled

144 = 2 x 2 x 2 x 2 x 3 x 3 ✓..

(3)

2. a) Complete the table of values for $y = x^2 - x - 2$ below

x	-3	-2	-1	0	1	2	3
y	10	4	0	-2	-2	0	4

(-2) x (-2) - (-2) - 2 = +4 + 2 -2 = 4

(0)x(0) - (0) - 2 = -2

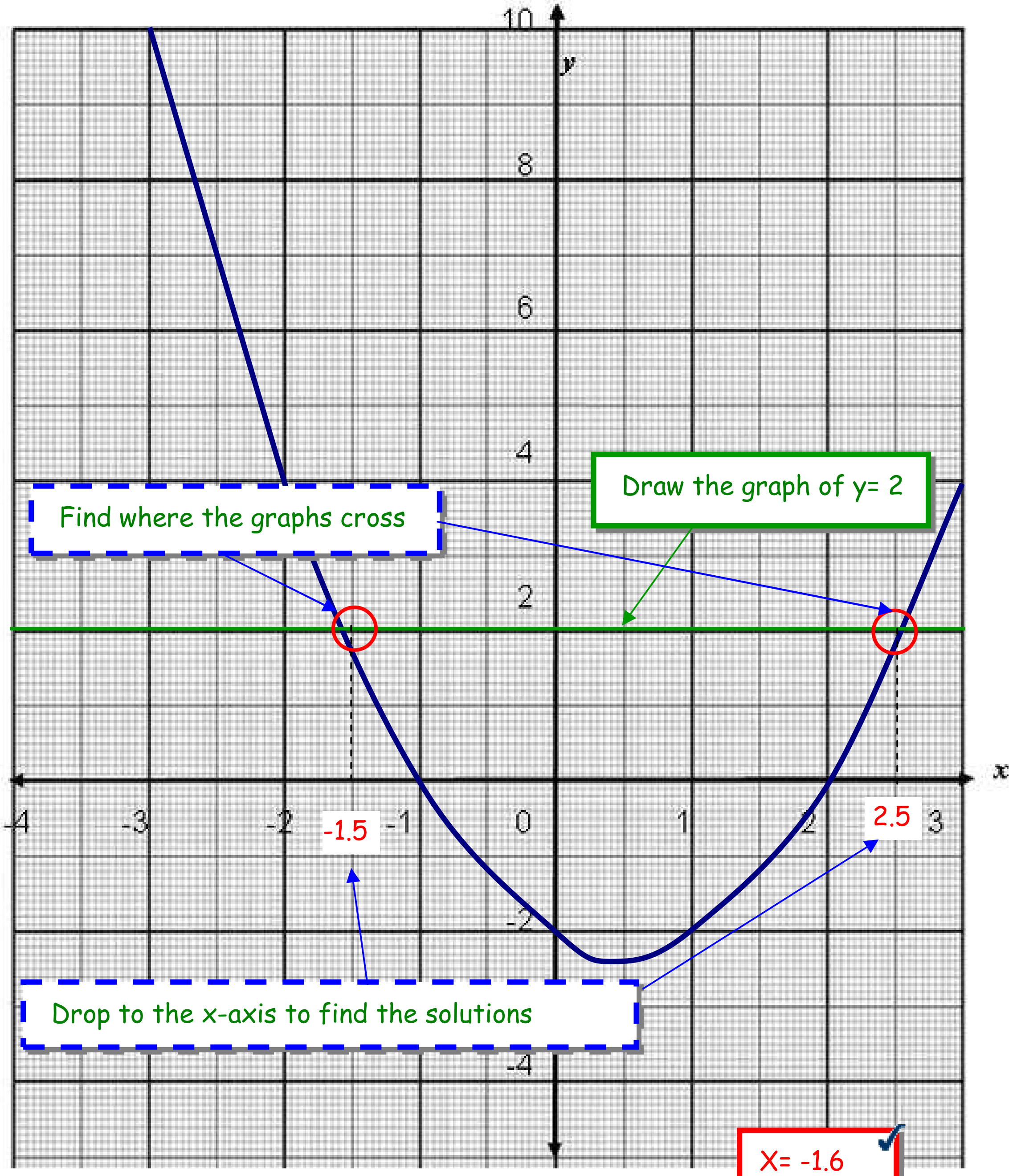
(2) x(2) - (2) - 2 = 0

x	x^2	$-x$	-2	y
-2	4	+2	-2	4
0	0	0	-2	-2
2	4	-2	-2	0

Use a table to work out y.
Don't forget - x - = +

(2)

b) Draw the graph for $y = x^2 - x$ -2 on the grid below **(2)**



Find where the graphs cross

Draw the graph of y= 2

Drop to the x-axis to find the solutions

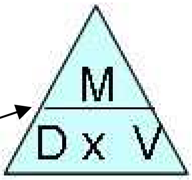
c) Use your graph to estimate the values of x when $y = 2$ x = ……… X= -1.6 X = 2.6 …..…**(2)**

3. A glass ball has a volume of 15 cm^3

The density of glass is 2.5grams per cm^3

Work out the mass of the glass ball

Use the mass density time triangle
Mass = Density x Volume

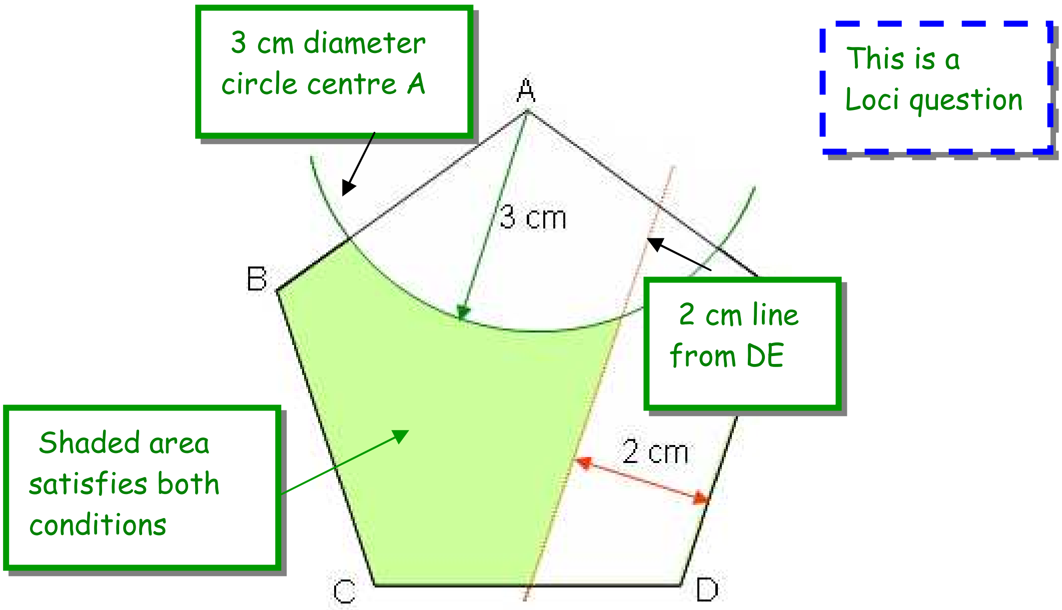


37.5 ✓ grams
(2)

Prime Mass = $15\ \cancel{cm^3} \times \frac{2.5\ gm}{\cancel{cm^3}} = 37.5\ gm$

Note how the units cancel

4.



