MathsMadeEasy

GCSE Mathematics

Non-Calculator

Higher Tier

Mock 3, paper 1

ANSWERS

1 hour 45 minutes

Legend used in answers

Blue dotted boxes – instructions or key points

Start with a column or row that has only one number missing

Green Box - Working out

5b means five times b
b = -3 so 5 x -3 = -15

Red Box and ✓ - Answer

48 % ✓

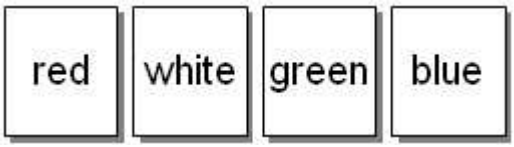
24

Marks shown in brackets for each question (2)

Authors Note

Every possible effort has been made to ensure that everything in this paper is accurate and the author cannot accept responsibility for any errors.

1. A box contains red, white, green or blue cards.



The table below show the probability of picking a particular coloured card if one is taken randomly from the box.

Colour	Red	White	Green	Blue
Probability	0.25	0.3		0.15

ALL probabilities add up to 1.

a) If a card is taken at random from the box what is the probability it will be green.

Probability (Green) = 1 – 0.25 – 0.3 – 0.15 = 0.3

..... 0.3 ... **(2)**

If the box contains 60 cards.

b) Work out how many white cards there are in the box.

Prob (white) = 0.3, so number of white cards = 60 x 0.3 = 18

18

..... **(2)**

2. In the sales David buys 2 pens and 4 pencils for £2.80
Jane buys 4 pens and 4 pencils for £4.00

What is the cost of one pen giving your answer in pence.

Looking at the question notice that the difference between what they bought (4 pens + 4 pencils) - (2 pens + 4 pencils) = 2 pens.
This difference in cost is £4.00 – £2.80 = £1.20
So 2 pens cost £1.20 → 1 pen costs £0.60

60

.....pence **(3)**

3. Stuart needs to collect information about the amount of time his friends spend playing computer games.
Design a suitable question he could use.

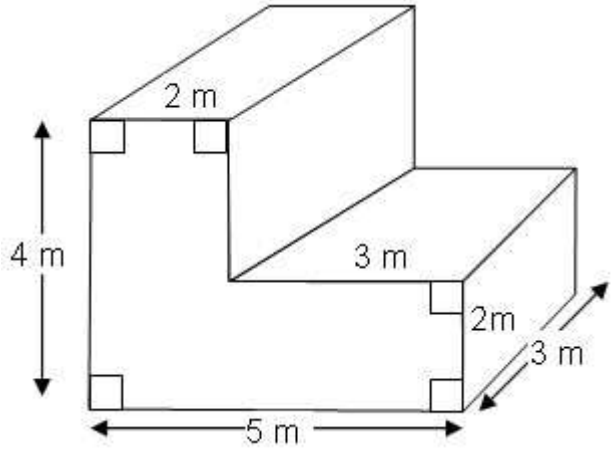
How much time have you spent playing computer games this week?

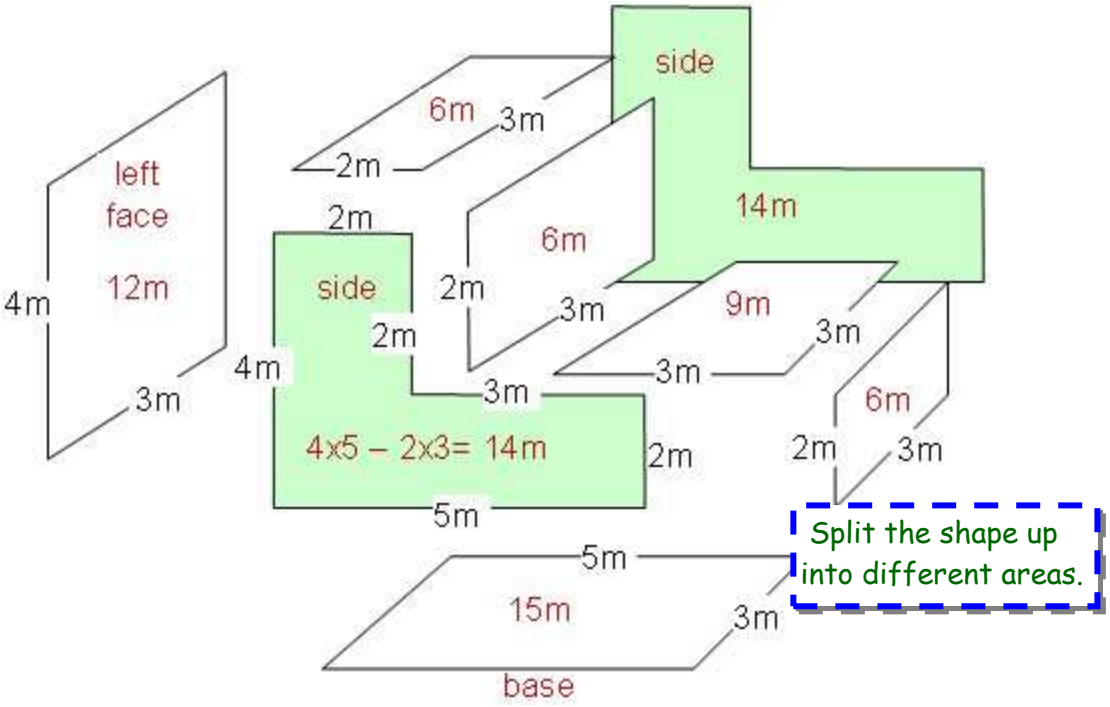
Make the options specific: use a tally

II	IIII	~~IIII~~ III	IIII	II
None	1-5 hours	6-10 hours	11-15 hours	More than 15 hours

(2)

4. Work out the total surface area of the L shaped block below,
Give the units in your answer.





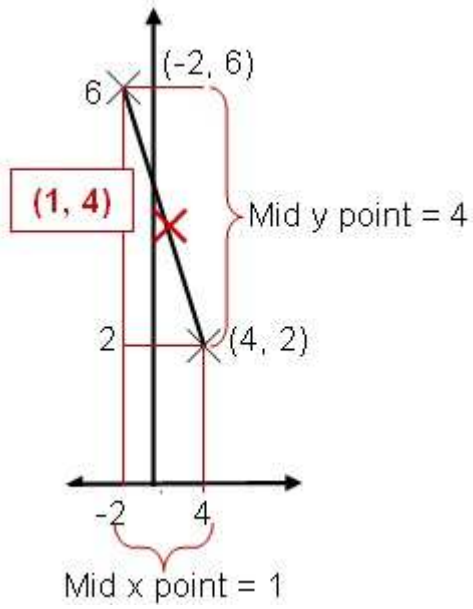
Total surface area = base + two sides + left face + stepped faces
= 15 + 14 + 14 + 12 + 6 + 6 + 9 + 6
= 82 cm^2

(4)

82 cm^2

.....

