FUNCTIONS - Copyright: www.pearson.com
https://qualifications.pearson.com/en/qualifications/edexcel-gcses/mathematics-2015.html

A7 WHERE APPROPRIATE, INTERPRET SIMPLE EXPRESSIONS AS FUNCTIONS WITH INPUTS AND OUTPUTS; INTERPRET THE REVERSE PROCESS AS THE 'INVERSE FUNCTION'; INTERPRET THE SUCCESSION OF TWO FUNCTIONS AS A 'COMPOSITE FUNCTION' (THE USE OF FORMAL FUNCTION NOTATION IS EXPECTED) (higher tier)

In GCSE Mathematics equations are written as shown below:

$$
y=3 x+4 \quad y=x^{2}+5
$$

Sometimes a different notation is used which is called function notation.
We often use the letters f and g and we write the above equations as

$$
\mathrm{f}(x)=3 x+4 \quad \mathrm{~g}(x)=x^{2}+5
$$

## EXAMPLE 1

Using the equation $y=3 x+4$, find the value of $y$ if
(a) $x=4$
(b) $x=-6$
(a) $y=3(4)+4=12+4=16$

Substitute for $x=4$ in the equation
(b) $y=3(-6)+4=-18+4=-14$

Substitute for $x=-6$ in the equation

## EXAMPLE 2

f is a function such that $\mathrm{f}(x)=3 x+4$
Find the values of
(a) $\mathrm{f}(4)$
(b) $\mathrm{f}(-6)$
(a) $\mathrm{f}(4)=3(4)+4=12+4=16$

Substitute for $x=4$ in the equation
(b) $\mathrm{f}(-6)=3(-6)+4=-18+4=-14$

Substitute for $x=-6$ in the equation

## EXAMPLE 3

g is a function such that $\mathrm{g}(x)=2 x^{2}-5$
Find the values of
(a) $\mathrm{g}(3)$
(b) $g(-4)$
(a) $\mathrm{g}(3)=2(3)^{2}-5=18-5=13 \quad$ Substitute for $x=3$ in the equation
(b) $\mathrm{g}(-4)=2(-4)^{2}-5=32-5=17$

Substitute for $x=-4$ in the equation

## EXAMPLE 4

The functions f and g are defined for all real values of $x$ and are such that

$$
\mathrm{f}(x)=x^{2}-4 \quad \text { and } \quad \mathrm{g}(x)=4 x+1
$$

Find
(a) $\mathrm{f}(-3)$
(b) $\mathrm{g}(0.3)$
(c) Find the two values of $x$ for which $\mathrm{f}(x)=\mathrm{g}(x)$.
(a) $f(-3)=(-3)^{2}-4=9-4=5$ Substitute for $x=-3$ in the equation $\mathrm{f}(x)$
(b) $\mathrm{g}(0.3)=4(0.3)+1=1.2+1=2.2$

Substitute for $x=0.3$ in the equation $\mathrm{g}(x)$
(c) $x^{2}-4=4 x+1$

$$
\text { Put } \mathrm{f}(x)=\mathrm{g}(x)
$$

$$
\begin{aligned}
& x^{2}-4-4 x-1=0 \\
& x^{2}-4 x-5=0 \\
& (x-5)(x+1)=0 \\
& x-5=0 \quad \text { or }
\end{aligned} \quad x+1=0, ~ \begin{array}{lll}
x=5 \quad \text { or } & x=-1
\end{array}
$$