3 cm diameter circle centre A

This is a Loci question

2 cm line from DE

Shaded area satisfies both conditions

ABCDE is a pentagon
Shade the area inside the pentagon which is both

more than 3 centimetres from A **and**
more than 2 centimetres from the line DE

(4)

5. David did a survey of the time in hours, people spent watching TV in a week.
He recorded his results in the following table.

Time (t hours)	Frequency
$0 < t \le 5$	10
$5 < t \le 10$	13
$10 < t \le 15$	16
$15 < t \le 20$	12
$20 < t \le 25$	9

A person is selected at random from David's survey

$15 < t \le 20$	12
$20 < t \le 25$	9

All these people watched > 15 hours of TV = 12 + 9 = 21

Total number people in survey is 10+ 13+16+ 12 + 9 = 60

Probability > 15 hrs TV = $\frac{\text{number people watching > 15 hrs TV}}{\text{Total people watching}}$

Probability > 15 hrs TV = $\frac{21}{60}$

P(>15hrs) = $\frac{21}{60}$ ✓

…………..…
(2)

6. Estimate the following:

$$\frac{809 \times 1.912}{0.395}$$

Estimate by rounding the numbers

809 is about 800;
1.912 is about 2
0.395 is about 0.4

CANCEL to make it easier

$\frac{800 \times 2}{0.4}$ → $\frac{800 \times \cancel{2}^{1}}{\cancel{0.4}^{0.2}}$ → $\frac{\cancel{800}^{400} \times 1}{\cancel{0.2}^{0.1}}$

$\frac{400}{0.1} = 4000$ ✓

…..
(3)

7. A glass ball has a mass of 258grams correct to the nearest gram.

a) What is the **greatest** possible mass for the ball?

If the glass Ball is 258.4gms to nearest gm we round down to 258gm
If ball is 258.5 gm strictly to nearest gm we round up to 259 gm
but this is allowed as the answer (or 258.499)

258.5 gm ✓

………grams
(1)

b) What is the **least** possible mass for the ball?

If the glass Ball is 257.4gms to nearest gm we round down to 257gm
If ball is 257.5 gm to nearest gm we round up to 258 gm

257.5 gm ✓

………grams
(1)

8. Laura wanted to know how much time students spent watching TV programs.

She used the question below on her questionnaire.

"How much TV did you watch this week?"

☐ Not much ☐ Quite a lot

This question is not good.

Design a better question that Laura can use to find out how much time students spend watching TV programs. Include some response boxes.

How much time have you spent watching TV this week? ✓

Make the options specific: ✓

II	IIII	~~IIII~~ III	IIII	II
None	1-5 hours	6-10 hours	11-15 hours	More than 15 hours

(2)

9. Write the following in standard form

Example 2.6×10^{4}

Standard form saves you writing lots of zeros

Only one number digit before decimal point

4 is the number of zeros

Count the number of decimal point jumps to get this

5 4 3 2 1

a) 6 . 1 8 0 0 0

Start at the end

We made 5jumps left so we get 10^{+5}

6.18×10^{5} ✓

………………………… (1)

Jump over digits going left until you are just before first digit

Note that we forget the +

Start at the decimal point

This time we reverse the jumping process – starting at the decimal point

1 2 3 4 5

b) 0 . 0 0 0 0 5 . 6

Jump over digits going Right until you are just before last digit

We made 5jumps Right so we get 10^{-5}

5.6×10^{-5} ✓

………………………… (1)

c) 18×10^{5}

1.8×10^{6} ✓

………………………… (1)

This is NOT standard form as there are two digits

Make one jump to the left and add that to the 10^{5} to get 10^{6}

10. a) Factorise $x^2 + 7x + 10$

This is a Quadratic equation because it has x^2 , x and a number. Factorising here means two sets of brackets

1. Start with:
(x A)(x B)

2. To find A and B look at the quadratic

3. A and B are two numbers which

$x^2 + 7x + 10$

make +7 by adding or subtracting

and multiply to make +10

Quadratics - find two numbers which multiply to make end number and can also make the middle one.

10 = 5x2 and
7 = 5 + 2

So A is 5
and B is 2

So (x + 5)(x + 2)

....

(2)

b) Solve $x^2 + 7x + 10 = 0$

$x^2 + 7x + 10 = (x + 5)(x + 2)$

So (x + 5)(x + 2) = 0

So either (x + 5) = 0
or (x + 2) = 0

If x + 5 = 0, x = -5
If x + 2 = 0, x = -2

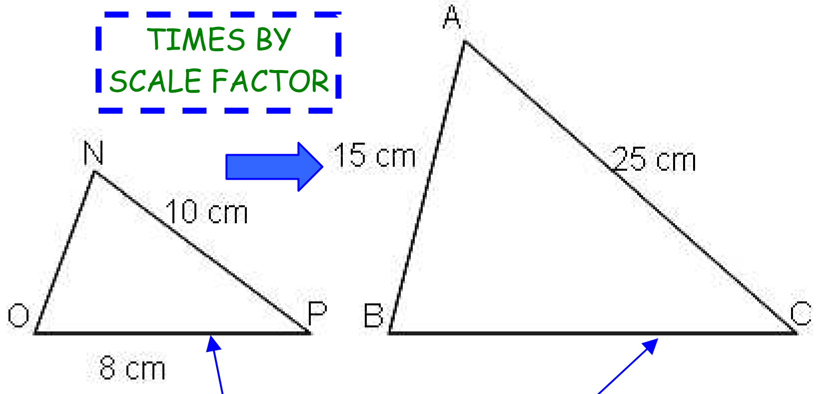
x= -5, x = -2

x =...........
x =...........

(1)

11.

TIMES BY SCALE FACTOR



Diagrams NOT drawn accurately

ABC is just a bigger version (scaled up) of NOP

To find the scale factor use sides from each triangle that are the similar sides.