5. A line is shown below drawn between two co-ordinates (-2, 6) and (4, 2). Find the co-ordinates of the midpoint.



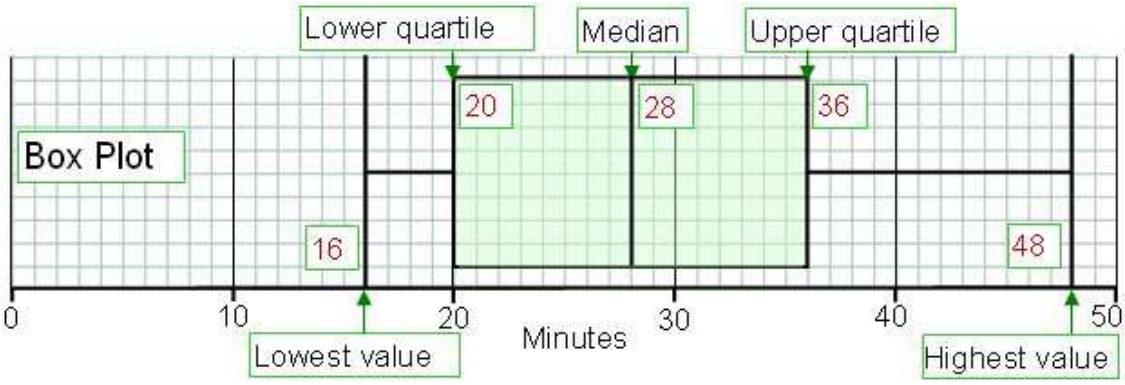
(1, 4)

(2)

6. Mrs Dew set her students some homework.
Each student recorded the time taken for them to do their homework.
Mrs Dew used this information to work out the following table.

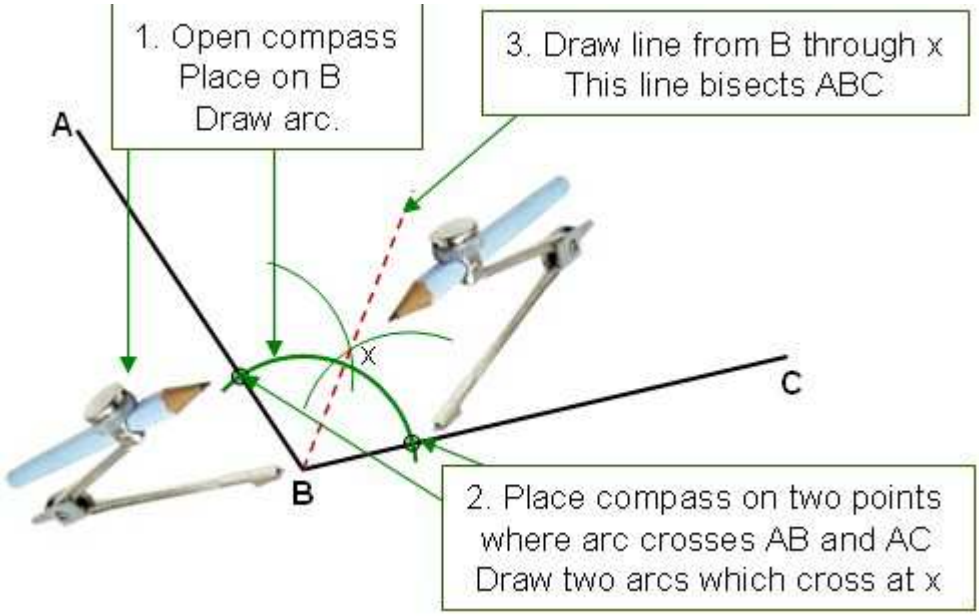
	Minutes
Shortest time	16
Lower quartile	20
Median	28
Upper Quartile	36
Longest time	48

Draw a box plot for this information on the grid below



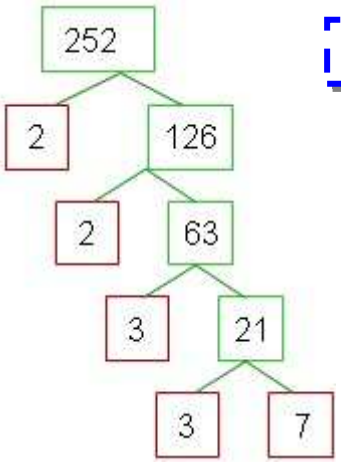
(2)

7. Construct a bisector of the angle ABC using a ruler and compasses.
Show all your construction lines



(2)

8. a) Write 252 as a product of its prime factors.



Make a Prime number tree

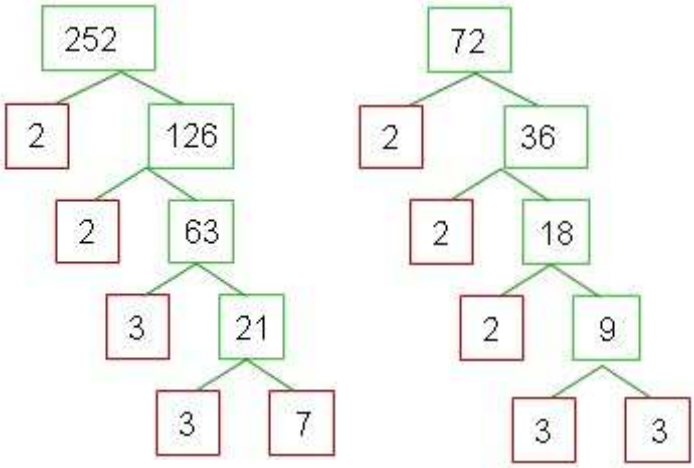
Keep dividing by the lowest prime number (2) until you have to go to the next prime number.