$$
\text { Rearrange the equation as a quadratic }=0
$$

Simplify
Solve the quadratic by factorising

## EXERCISE 1:

1. The function f is such that $\mathrm{f}(x)=5 x+2$
Find
(a) $\mathrm{f}(3)$
(b) $\mathrm{f}(7)$
(c) $\mathrm{f}(-4)$
(d) $\mathrm{f}(-2)$
(e) $\mathrm{f}(-0.5)$
(f) $\mathrm{f}(0.3)$
2. The function f is such that $\mathrm{f}(x)=x^{2}-4$
Find
(a) $\mathrm{f}(4)$
(b) $\mathrm{f}(6)$
(c) $\mathrm{f}(-2)$
(d) $\mathrm{f}(-6)$
(e) $\mathrm{f}(-0.2)$
(f) $f(0.9)$
3. The function g is such that $\mathrm{g}(x)=x^{3}-3 x^{2}-2 x+1$
Find
(a) $g(0)$
(b) $\mathrm{g}(1)$
(c) $\mathrm{g}(2)$
(d) $\mathrm{g}(-1)$
(e) $\mathrm{g}(-0.4)$
(f) $g(1.5)$
4. The function f is such that $\mathrm{f}(x)=\sqrt{2 x+5}$
Find
(a) $f(0)$
(b) $\mathrm{f}(1)$
(c) $f(2)$
(d) $\mathrm{f}(-1)$
(e) $\mathrm{f}(-0.7)$
(f) $\mathrm{f}(1.5)$
5. $\mathrm{f}(x)=3 x^{2}-2 x-8$

Express $\mathrm{f}(x+2)$ in the form $a x^{2}+b x$
6. The functions f and g are such that

$$
\mathrm{f}(x)=3 x-5 \quad \text { and } \quad \mathrm{g}(x)=4 x+1
$$

(a) Find (i) $\mathrm{f}(-1)$
(ii) $\mathrm{g}(2)$
(b) Find the two values of $x$ for which $\mathrm{f}(x)=\mathrm{g}(x)$.
7. The functions f and g are such that

$$
\mathrm{f}(x)=2 x^{2}-1 \quad \text { and } \quad \mathrm{g}(x)=5 x+2
$$

(a) Find $\mathrm{f}(-3)$ and $\mathrm{g}(-5)$
(b) Find the two values of $x$ for which $\mathrm{f}(x)=\mathrm{g}(x)$.

## COMPOSITE FUNCTIONS

A composite function is a function consisting of 2 or more functions.

The term composition is used when one operation is performed after another operation. For instance:


This function can be written as $\mathrm{f}(x)=5(x+3)$
Suppose $\mathrm{f}(x)=x^{2}$ and $\mathrm{g}(x)=2+3 x$
What is $\mathbf{f g}(x)$ ? Now $\mathrm{fg}(x)=\mathrm{f}[\mathrm{g}(x)]$
This means apply $g$ first and then apply $f$.


## What is $\operatorname{gf}(x)$ ?

This means apply f first and then apply g .


NOTE: The composite function $\mathbf{g f}(\boldsymbol{x})$ means apply f first followed by g .

NOTE: The composite function $\mathbf{f g}(\boldsymbol{x})$ means apply g first followed by f .

NOTE: $\mathrm{fg}(x)$ can be written as fg and $\mathrm{gf}(x)$ can be written as $\mathrm{gf} ; \mathrm{fg}$ is not the same as gf .

## EXAMPLE 5

f and g are functions such that $\mathrm{f}(x)=\frac{1}{x} \quad$ and $\quad \mathrm{g}(x)=3-2 x$
Find the composite functions (a) fg (b) gf
(a) $\operatorname{fg}=\operatorname{fg}(x)=\mathrm{f}(3-2 x)$
$\longleftarrow$ Do g first: Put $(3-2 x)$ instead of $\mathrm{g}(x)$
$=\frac{1}{3-2 x}$
Substitute $(3-2 x)$ for $x$ in $\frac{1}{x}$
(b) $\mathrm{gf}=\operatorname{gf}(x)=\mathrm{g}\left(\frac{1}{x}\right)$
Do f first: Put $\frac{1}{x}$ instead of $\mathrm{f}(x)$
$=3-2 \frac{1}{x}=3-\frac{2}{x}$
Substitute $\frac{1}{x}$ for $x$ in $(3-2 x)$


## EXAMPLE 7

$\mathrm{f}(x)=7-2 x$
$\mathrm{g}(x)=4 x-1$
$\mathrm{h}(x)=3(x-1)$

Evaluate
(a) $f g(5)$
(b) $\mathrm{ff}(-2)$
(c) $\operatorname{ghf}(3)$
(a) $\mathrm{fg}(5)=\mathrm{f}(20-1)=\mathrm{f}(19)$ Substitute for $x=5$ in $4 x-1$

$$
=7-2(19)=-31
$$

Substitute for $x=19$ in $7-2 x$
(b) $\mathrm{ff}(-2)=\mathrm{f}[7-2(-2)]=\mathrm{f}(11)$