The two triangles NOP and ABC are mathematically similar.
Angle N = angle A
Angle P = angle C
OP = 8 cm; NP = 10 cm
AC = 25 cm; AB = 15 cm

If two shapes are similar, one is an enlargement of the other with the same angles

a) What is the length of BC

When we are getting BIGGER, work out the scale factor this way

$$\text{Scale factor} = \frac{\text{Big side}}{\text{Small side}} = \frac{25}{10}$$

Side BC can be found from side OP using the Scale Factor

$$BC = OP \times \frac{25}{10} = 8 \times \frac{\cancel{25}^{5}}{\cancel{10}^{2}} = \frac{40}{2}$$

20 ✓

……………………cm
(2)

b) What is the length of NO?

When we are getting Smaller work out the scale factor this way

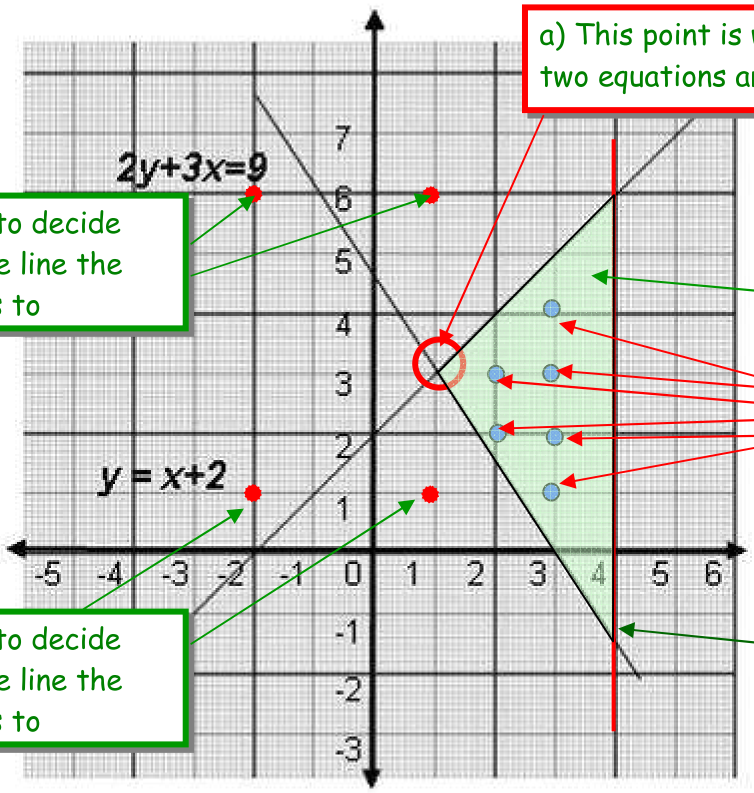
$$\text{Scale factor} = \frac{\text{Small side}}{\text{Big side}} = \frac{10}{25}$$

Side NO can be found from side AB using the Scale Factor

$$NO = AB \times \frac{10}{25} = 15 \times \frac{\cancel{10}^{2}}{\cancel{25}^{5}} = \frac{30}{5}$$

.. 6 ✓ ..cm
(2)

12. On the grid below are two straight lines with equations 2y + 3x = 9 and y=x +2.



a) This point is where the two equations are equal

Use two points to decide which side of the line the inequality refers to

This area satisfies all inequalities

These 6 integer points satisfy all inequalities ✓

Use two points to decide which side of the line the inequality refers to

Draw x=4 Points are to the left of it

a) Using the graphs find the solution to the simultaneous equations

$$2y + 3x = 9$$
$$y = x + 2$$

Simultaneous equations are Two equations which have the same answers for x and y

On the graph the only place where x and y are equal is where they cross at (1,3)

x = … 1 ✓ ……

y = … 3 ……

(1)

b) 2y + 3x > 9   y< x + 2   x<4   and x and y are integers

Find and mark the six points on the grid which satisfy all these three inequalities

(3)

These are plotted already. But you need to know which side of the line the inequality refers to

For y < x + 2 use two points to decide

Point (-2, 1)
y < x+ 2 → 1 < -2 + 2
FALSE

Point (1, 1)
y +< x + 2 → 1 < 1 + 2
TRUE

Inequality y < x + 2 = points below the line

For y < x + 2 use two points to decide

Point (-2, 6)
2y +3x >9→ 12 -6 >9
FALSE

Point (1, 6)
2y + 3x > 9→ 12+3>9
TRUE

Inequality 2y+ 3x > 9 = points to right of line

Plot the line x<4

Points are to Left of line

13. Jane’s weekly pay this year is £360.
This is 25% more than her weekly pay last year.

Matthew says “Jane’s weekly pay last year must have been £270”.
Matthew is wrong

a) Explain why

25% of £360 is £90 and £360 - £90 = £270! ✓
But we needed to do 25% of whatever her
weekly pay was last year, NOT this year

(1)

b) Work out Jane’s weekly pay last year.

Let’s say she was earning £A last year.
Then she got a 25% increase or 25% x A

£ 288 ✓

(2)

We can simplify this as: £A + 0.25x£A
and factorise this to get £A (1 + 0.25) = £A x 1.25

This equals what she is earning this year.
so £A x 1.25 = £360

Solve this equation by dividing both sides
By 1.25

$$\frac{£A \times \cancel{1.25}}{\cancel{1.25}} = \frac{£360}{1.25} = \frac{360}{5/4}$$

$\frac{360}{5/4}$ is the same as $\cancel{360}^{72} \times \frac{4}{\cancel{5}}$

and easy to cancel by 5

GENERALLY
When you have a question like this just divide the amount you have now by 1 + (percentage as a decimal) to get last years amount.
i.e. if you have £450 and it is 50% (0.5) more than last year, last year you had £450 ÷ 1.5 = £300

14. A survey of 80 children was made to see how long they spent playing computer games in a week

The table below shows how long in hours the children spent.

Time (t hours)	Frequency
$5 \le t < 10$	10
$10 \le t < 15$	16
$15 \le t < 20$	30
$20 \le t < 25$	21
$25 \le t < 30$	3

a) Complete the cumulative frequency table

Cumulative means add find a new total as you go along by adding on each new number

(1)

Time (t hours)	Cumulative Frequency
$5 \le t < 10$	10
$5 \le t < 15$	**26**
$5 \le t < 20$	**56**
$5 \le t < 25$	**77**
$5 \le t < 30$	**80**