(2)

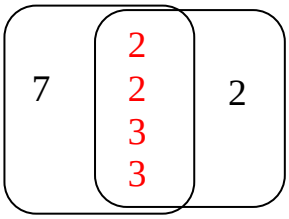
2 x 2 x 3 x 3 x 7

(2)

b) Find the highest Common Factor (HCF) of 72 and 252

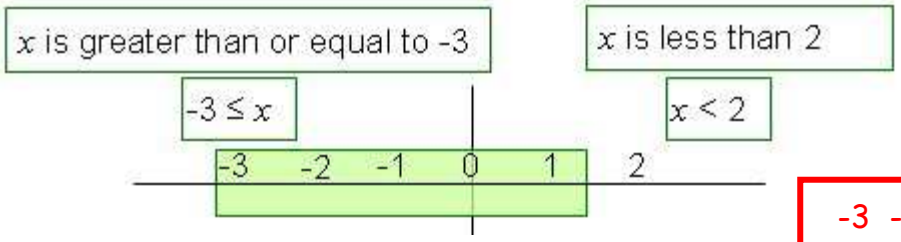


Use the prime factor trees to find the common factors.
Common factors in each of above are 2 x 2 x 3 x 3 = 36



36

9. (a) x is an integer where $-3 \leq x < 2$
Write down all the possible values of x.



-3 -2 -1 0 1 **(2)**

b) (i) Solve the inequality $4x \geq x + 5$

Treat this as though it was written as $4x = x + 5$
Subtract x from both sides → $4x = 5$
Divide both sides by 3 → $x = \frac{5}{3}$
Now replace the ≥ sign → $x \geq \frac{5}{3}$

$x \geq \frac{5}{3}$ .....

Write down the smallest whole value of x that satisfies $4x \geq x + 5$

2 ..... **(3)**

10. Write as a power of 6

a) $6^9 \div 6^7$

When powers divide subtract them.

$6^9 \div 6^7 = 6^{(9-7)} = 6^2$

6^2 ..... **(3)**

b) $\frac{6^7 \times 6^3}{6}$

When powers multiply add them.

$6 = 6^1$

$6^7 \times 6^3 = 6^{(7+3)} = 6^{10}$
$6^{10} \div 6^1 = 6^{(10-1)} = 6^9$

6^9 .....

c) Write down the reciprocal of 3

Reciprocal means one (1) over your number

$\frac{1}{3}$ .....

11. a) Make p the subject of the formula $s = 3p - 16$

Always do the same to both sides of the equation

Add 16 to both sides → $s + 16 = 3p - 16 + 16 \rightarrow s + 16 = 3p$

Divide by 3 → $s + 16 = 3p \rightarrow \frac{s + 16}{3} = \frac{3p}{3}$

$\frac{s + 16}{3}$

p =..... **(2)**

b) Make b the subject of the formula $3(b - 3c) = 2b + 6$

Expand the left side → $3b - 9c = 2b + 6$

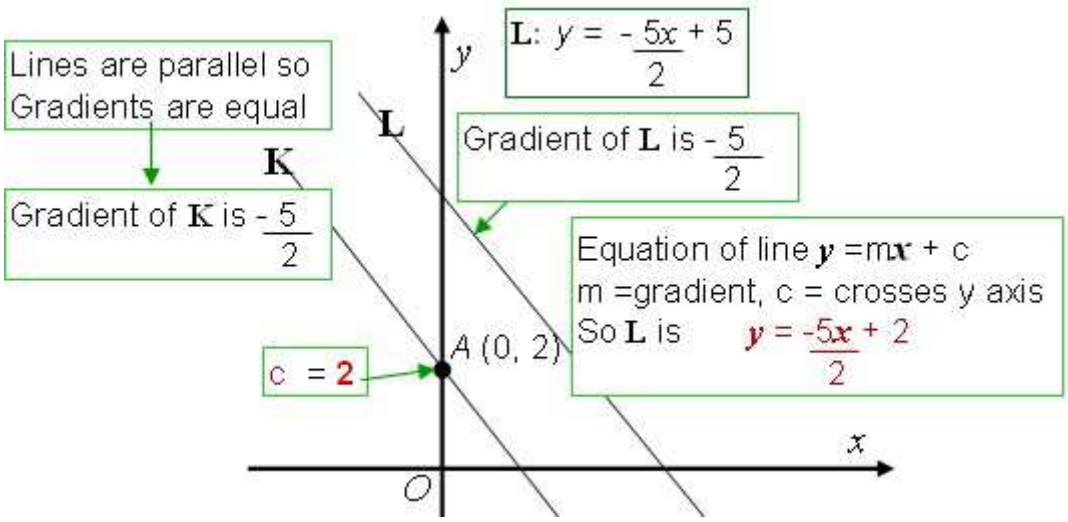
Get b's on left – subtract 2b from both sides → $3b - 2b - 9c = 2b - 2b + 6 \rightarrow b - 9c = 6$

Add 9c to both sides → $b - 9c + 9c = 6 + 9c \rightarrow b = 6 + 9c$

$b = 6 + 9c$ ✓

b =..... **(3)**

12.



The straight line **L** has the equation $y = -\frac{5}{2}x + 5$

The straight line **K** is parallel to **L** and passes through the point (0,2)
Write down the equation for the line **K.**

$y = -\frac{5}{2}x + 2$

..... **(2)**

13. (a) Work out the value of $1\frac{3}{4} \times 2\frac{4}{7}$

Give your answer as a fraction in its simplest form.

Only add the top numbers
You have 7 quarters

Convert both to an improper fraction

$1\frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4}$

$2\frac{4}{7} = \frac{14}{7} + \frac{4}{7} = \frac{18}{7}$

Now multiply both together

$\frac{\cancel{7}}{4} \times \frac{18}{\cancel{7}}$

$\frac{\cancel{18}^{9}}{\cancel{4}_{2}}$

$\frac{9}{2}$ ✓

(3)

.....

14. Solve the simultaneous equations

$$6x - 4y = 19$$
$$12x + 12y = 18$$

Simultaneous equations are two equations with the same values for x and y

Often we can just add or subtract equations to eliminate x or y. But this time its harder - first we have multiply one equation so x or y is the same as the other equation

x 3 → $6x - 4y = 19$

$18x - 12y = 57$

Multiply by 3 so $4y$ becomes $12y$ then we can add to eliminate it

$12x + 12y = 18$

$30x = 75$

Now add the two equations to eliminate y

So x = 75÷ 30 = 2 ½

SUBSTITUTE x= $2\frac{1}{2}$

$6x - 4y = 19$

$6 \times 2\frac{1}{2} - 4y = 19$

$15 - 4y = 19$

Take 15 from both sides

$15 - 4y - 15 = 19 - 15$

$-4y = 4$ so $y = -1$

x…… $2\frac{1}{2}$ ✓

……..

y…… - 1 ✓

……..

(3)

15.