Substitute for $x=-2$ in $7-2 x$ and simplify

$$
=7-2(11)=-15
$$

Substitute for $x=11$ in $7-2 x$ and simplify
(c) $\operatorname{ghf}(3)=\operatorname{gh}(7-6)=\operatorname{gh}(1)$

Substitute for $x=3$ in $7-2 x$ and simplify

$$
\begin{aligned}
& =g[3(1-1)]=g(0) \\
& =4(0)-1=-1
\end{aligned}
$$

Substitute for $x=1$ in $3(x-1)$ and simplify

Substitute for $x=0$ in $4 x-1$ and simplify


## EXAMPLE 9

## (more challenging question)

Functions $f, g$ and $h$ are such that
f: $x \rightarrow 4 x-1$
$\mathrm{g}: x \rightarrow \frac{1}{x+2}, x \neq-2$
$\mathrm{h}: x \rightarrow(2-x)^{2}$

Find (a)(i) $\operatorname{fg}(x)$
(ii) $\operatorname{hh}(x)$
(b) Show that $\operatorname{fgh}(x)=\frac{8 x+1}{4 x+1}$
(a) $\operatorname{fg}(x)=\mathrm{f}\left(\frac{1}{x+2}\right)$

$=4\left(\frac{1}{x+2}\right)-1$
f's operation is $\times 4-1$
$=\frac{4-1(x+2)}{x+2}$
Simplify using a common denominator of $x+2$
$=\frac{2-x}{x+2}$
(b) $\mathrm{hh}(x)=\mathrm{h}(2-x)^{2}$
$\longleftarrow \quad$ Substitute for $\mathrm{h}(x)$

$$
\begin{array}{ll}
=\left[2-(2-x)^{2}\right]^{2} & \longleftarrow \text { h's operation is subtract from } 2 \text { and then square } \\
=\left[2-\left(4-2 x+x^{2}\right)\right]^{2} & \longleftarrow{(2-x)^{2}=(2-x)(2-x)=4-2 x+x^{2}}^{=\left(-2+4 x-2 x^{2}\right)^{2}}
\end{array}
$$

(c) $\operatorname{hgf}(x)=\operatorname{hg}(4 x-1)$

$=\mathrm{h}\left(\frac{1}{4 x-1+2}\right)$
Substitute $(4 x-1)$ for $x$ in $g(x)$
$=\left(2-\frac{1}{4 x+1}\right)^{2}$

$=\left[2\left(\frac{4 x+1}{4 x+1}\right)-\frac{1}{4 x+1}\right]^{2} \longleftarrow \quad$ Simplify using a common denominator of $4 x+1$
$=\left(\frac{8 x+2-1}{4 x+1}\right)^{2}$
$=\frac{8 x+1}{4 x+1}$

## EXERCISE 2:

1. Find an expression for $\mathrm{fg}(x)$ for each of these functions:
(a) $\mathrm{f}(x)=x-1$ and $\mathrm{g}(x)=5-2 x$
(b) $\mathrm{f}(x)=2 x+1$ and $\mathrm{g}(x)=4 x+3$
(c) $\mathrm{f}(x)=\frac{3}{x} \quad$ and $\quad \mathrm{g}(x)=2 x-1$
(d) $\mathrm{f}(x)=2 x^{2} \quad$ and $\quad \mathrm{g}(x)=x+3$
2. Find an expression for $\mathrm{gf}(x)$ for each of these functions:
(a) $\mathrm{f}(x)=x-1$ and $\mathrm{g}(x)=5-2 x$
(b) $\mathrm{f}(x)=2 x+1$ and $\mathrm{g}(x)=4 x+3$
(c) $\mathrm{f}(x)=\frac{3}{x} \quad$ and $\quad \mathrm{g}(x)=2 x-1$
(d) $\mathrm{f}(x)=2 x^{2} \quad$ and $\quad \mathrm{g}(x)=x+3$
3. The function f is such that $\mathrm{f}(x)=2 x-3$