26 is 10 + 16

56 is 26 + 30

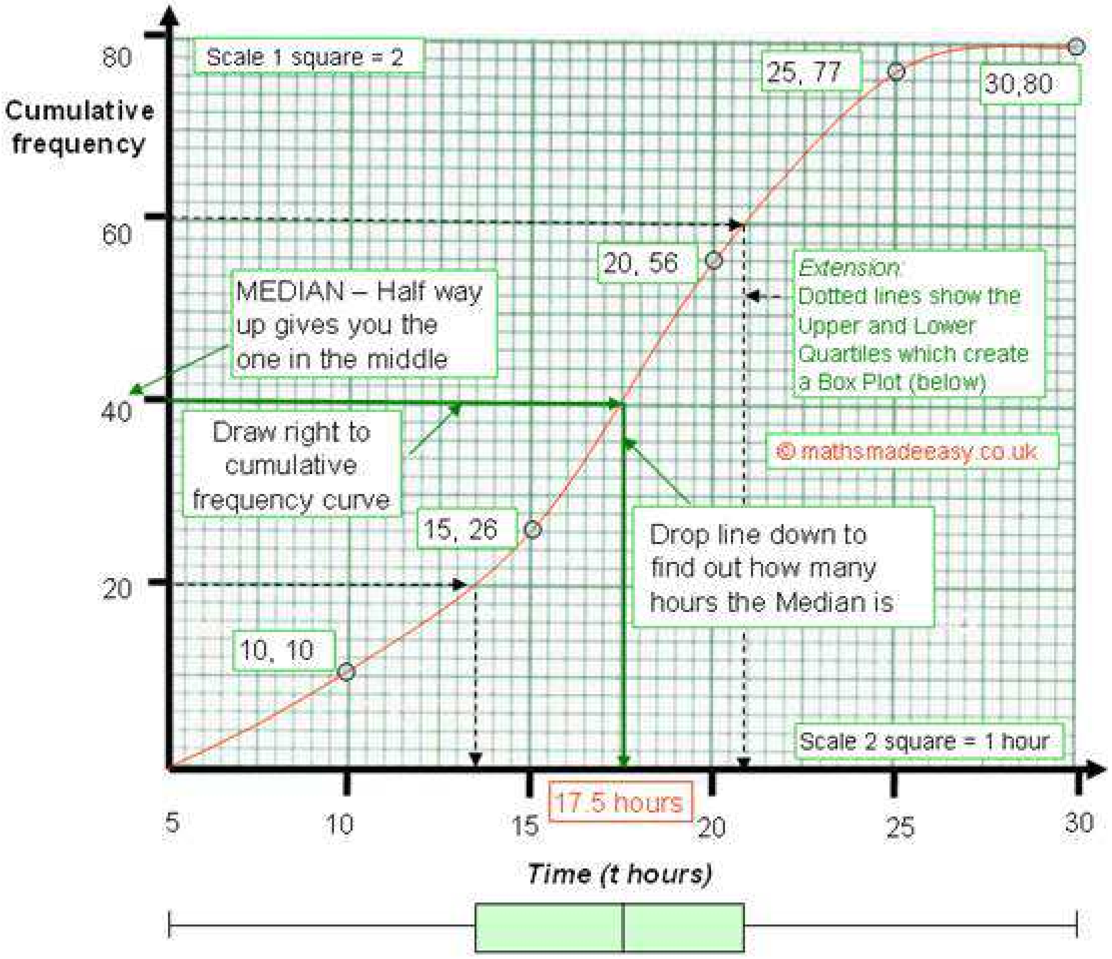
Notice that we start from 5 each time

Here we want everything between 5 and 20

b) Using your completed table draw a cumulative frequency graph on the grid

(2)

When plotting points use the **upper number** in each time interval e.g. plot 30, 80 for last point



c) Using the completed graph estimate the median time
Remember to state the units in your answer

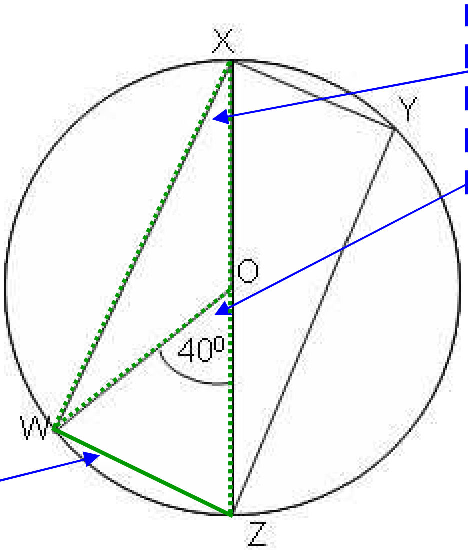
17.5 hours ✓

(2)

17 to 18 hours are also OK

15.

Diagram NOT
drawn accurately



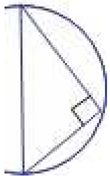
From the same arc, the angle at the centre is twice the angle at the circumference

Arc

W, X, Y and Z are points on the circumference of a circle, centre O.
XOZ is a straight line and angle WOZ is 40^0

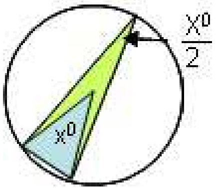
a) What is the size of angle XYZ giving a reason for your answer?

The angle in a semi-circle is always 90^0



....90.....^{0}
(2)

b) What is the size of angle WXZ giving a reason for your answer?



From same points on a circle the angle at circumference is twice that at centre

....20.....^{0}
(2)

16. The resistance R ohms of a wire is directly proportional to the length l cm of the wire

When $l = 150$, $R = 750$

a) Find R when $l = 450$

If A is *directly proportional* to B it is written as A / B

It can be made into an equation by using a constant k.
So A / B becomes A = k x B

R / l → R = k x l

You can find the constant k from the values they give you in the question. → 750 = k x 150 → $k = \frac{\cancel{750}^{5}}{\cancel{150}}$

Replace K with the value found. Then use the new value given → R = 5 x l → R = 5 x 450 = 2250

R = ………2250 ✓……… (3)

The resistance R ohms of a wire is inversely proportional to the cross sectional area A cm^2 of the wire.

When $A = 0.1$, $R = 180$

a) Find R when $A = 0.09$

If A is *inversely proportional* to B it is written as A / 1/B

Make it into an equation by using a constant k. So $A = \frac{k}{B}$

You can find the constant k from the values they give you in the question. → $R = \frac{k}{l}$ → k = 180 x 0.1 = 18

Replace K with the value found. Then use the new value given → $R = \frac{18}{0.09} = \frac{\cancel{1800}^{200}}{\cancel{9}^{1}} = 200$

200 ✓

R = ……………………………… (3)

17.