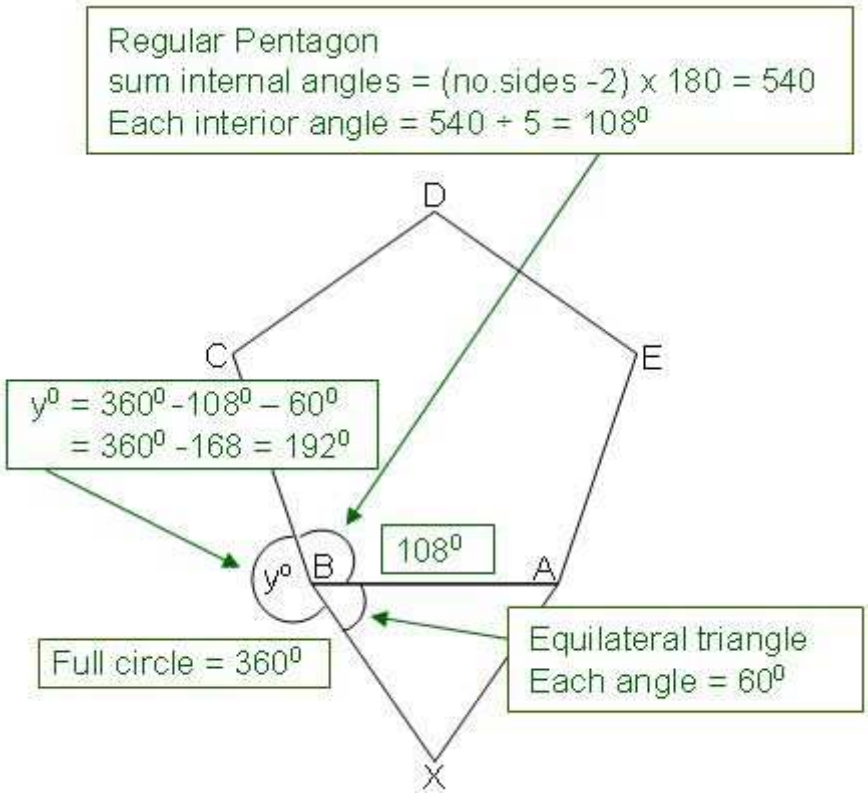


Diagram not drawn accurately

ABCDE is a regular pentagon and ABX is an equilateral triangle.
Work out the value of angle *y*

192

(4)

y = ...

16. A survey of 100 adults was made to see how long they spent watching TV each week.

The table below shows how long in hours the adults spent.

Time (t hours)	Frequency
$0 < t \leq 5$	8
$5 < t \leq 10$	18
$10 < t \leq 15$	26
$15 < t \leq 20$	28
$20 < t \leq 25$	14
$25 < t \leq 30$	6

Cumulative means find a new total as you go along by adding on each new number

b) Complete the cumulative frequency table. **(1)**

Time (t hours)	Frequency
$0 < t \leq 5$	8
$5 < t \leq 10$	26 ✓
$10 < t \leq 15$	52 ✓
$15 < t \leq 20$	80 ✓
$20 < t \leq 25$	94 ✓
$25 < t \leq 30$	100 ✓

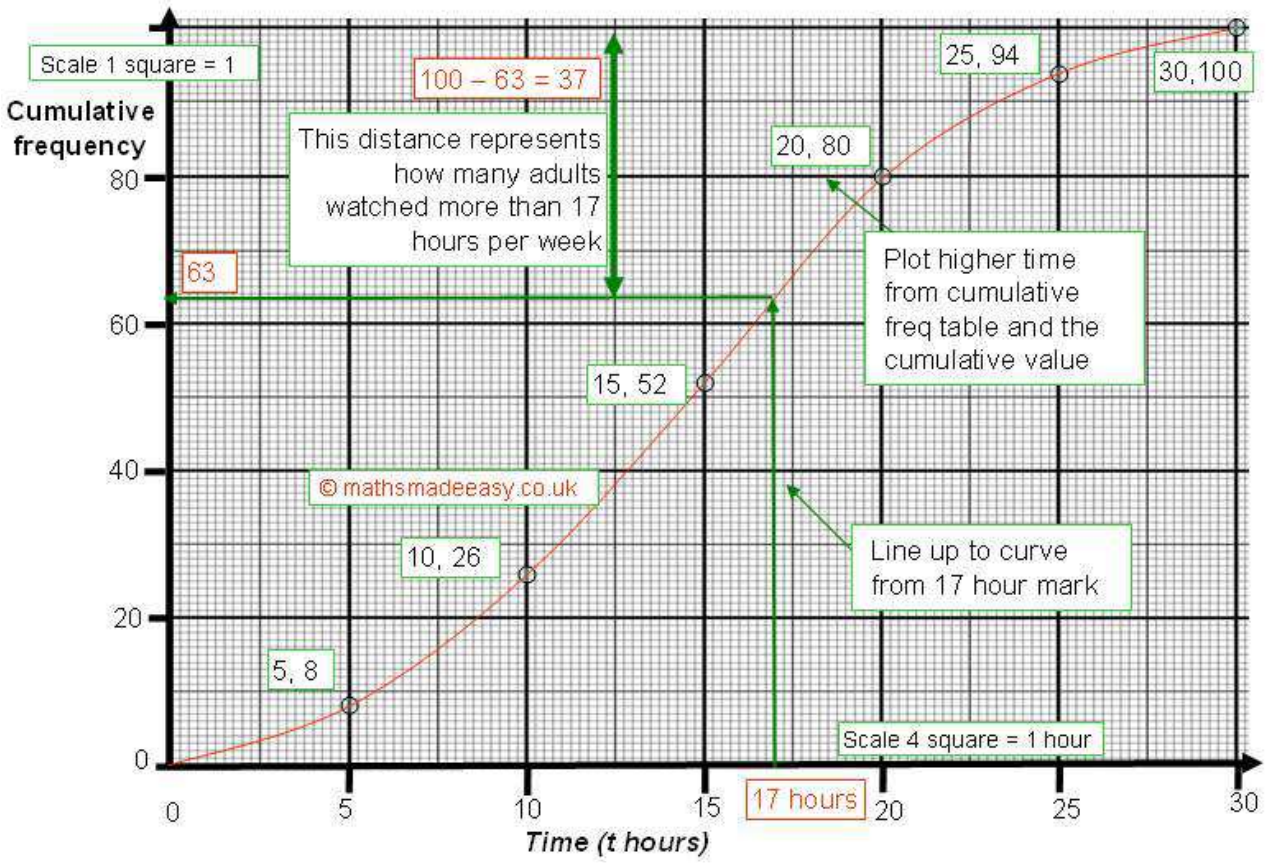
8 + 18 = 26

26 + 26 = 52

52 + 28 = 80

80 + 14 = 94

94 + 6 = 100



b) On the grid draw a cumulative frequency graph for your table

(2)