Find (i) $\mathrm{ff}(2)$ (ii) Solve the equation $\mathrm{ff}(a)=a$
4. Functions f and g are such that

$$
\mathrm{f}(x)=x^{2} \quad \text { and } \quad \mathrm{g}(x)=5+x
$$

Find (a)(i) $\operatorname{fg}(x)$ (ii) $\operatorname{gf}(x)$
(b) Show that there is a single value of $x$ for which $\operatorname{fg}(x)=\operatorname{gf}(x)$ and find this value of $x$.
5. Given that $\mathrm{f}(x)=3 x-1, \mathrm{~g}(x)=x^{2}+4$ and $\mathrm{fg}(x)=\operatorname{gf}(x)$, show that $x^{2}-x-1=0$
6. The function f is defined by $\mathrm{f}(x)=\frac{x-1}{x}, x \neq 0$

Solve $\mathrm{ff}(x)=-2$
7. The function g is such that $\mathrm{g}(x)=\frac{1}{1-x}$ for $x \neq 1$
(a) Prove that $\operatorname{gg}(x)=\frac{x-1}{x}$
(b) Find $\operatorname{ggg}(3)$
8. Functions $\mathrm{f}, \mathrm{g}$ and h are such that $\mathrm{f}(x)=3-x, \mathrm{~g}(x)=x^{2}-14$ and $\mathrm{h}(x)=x-2$ Given that $\mathrm{f}(x)=\operatorname{gfh}(x)$, find the values of $x$.

## INVERSE FUNCTIONS

The function $\mathrm{f}(x)=3 x+5$ can be thought of as a sequence of operations as shown below


Now reversing the operations


The new function, $\frac{x-5}{3}$, is known as the inverse function.

Inverse functions are denoted as $\mathrm{f}^{-1}(x)$.

## EXAMPLE 10

Find the inverse function of $\mathrm{f}(x)=3 x-4$

$$
y=3 x-4
$$

$$
x=3 y-4
$$

$$
x+4=3 y
$$

$$
\left.\frac{x+4}{3}=y\right\}
$$

$$
\mathrm{f}^{-1}(x)=\frac{x+4}{3}
$$

Step 1: Write out the function as $y=\ldots$
Step 2: Swap the $x$ and $y$
Step 3: Make $y$ the subject

Step 4: Instead of $y=$ write $\mathrm{f}^{-1}(x)=$

## EXAMPLE 11

Find the inverse function of $\mathrm{f}(x)=\frac{x-2}{7}$

$$
\begin{aligned}
& y=\frac{x-2}{7} \\
& x=\frac{y-2}{7} \\
& 7 x=y-2 \\
& 7 x+2=y
\end{aligned}
$$

## EXAMPLE 12

Find the inverse function of $\mathrm{f}(x)=\sqrt{x+4}$
$y=\sqrt{x+4}$
Step 1: Write out the function as $y=\ldots$
$x=\sqrt{y+4}$
Step 2: Swap the $x$ and $y$
$\left.\begin{array}{l}x^{2}=y+4 \\ x^{2}-4=y\end{array}\right\}$
Step 3: Make $y$ the subject
$\mathrm{f}^{-1}(x)=x^{2}-4$
Step 4: Instead of $y=$ write $\mathrm{f}^{-1}(x)=$

## RULES FOR FINDING THE INVERSE $\mathrm{f}^{-1}(x)$ :

Step 1: Write out the function as $y=\ldots$
Step 2: Swap the $x$ and $y$
Step 3: Make $y$ the subject
Step 4: Instead of $y=$ write $\mathrm{f}^{-1}(x)=$

## EXERCISE 3:

1. Find the inverse function, $\mathrm{f}^{-1}(x)$, of the following functions:
(a) $\mathrm{f}(x)=3 x-1$
(b) $\mathrm{f}(x)=2 x+3$
(c) $\mathrm{f}(x)=1-2 x$
(d) $\mathrm{f}(x)=x^{2}+5$
(e) $\mathrm{f}(x)=6(4 x-1)$
(f) $\mathrm{f}(x)=4-x$
(g) $\mathrm{f}(x)=3 x^{2}-2$
(h) $\mathrm{f}(x)=2(1-x)$
(i) $\mathrm{f}(x)=\frac{2}{x+1}$
(j) $\mathrm{f}(x)=\frac{x+1}{x-2}$
2. The function f is such that $\mathrm{f}(x)=7 x-3$
(a) Find $\mathrm{f}^{-1}(x)$.
(b) Solve the equation $\mathrm{f}^{-1}(x)=\mathrm{f}(x)$.
3. The function f is such that $\mathrm{f}(x)=\frac{8}{x+2}$
(a) Find $\mathrm{f}^{-1}(x)$.
(b) Solve the equation $\mathrm{f}^{-1}(x)=\mathrm{f}(x)$.
4. The function f is such that $\mathrm{f}(x)=\frac{1}{x+4}, \quad x \neq-4$.