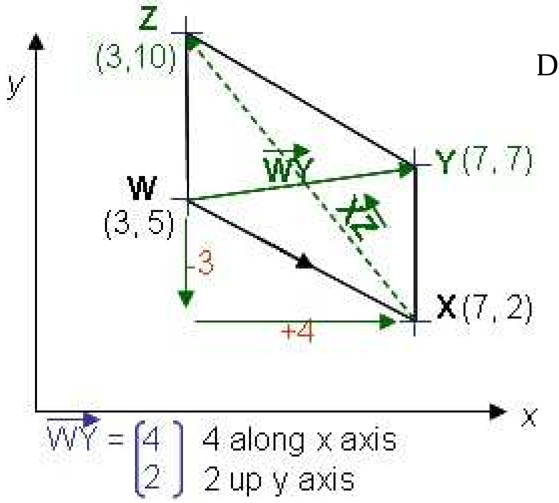


Diagram not drawn accurately

The diagram above shows two points W and X
W is the point (3, 5)
X is the point (7, 2)

The column vector tells you how far you move along the x and y axis

a) Write down the vector $\overrightarrow{WX}$ as a column vector $\begin{pmatrix}x\\y\end{pmatrix}$

From W to X we move +4 along the x-axis and -3 along the y-axis

$\begin{pmatrix}+4\\-3\end{pmatrix}$ ✓

(2)

WXYZ is a parallelogram

$\overrightarrow{WY} = \begin{pmatrix}4\\2\end{pmatrix}$

This gives us point Y from point W (3,5)
By increasing *x* by 4 and increasing *y* by 2

Point Y is (3, 5) + (4, 2) → (7, 7)

b) Find the vector $\overrightarrow{XZ}$ and write as a column vector $\begin{pmatrix}x\\y\end{pmatrix}$

To find point WZ we need point Z.
Use vector WZ to find Z. It will be the same as XY since we have a parallelogram

XY goes from (7,2) to (7, 7) → $\begin{pmatrix}0\\5\end{pmatrix}$ so from W (3, 5) we keep the same x co-ordinate but increase the y co-ordinate by 5 to get to point Z

Point Z is (3, 5) + (0, 5) → (3, 10)

Vector XZ is from (7, 2) to (3, 10) → (-4, 8)

$\begin{pmatrix}-4\\+8\end{pmatrix}$ ✓

……………

(2)

18. a) Solve $\frac{5}{4a} + \frac{5}{a} = 4$

Treat this as though it was like adding fractions by finding a common denominator

$$\frac{5}{4a} + \frac{5}{a} = \frac{5a}{4a^2} + \frac{20a}{4a^2} = \frac{25a}{4a^2} = 4$$

One way to find a new denominator is to multiply both old denominator together i.e. 4a x a = $4a^2$

The numbers at the top must also be changed. Cross multiply to get the ones.

$$\frac{25\cancel{a}}{4\cancel{a^2}^{a}} = 4 \rightarrow \frac{25}{4a} = 4 \rightarrow \frac{25}{16} = a$$

Cancel the a's

Rearrange to get *a* on the right and numbers on left

a = $\frac{25}{16}$ ✓ ……………………

(2)

c) Using your answer to part (a) or otherwise,

Solve $\frac{5}{4(b-1)^2} + \frac{5}{(b-1)^2} = 4$

If it says using your previous answer look for similarities between the two

Notice this is like the first part except *a* has been replaced by $(b - 1)^2$

we found a = $\frac{25}{16}$ → So $(b - 1)^2 = \frac{25}{16}$ → So $\sqrt{((b-1)^2)} = \frac{\sqrt{25}}{\sqrt{16}}$

So $(b - 1) = \frac{\pm 5}{\pm 4}$ → So $b = \frac{\pm 5}{\pm 4} + 1$ → So $b = \frac{+5}{4} + 1$ or $\frac{-5}{4} + 1$

b = ….. $\frac{9}{4}$ or $\frac{-1}{4}$ ✓

or b = …..

(3)

19. The table and histogram show information about the time it took 235 students to complete their homework.

Time (t minutes)	Frequency
$20 < t \leq 40$	40
$40 < t \leq 50$	50
$50 < t \leq 55$	45
$55 < t \leq 57.5$	30
$57.5 < t \leq 60$	25
$60 < t \leq 75$	45

Work out the vertical scale on histogram using given frequencies

Remember
Freq = class width x Freq density

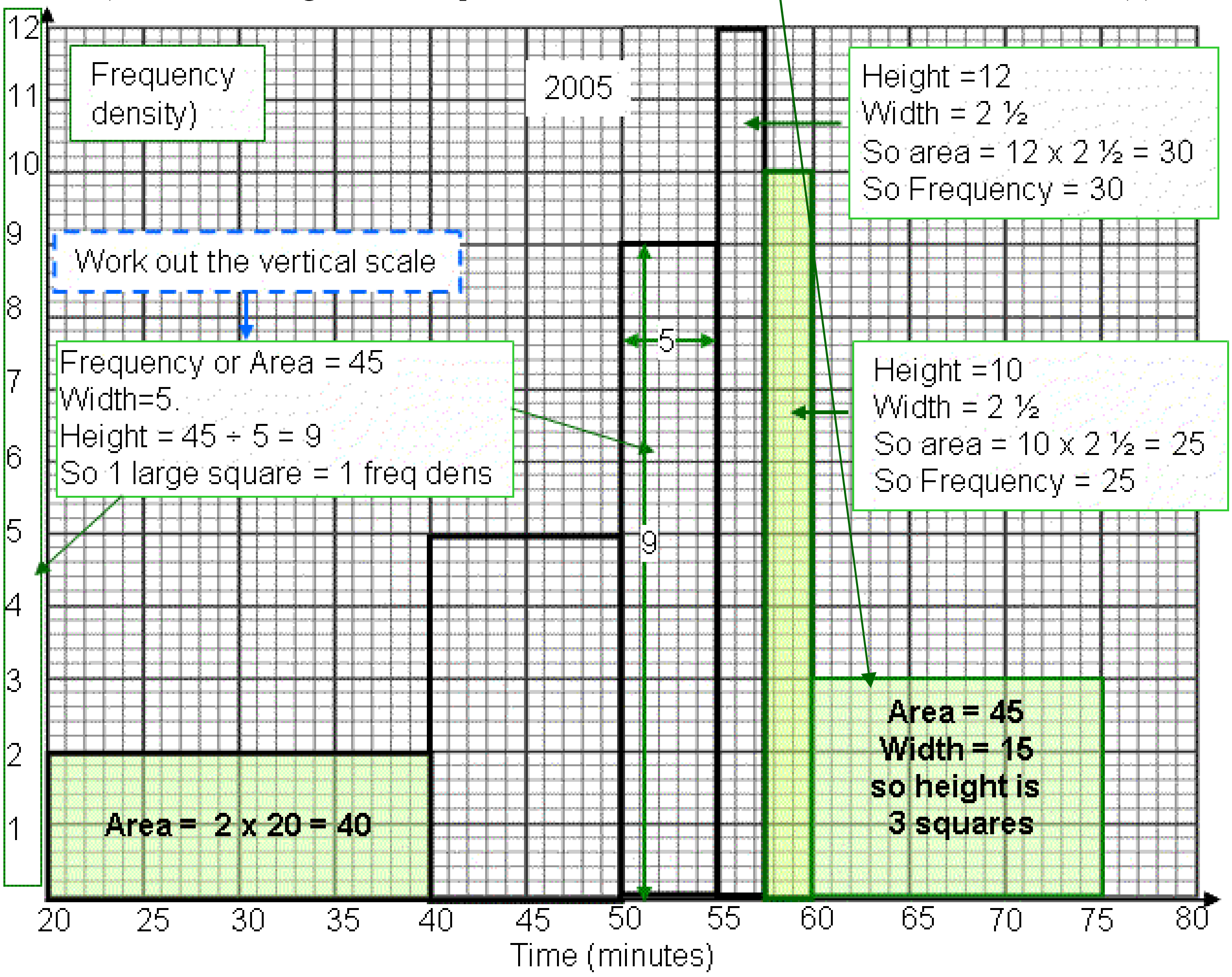
Freq density scale
1 large square = 1

1^{st} bar has area 40
So freq = 40

None of the students took longer than 75 minutes.