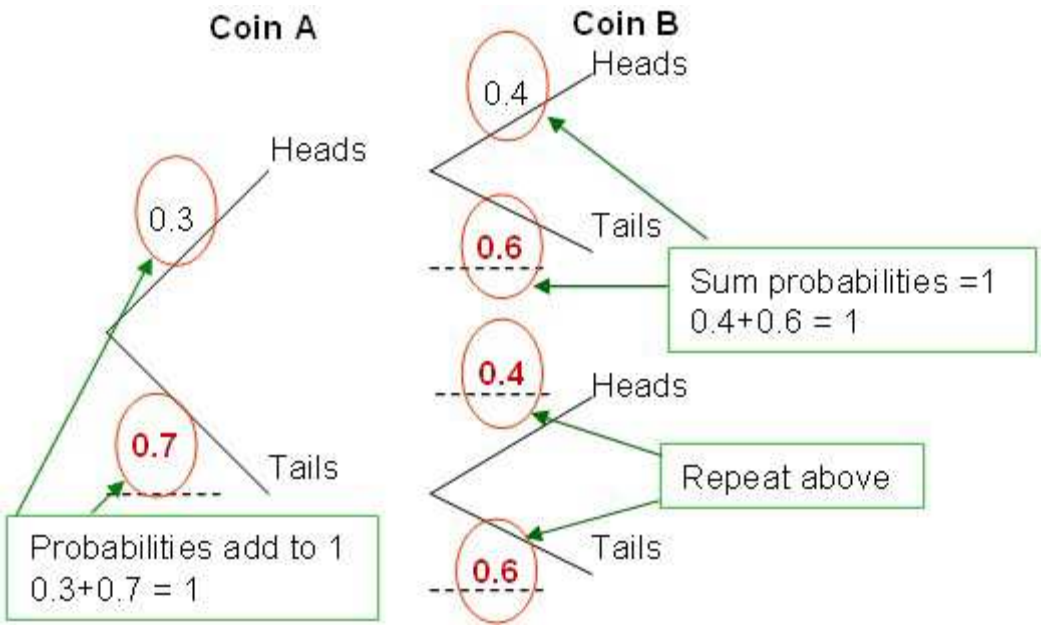
c) Use your graph to estimate how many adults watched ***more*** than 17 hours TV per week.

Remember: Take the 63 from 100 to find how many watched more than 17 hours.

..... 37 ✓

17. Two biased coins A and B are thrown one after the other.
The probability that coin A will land on a head is 0.3.
The probability that coin B will land on a head is 0.4.

a) Complete the probability Tree diagram.



(2)

b) Work out the probability that both coins will land on a head

Prob (head on A and head on B) = 0.3 x 0.4 = 0.12

0.12

(2)

c) Work out the probability that exactly one coin will be heads

This means we get: heads then tails
OR tails then heads
So we must add up two probabilities

Prob (head on A and tail on B) = 0.3 x 0.6 = 0.18
Prob (tail on A and head on B) = 0.7 x 0.4 = 0.28

Add these two probabilities together 0.18+ 0.28 =0.46

0.46

(3)

18. Prove that the recurring decimal $0.\dot{3}\dot{6} = \frac{4}{11}$

Multiply by 100
Then take 0.363636 away

$0.\dot{3}\dot{6} \times 100 = 36.3636$

So $0.\dot{3}\dot{6} \times 100 - 0.\dot{3}\dot{6} \times 1 = 36.0$
Or $0.\dot{3}\dot{6} \times 99 = 36 \rightarrow 0.\dot{3}\dot{6} = \frac{36}{99} = \frac{4}{11}$ ✓

…… **(3)**

19. Expand and simplify $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

Remember: Two Double Brackets have FOUR multiplications!!!

$\sqrt{5} \times \sqrt{5} = +5$

$-\sqrt{3} \times \sqrt{3} = -3$

$(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

It looks like a face!

$\sqrt{5} \times \sqrt{3} = +\sqrt{15}$

$\sqrt{5} \times -\sqrt{3} = -\sqrt{15}$

Use **FOIL** to help you remember the 4 multiplications:
First terms
Outer terms
Inner terms
Last terms

Simplify – collect like terms together

$5 - 3 + \sqrt{15} - \sqrt{15} = 2$

OR use a 2x2 grid
Then simplify

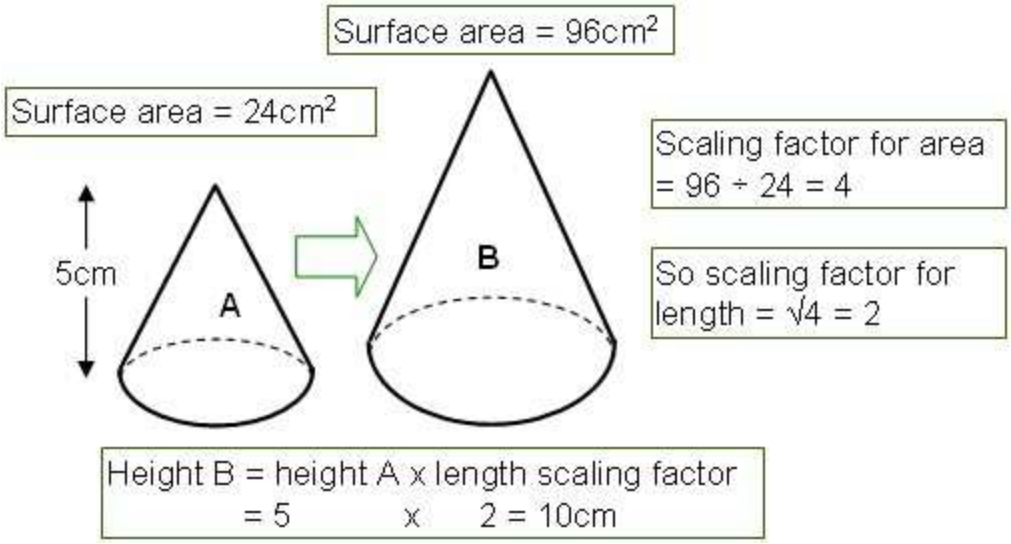
×	$\sqrt{5}$	$-\sqrt{3}$
$\sqrt{5}$	5	$-\sqrt{5} \times \sqrt{3}$
$+\sqrt{3}$	$+\sqrt{5} \times \sqrt{3}$	-3

These two cancel each other

2 ✓

…… **(2)**

20.