Evaluate $\mathrm{f}^{-1}(x)$.
[Hint: First find $\mathrm{f}^{-1}(x)$ and then substitute for $x=-3$ ]
5. $\mathrm{f}(x)=\frac{x}{x+3}, \quad x \in \mathrm{R}, \quad x \neq-3$
(a) If $\mathrm{f}^{-1}(x)=-5$, find the value of $x$.
(b) Show that $\mathrm{ff}^{-1}(x)=x$
6. Functions f and g are such that

$$
\mathrm{f}(x)=3 x+2 \quad \mathrm{~g}(x)=x^{2}+1
$$

Find an expression for $(\mathrm{fg})^{-1}(x)$
[Hint: First find $\mathrm{fg}(x)$ ]

## MIXED EXERCISE:

1. Here is a number machine.

(a) Work out the output when the input is 4
(b) Work out the input when the output is 11
(c) Show that there is a value of the input for which the input and the output have the same value.
2. Functions f and g is such that $\mathrm{f}(x)=2 x-1$ and $\mathrm{g}(x)=\frac{3}{x}$
(a) Find the value of
(i) $f(3)$
(ii) $f g(6)$
(b) Express the inverse function in the form $\left.\mathrm{f}^{-1}(x)\right)=\ldots .$.
(c) Express the composite function gf in the form $\operatorname{gf}(x)=\ldots$..
3. The function f is such that $\mathrm{f}(x)=4 x-1$
(a) Find $\mathrm{f}^{-1}(x)$

The function g is such that $\mathrm{g}(x)=k x^{2}$ where $k$ is a constant.
(b) Given that $\operatorname{fg}(2)=12$, work out the value of $k$
4. Functions f and g are such that $\mathrm{f}(x)=3(x-4)$ and $\mathrm{g}(x)=\frac{x}{5}+1$
(a) Find the value of $f(10)$
(b) Find $\mathrm{g}^{-1}(x)$
(c) Show that $\mathrm{ff}(x)=9 x-48$
5. Given that $\mathrm{f}(x)=x^{2}$ and $\mathrm{g}(x)=x-6$, solve the equation $\mathrm{fg}(x)=\mathrm{g}^{-1}(x)$
6. f and g are functions such that $\mathrm{f}(x)=2 x-3$ and $\mathrm{g}(x)=1+\sqrt{x}$
(a) Calculate $\mathrm{f}(-4)$
(b) Given that $\mathrm{f}(a)=5$, find the value of $a$.
(c) Calculate $\operatorname{gf}(6)$.
(d) Find the inverse function $\mathrm{g}^{-1}(x)$.
7. Functions f and g are such that
$\mathrm{f}(x)=\frac{1}{x+2}$ and $\mathrm{g}(x)=\sqrt{x-1}$
(a) Calculate $\mathrm{fg}(10)$
(b) Find the inverse function $\mathrm{g}^{-1}(x)$.
8. Functions f and g are such that
$\mathrm{f}(x)=2 x+2$ and $\mathrm{g}(x)=2 x-5$
(a) Find the composite function fg .

Give your answer as simply as possible.
(b) Find the inverse function $\mathrm{f}^{-1}(x)$.
(c) Hence, or otherwise, solve $\mathrm{f}^{-1}(x)=\mathrm{g}^{-1}(x)$.
9. The function f is such that $\mathrm{f}(x)=\frac{1}{x+3}$
(a) Find the value of $f(2)$
(b) Given that $\mathrm{f}(a)=\frac{1}{10}$, find the value of $a$.

The function g is such that $\mathrm{g}(x)=x+2$
(c) Find the function gf.