a) Use the table to complete the histogram (2)

b) Use the histogram to complete the table (2)



Work out the vertical scale

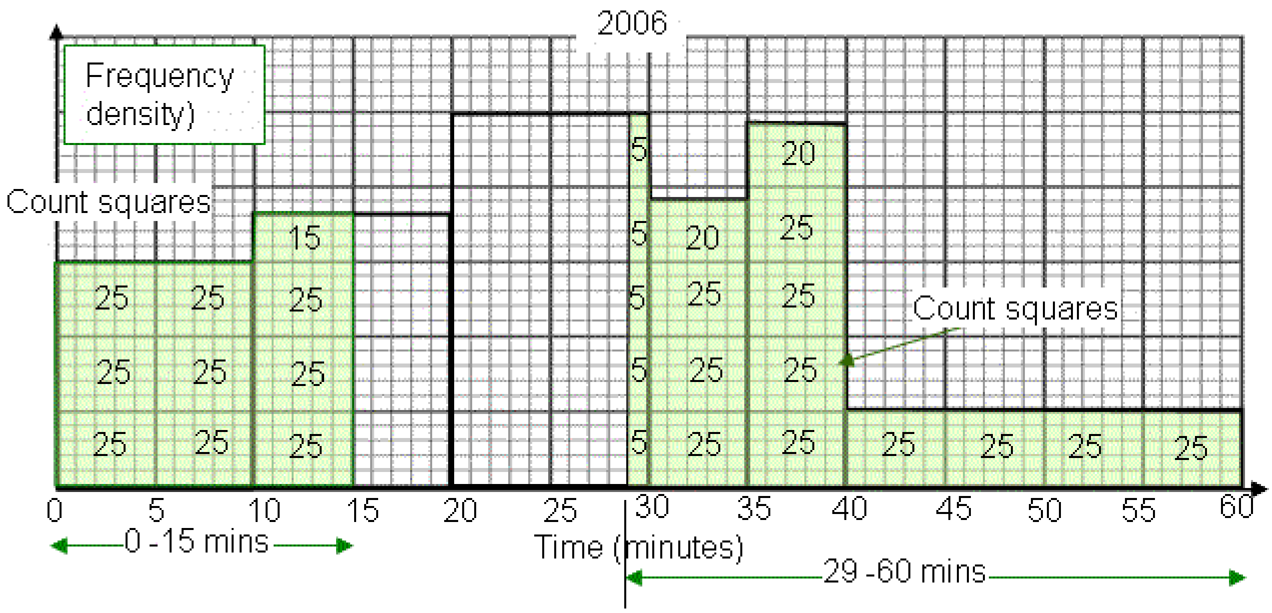
Frequency or Area = 45
Width=5.
Height = 45 ÷ 5 = 9
So 1 large square = 1 freq dens

Height =12
Width = 2 ½
So area = 12 x 2 ½ = 30
So Frequency = 30

Height =10
Width = 2 ½
So area = 10 x 2 ½ = 25
So Frequency = 25

Area = 2 x 20 = 40

Area = 45
Width = 15
so height is
3 squares



The histogram above shows information about the time it took some adults to read their e-mails at work.
None of them took more than 60 minutes.

120 people took up to 15 minutes to read their e-mails.

c) Using the histogram work out an estimate for the number of adults who took 29 minutes or longer to read their e-mails

Count the number of small squares to the 15 minute point
There are 240 and represents 120 adults

So 2 small squares = 1 adult

Now count the number of small squares
from the 29 minute point up to 60

There are 340 small squares
This represents 170 adults

170 adults ✓

……………………..
(3)

20. a) Work out the value of $64^{\frac{1}{3}}$

Fractional Powers means Roots

$\frac{1}{2}$ means square root

$\frac{1}{3}$ means cubed root

What is the cubed root of 64 or what number times itself three times is 64 = 4 x 4 x 4

……… $\boxed{4}$ ✓ ….

(1)

b) $12\sqrt{12}$ can be written as 12^{m}
Find the value of m

12 on its own is like 12^{1} because any number to 1 is always itself

&

$\sqrt{12}$ can be written as $12^{1/2}$

So we have $12^{1} \times 12^{1/2}$

When you multiply powers we ADD them $12^{1} \times 12^{1/2} = 12^{3/2}$

m =.. $\boxed{\frac{3}{2}}$ ✓ ….

(1)

$12\sqrt{12}$ can also be expressed in the form $p\sqrt{3}$ where P is a positive integer

c) Express $12\sqrt{12}$ in the form $p\sqrt{3}$

$12\sqrt{12} = 12\sqrt{4x3} = 12 \times 2\sqrt{3} = 24\sqrt{3}$

Rule: $\sqrt{axb} = \sqrt{a} \times \sqrt{b}$

$\sqrt{4} = 2$ so take outside square root

……… $\boxed{24\sqrt{3}}$ ✓ .

(2)

Rationalise the denominator of $\frac{1}{12\sqrt{12}}$

To rationalise times by $\frac{\sqrt{12}}{\sqrt{12}}$

Give your answer in the form $\frac{\sqrt{3}}{m}$ where m is a positive integer

$$\frac{1}{12\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{12}}{12 \times 12} = \frac{\sqrt{(4 \times 3)}}{12 \times 12} = \frac{\cancel{2}^{1}\sqrt{3}}{\cancel{12}^{6} \times 12} = \frac{\sqrt{3}}{72}$$

. $\boxed{\frac{\sqrt{3}}{72}}$ ✓ ………

(2)

21.