Two cones A and B are mathematically similar.
The total surface area of cone A is 12 cm^2
The total surface area of cone B is 48 cm3
The height of cone A is 5cm.

a) Work out the height of cone B

10

…….cm **(2)**

The volume of cone A is 6 cm^3

b) Work out the volume of cone B

Volume scaling factor = (length scaling factor)^3 = 23 = 8

Volume of B =volume A x volume scaling factor = 6 x 8 = 48

48

…….cm^3 **(2)**

21. (a) Expand $p\ (4q - 3p^3)$

p x $-3p^3$ = $-3p^4$

Multiply each term in the bracket by the term outside

p x 4q = 4pq → 4pq $-3p^4$

4pq $-3p^4$ ✓ ..... **(1)**

A factor is something that will go into both terms

2y x x^2 = $2yx^2$ or 2 x^2y

b) Factorise completely $2x^2 y - 8y^4$

2 and y are both factors → 2y. Take 2y outside the ( )

2y (x^2 - $4y^3$) ✓ **(1)**

2y x $4y^3$ = $8y^4$

c) Simplify $\dfrac{16\ a^3\ b^3}{4\ a^2\ b^4}$

Cancel terms top and bottom

$\dfrac{\cancel{16}^{4}\ \cancel{a^3}^{a}\ \cancel{b^3}}{\cancel{4}\ \cancel{a^2}\ \cancel{b^4}^{b}}$

$\dfrac{4\ a}{b}$ ✓ ... **(1)**

d) Expand $(x - y)^2$

$(x - y)^2$ = (x - y)(x y) so don't forget to multiply out fully.

$x^2 + y^2 - 2xy$ **(1)**

e) Solve $x^2 + 1 = 5$

$x^2 = 4$

$x = \sqrt{4}$

± 2 **(1)**

22.

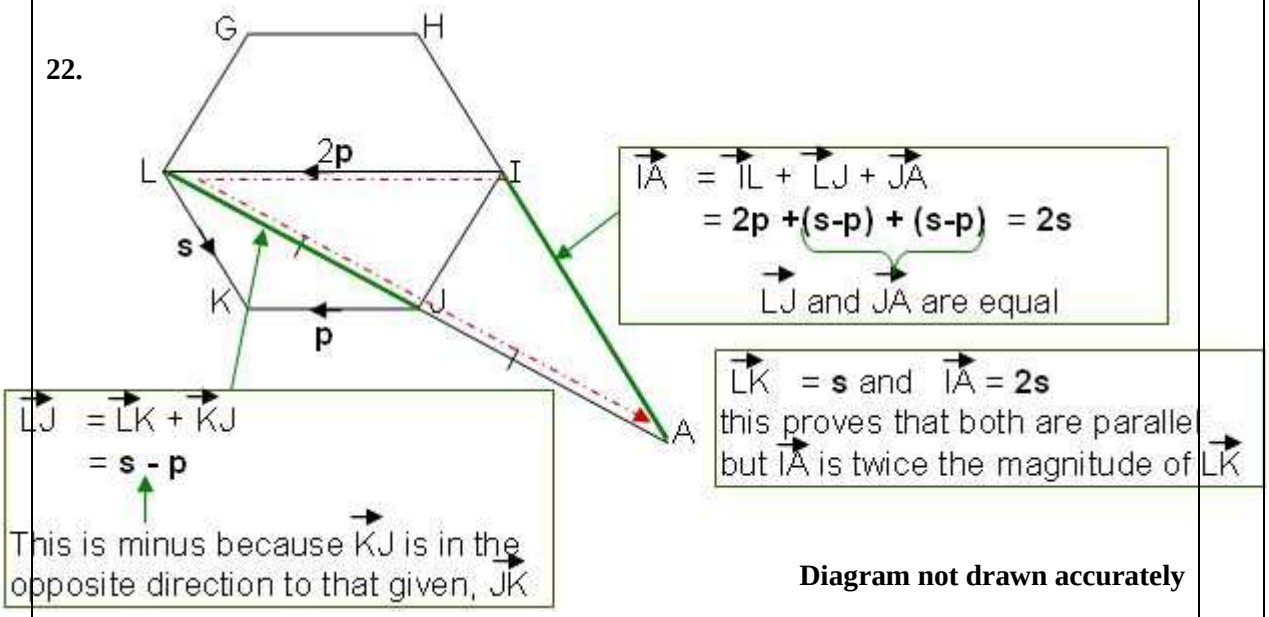


Diagram not drawn accurately

(1)

GHIJKL is a regular hexagon.

$\overrightarrow{LK} = \mathbf{s}$ $\overrightarrow{JK} = \mathbf{p}$ $\overrightarrow{IL} = 2\mathbf{p}$

a) What is the vector $\overrightarrow{LJ}$ in terms of **s** and **p**

s - p

$\overrightarrow{LJ} = \overrightarrow{JA}$

b) Prove that LK is parallel to IA

LK = s
IA = 2s

(3)

23.

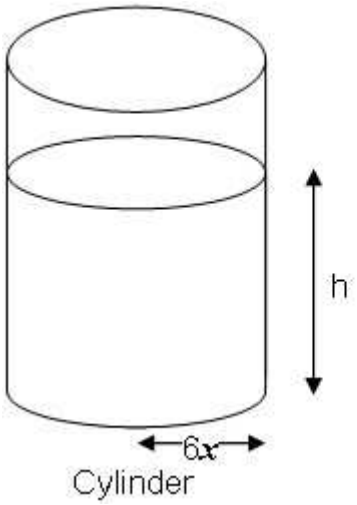


Diagram not drawn accurately

The radius of the base of a cylinder is $6x$ cm.
The cylinder is filled with water to a height of h cm.
The radius of a sphere is $3x$ cm.
The sphere is dropped into the cylinder and is completely immersed.

Find, in terms of x, the increase in the height of the water in the cylinder.
Give your answer in its simplest form

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3x)^3 = \frac{4}{3} \times \pi \times 27x^3$

Volume of cylinder = $\pi r^2 h = \pi (6x)^2 = \pi \times 36x^2 \times h$

The increase in volume of the cylinder = volume of sphere

$\frac{4}{3} \times \pi \times 27x^3 = \pi \times 36x^2 \times h$ Cancel π's and x^2 → $\frac{4}{3} \times 27x = 36 \times h$

Rearrange for h , $\frac{4}{3} \times 27x = 36 \times h$ → $h = \frac{4 \times 27^{3} x}{3 \times 36^{9}} = x$

h = x ✓

h = ... **(3)**

24. i) Expand and simplify

$$n^2 + (n + 2)^2$$

$(n + 2)^2 = (n + 2)(n + 2) = n^2 + 4n + 4$ add $n^2 \rightarrow 2n^2 + 4n + 4$

$2n^2 + 4n + 4$ ✓

n is a whole number.

A number x 2 or 4 or any even number must always be even

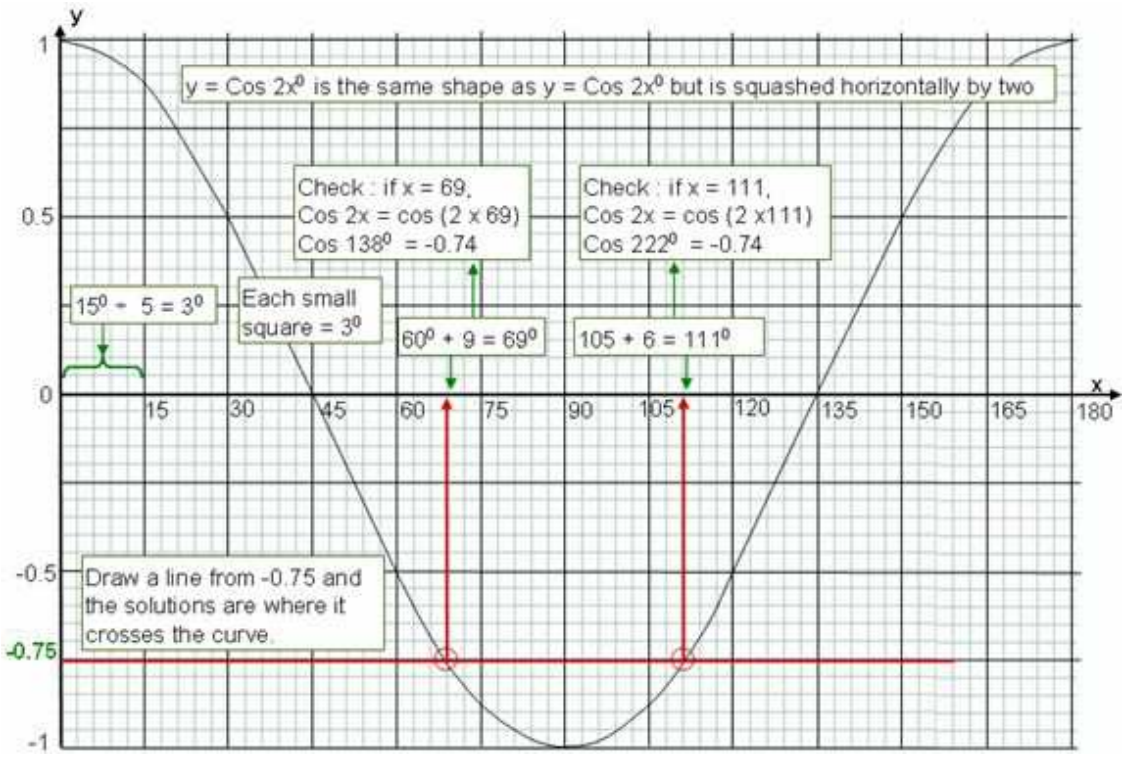
ii) Prove that $n^2 + (n + 2)^2$ is always an even number

If n is odd $2n^2$ is even, 4n is even and 4 is even so the sum is even

If n is even $2n^2$ is even, 4n is even and 4 is even so the sum is even

(4)

25. The graph of $y = \cos 2x^0$ for $0 \leq x \leq 180$ is shown below.



Use the graph to solve $\cos 2x^\circ = -0.75$ for $0 \leq x \leq 180$

$x = 69^\circ$ and 111° ✓

(2)

26. For all values of x:

$x^2 - 4x + 11 = (x - a)^2 + b$

This form is what you get when you Complete the Square

a) Find the value of a and b

$x^2 - 4x + 11 \rightarrow (x - 2)^2 - 4 + 11 \rightarrow (x - 2)^2 + 7$

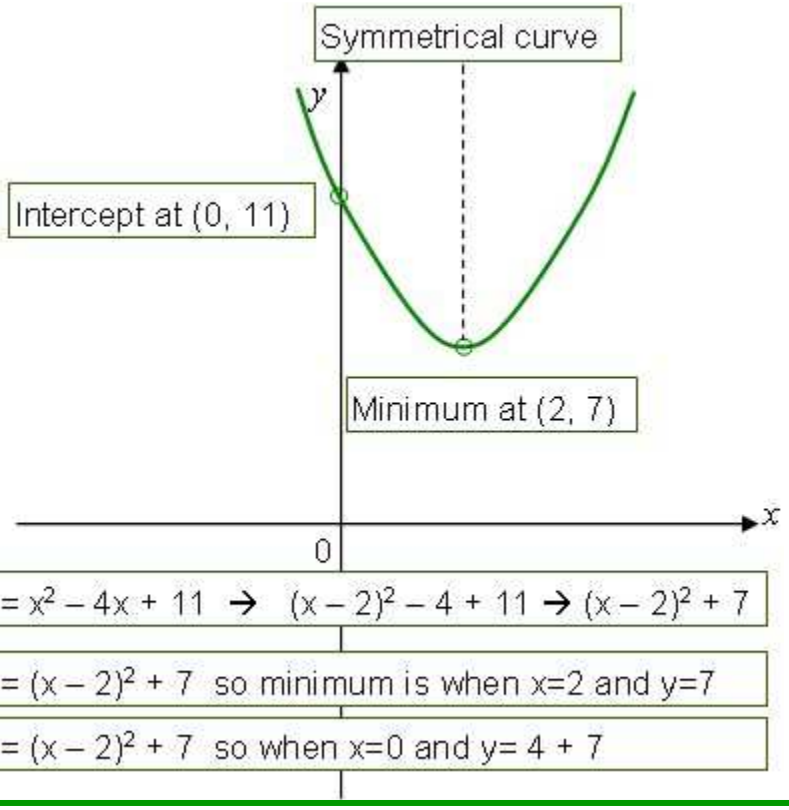
Half of 4

The square term will create an addition 4 (-2x-2) which we have to remove

a= 2 7

(2)

b) On the axes below, sketch the graph $x^2 - 4x + 11$



$y = x^2 - 4x + 11 \rightarrow (x - 2)^2 - 4 + 11 \rightarrow (x - 2)^2 + 7$

$y = (x - 2)^2 + 7$ so minimum is when x=2 and y=7

$y = (x - 2)^2 + 7$ so when x=0 and y= 4 + 7

$y = (x - 2)^2 + 7$ is the curve $y = x^2$ shifted by 2 places to the right along the x-axis (the -2 within the bracket does this) and shifted up by 7 places along the y-axis .