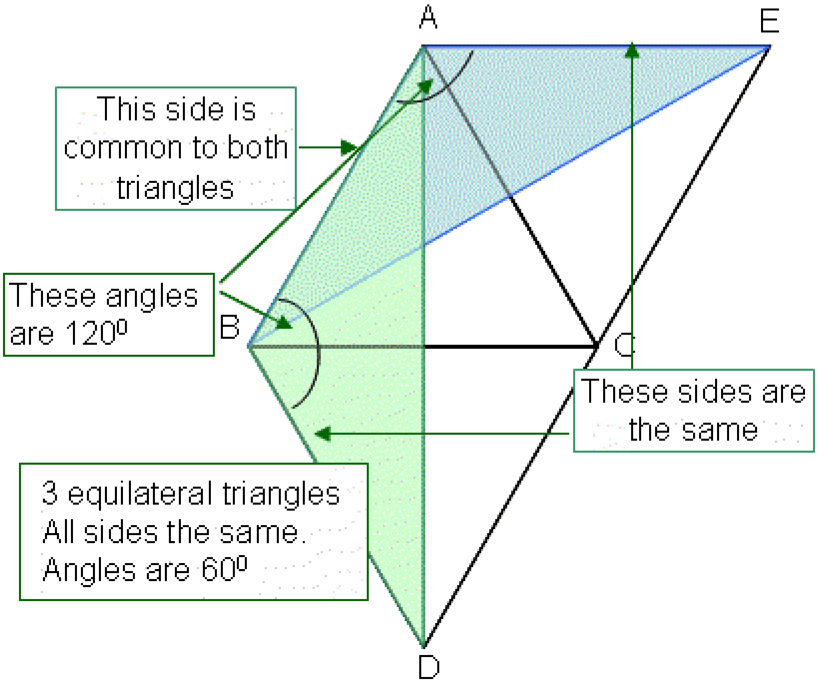


Diagram NOT
drawn accurately

ABC is an equilateral triangle
BDC is an equilateral triangle
AEC is an equilateral triangle

Congruent means the same

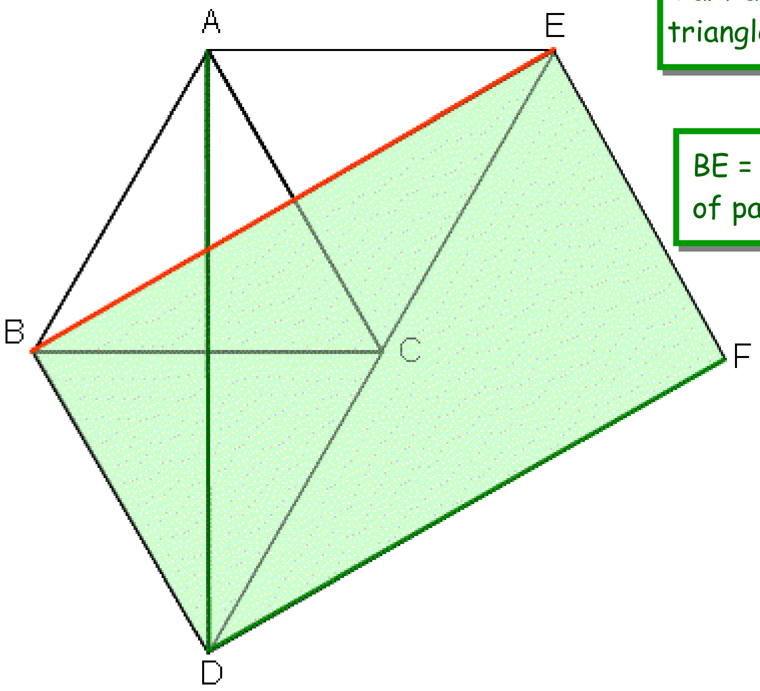
a) Prove that triangle ABE and Triangle ABD are congruent

Two sides are the same and one angle is, as shown.

……….
(3)

F is a point such that EBDF is a parallelogram.

b) Prove that DF = AD



Part a) showed BE = AD because triangles are congruent

BE = DF because opposite sides of parallelogram are equal

So BE=DF=AD or DF = AD

……………………
(2)

22.

$$y = \frac{x^2 - 2z}{x + 2z}$$

Rearrange the formula to make z the subject

1. Get the x+ 2z to the left side by multiplying both sides by x + 2z

$y \times (x + 2z) = \frac{x^2 - 2z}{\cancel{x + 2z}} \times \cancel{(x + 2z)}$

$y \times (x + 2z) = yx + 2yz = x^2 - 2z$

2. Move 2z to left side by adding 2z to both sides

$yx + 2yz + 2z = x^2 - 2z + 2z$

$yx + 2z + 2yz = x^2$

3. Move yx to right by taking yx from both sides

$yx + 2z + 2yz - yx = x^2 - yx$

$2z + 2yz = x^2 - yx$

4. Factorise left to isolate z and divide

$z(2 + 2y) = x^2 - yx$

$z\,(2 + 2y) = x^2 - yx$

z = … $\boxed{\frac{x^2 - yx}{2 + 2y}}$ ✓ …

(4)

23. a) Factorise $2x^2 - 10x + 8$

2 x 8 = 16

This is a harder quadratic equation because the value before the x^2 is not 1

1.Find two numbers which multiply to make 16

and also makes -10 by adding or subtracting

16 = -8 x -2 and -10 = - 8 - 2

2. Rewrite -10x as -2x - 8x in the equation

$2x^2 - 10x + 8 \rightarrow 2x^2 - 2x - 8x + 8$

Notice that we put -2x next to the $2x^2$ and -8x next to 8

3. Factorise $2x^2 - 2x - 8x + 8 \rightarrow 2x(x - 1) - 8(x - 1)$

4. Simplify $2x(x - 1) - 8(x - 1) \rightarrow (2x - 8)(x - 1)$

We know we are OK when factors are the same

$\boxed{(2x - 8)(x - 1)}$ ✓ ….

(2)

b) i) Factorise fully $(p^2 - q^2) - (p - q)^2$

1. work out this part first

x	p	-q
p	p^2	-pq
-q	-pq	$+q^2$

Using a grid is one approach - $p^2 - pq - pq + q^2 = p^2 - 2pq + q^2$

Substitute: $(p^2 - q^2) - (p^2 - 2pq + q^2) = 2pq - 2q^2$

Careful with the signs

Factorise: $2pq - 2q^2 = 2q(p - q)$

$2q(p - q)$ ✓ ..

p and q are integers

ii) Explain why $(p^2 - q^2) - (p - q)^2$ is always an even integer

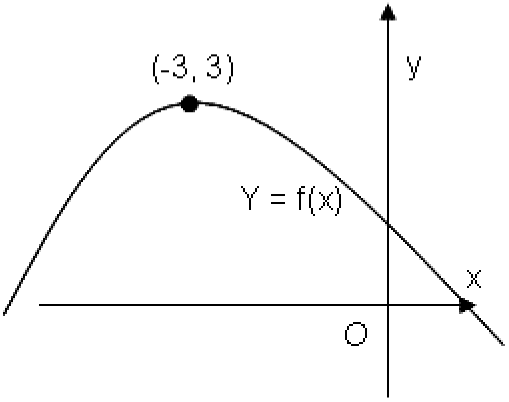
$2q(p - q)$ ....