(2)

26b. For all values of x:

$$6x^2 - 3x - \frac{5}{8} = a(x-b)^2 - c$$

a) Find the value of a, b and c

$6x^2 - 3x - \frac{5}{8}$ $= 6(x^2 - \frac{1}{2}x) - \frac{5}{8}$

$6(x^2 - \frac{1}{2}x) - \frac{5}{8}$ $= 6[(x - -\frac{1}{4})^2 - \frac{1}{16}] - \frac{5}{8}$

$6[(x - -\frac{1}{4})^2 - \frac{1}{16}] - \frac{5}{8}$ $= 6(x - -\frac{1}{4})^2 - \frac{6}{16} - \frac{5}{8}$

$6(x - -\frac{1}{4})^2 - \frac{6}{16} - \frac{5}{8}$ $= 6(x - -\frac{1}{4})^2 - 1$

a= 6 b= $\frac{1}{4}$ c= 1 **(3)**

b) Hence solve $6x^2 - 3x - \frac{5}{8} = 0$ leaving your answer in surd form.

$6(x - -\frac{1}{4})^2 - 1 = 0$

$(x - -\frac{1}{4})^2 = \frac{1}{6}$

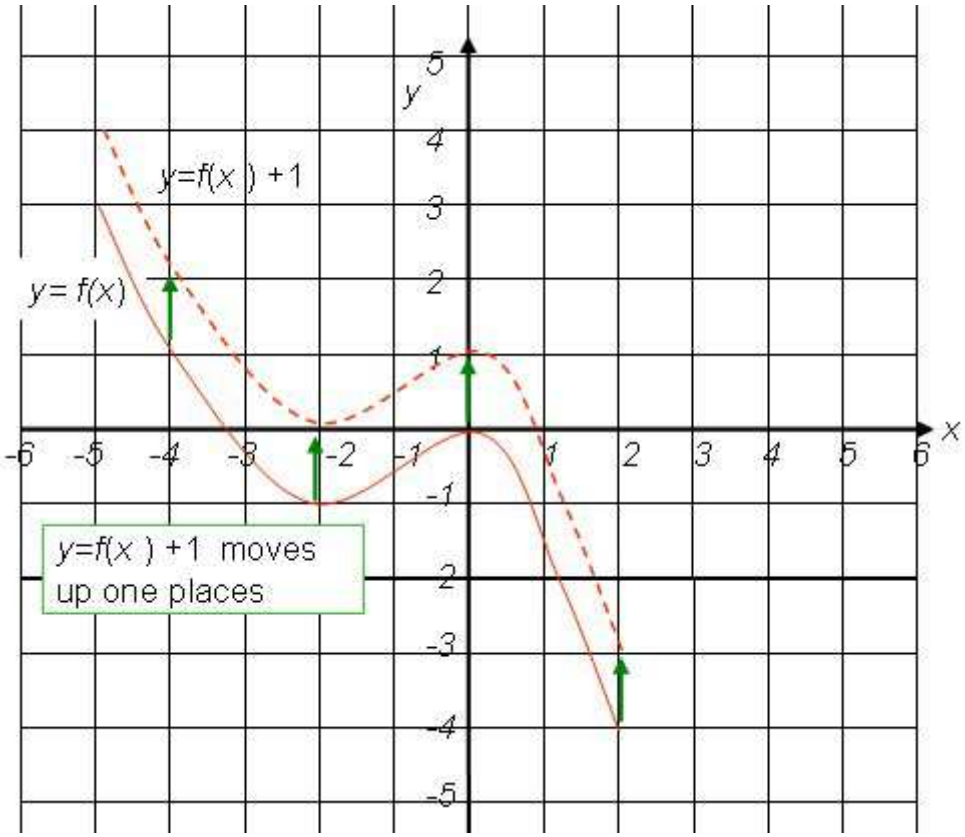
$x = \frac{1}{4} \pm \sqrt{\frac{1}{6}}$ or $= \frac{1}{4} \pm \frac{1}{\sqrt{6}}$

$\frac{1}{4} \pm \sqrt{\frac{1}{6}}$ or $= \frac{1}{4} \pm \frac{1}{\sqrt{6}}$ **(2)**

x = ...

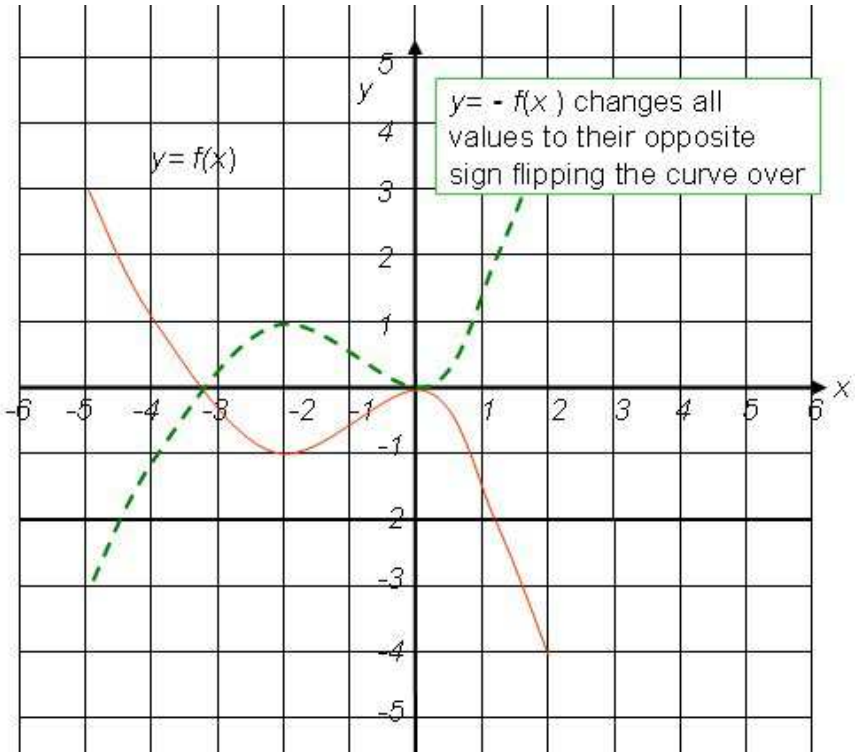
27. The graph is shown below of y = f(x).

a) Sketch the graph of **y = f(x) + 1** on the grid.



(2)

b) On the grid below draw the graph **y = – f(x)**



(2)