....

If q is even and p is even we get even x (even - even) = even x even = even
If q is odd and p is odd we get (2*odd) even x (odd - odd) = odd x even = even
If q is odd and p is even we get (2*odd) even x (even - odd) = even x odd = even
If q is even and p is odd we get even x (odd- even) = even x odd = even

...
(4)

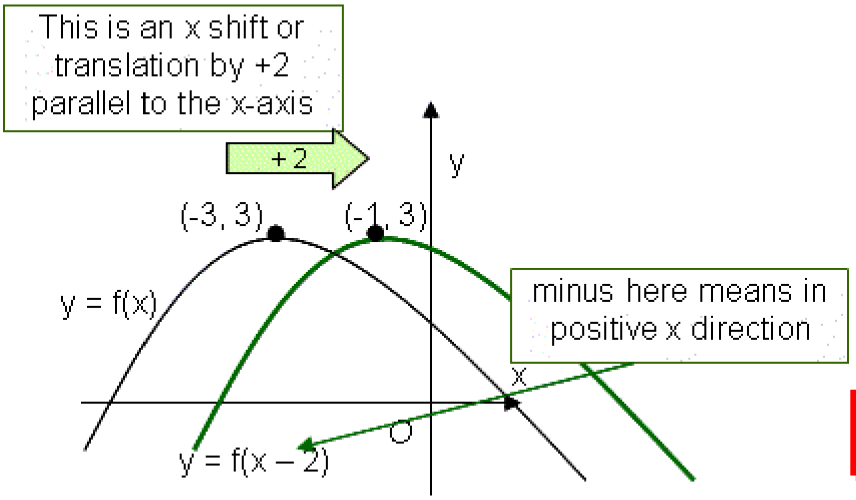
24.



The diagram above shows part of a curve with the equation y = f(x)
The maximum point is (-3, 3)

a) What are the co-ordinates of the maximum point of the curve with the equations below:

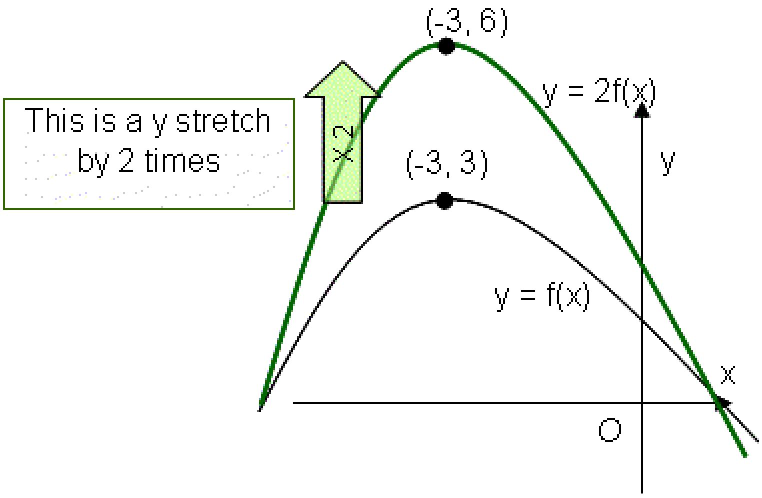
i) y = f(x – 2)



(-1,3) ✓

(1)

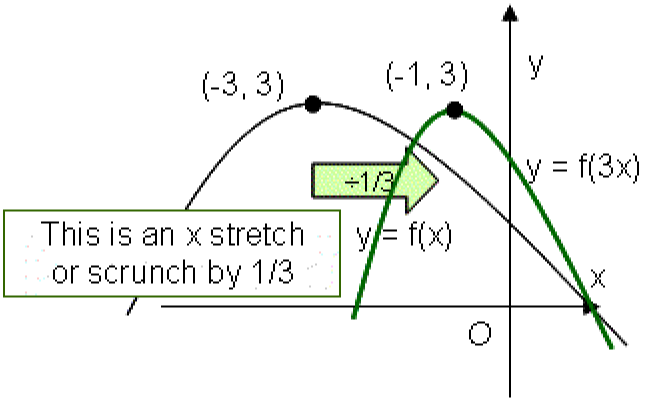
ii) y = 2f(x)



(-3,6) ✓

(1)

iii) $y = f(3x)$



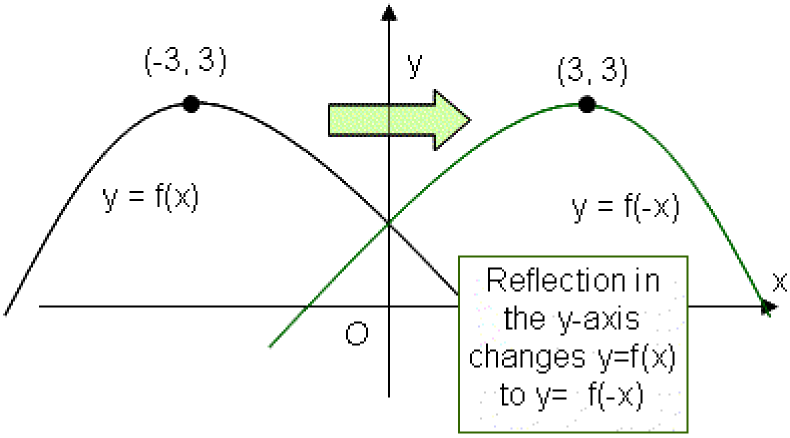
This is an x stretch or scrunch by 1/3

(-1,3) ✓

(1)

The curve $y = f(x)$ is reflected in the y axis

b) Find the equation of the curve after the reflection.



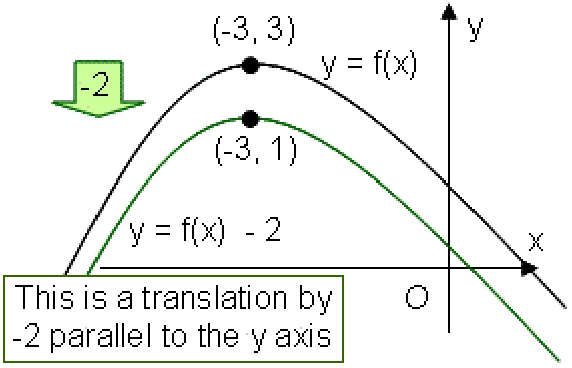
Reflection in the y-axis changes y=f(x) to y= f(-x)

y = f(-x) ✓ ……………………

(1)

The curve with the equation $y = f(x)$ has been transformed to the curve with the equation $y = f(x) - 2$

c) Describe the transformation



This is a translation by -2 parallel to the y axis

Translation by -2 parallel to y-axis ✓

(1)

TOTAL FOR PAPER: 100 MARKS

END