Give your answer as a single algebraic fraction in its simplest form.
10. Functions f and g are such that $\mathrm{f}(x)=x^{2}$ and $\mathrm{g}(x)=x-3$
(a) Find $\operatorname{gf}(x)$.
(b) Find the inverse function $\mathrm{g}^{-1}(x)$.
(c) Solve the equation $\operatorname{gf}(x)=\mathrm{g}^{-1}(x)$.
11. The function f is such that $\mathrm{f}(x)=(x-1)^{2}$
(a) Find $f(8)$

The function g is such that $\mathrm{g}(x)=\frac{x}{x-1}$
(b) Solve the equation $\mathrm{g}(x)=1.2$
(c) (i) Express the inverse function $\mathrm{g}^{-1}$ in the form $\mathrm{g}^{-1}(x)=$. $\qquad$
(ii) Hence write down $\operatorname{gg}(x)$ in terms of $x$.
12. f is a function such that $\mathrm{f}(x)=\frac{1}{x^{2}+1}$
(a) Find $f\left(\frac{1}{2}\right)$
g is a function such that $\mathrm{g}(x)=\sqrt{x-1}, x \geq 1$
(b) Find $\operatorname{fg}(x)$

Give your answer as simply as possible.
13. The function f is such that $\mathrm{f}(x)=\frac{x-6}{2}$
(a) Find $\mathrm{f}(8)$
(b) Express the inverse function $\mathrm{f}^{-1}$ in the form $\mathrm{f}^{-1}(x)=\ldots$

The function g is such that $\mathrm{g}(x)=\sqrt{x-4}$
(c) Express the function gf in the form $\operatorname{gf}(x)=\ldots$

Give your answer as simply as possible.
14. Functions f and g are such that $\mathrm{f}(x)=3 x-2$ and $\mathrm{g}(x)=\frac{10}{x+2}$
(a) Express the inverse function $\mathrm{f}^{-1}$ in the form $\mathrm{f}^{-1}(x)=\ldots$
(b) Find $\operatorname{gf}(x)$

Simplify your answer.
15. Functions f and g are such that $\mathrm{f}(x)=\frac{2}{x}$ and $\mathrm{g}(x)=\frac{x+1}{x}$
(a) Solve $\operatorname{gf}(a)=3$
(b) Express the inverse function $\mathrm{g}^{-1}$ in the form $\mathrm{g}^{-1}(x)=\ldots$
16. Functions g and h are such that $\mathrm{g}(x)=\frac{x}{2 x-5}$ and $\mathrm{h}(x)=x+4$
(a) Find the value of $g(1)$
(b) Find $\operatorname{gh}(x)$

Simplify your answer.
(c) Express the inverse function $\mathrm{g}^{-1}$ in the form $\mathrm{g}^{-1}(x)=\ldots$

## ANSWERS

## Exercise 1

1. 

(a) 17
(b) 37
(c) -18
(d) -8
(e) -0.5
(f) 3.5
2.
(a) 12
(b) 32
(c) 0
(d) 32
(e) -3.96
(f) -3.19
3.
(a) 1
(b) -3
(c) -7
(d) -1
(e) 1.256
(f) -5.375
4.
(a) $\sqrt{5}$
(b) $\sqrt{3}$
(c) 3
(d) $\sqrt{3}$
(e) $\frac{3 \sqrt{10}}{5}$
(f) $2 \sqrt{2}$
5. $3 x^{2}+10 x$
6.
(a) $\mathrm{f}(-1)=8$ and $\mathrm{g}(2)=9$
(b) $x=-6$
7.
(a) $f(-3)=17$ and $g(-5)=-23$
(b) $x=-\frac{1}{2}$ and $x=3$

## Exercise 2

1. 

(a) $4-2 x$
(b) $8 x+17$
(c) $\frac{3}{2 x-1}$
(d) $2(x+3)^{2}$
2.
(a) $7-2 x$
(b) $8 x+7$
(c) $\frac{6}{x}-1$
(d) $2 x^{2}+3$
3.
(a) -1
(b) $a=3$
4.
(a)(i) $(5+x)^{2}$
(ii) $5+x^{2}$
(b) $x=-2$
5. $3\left(x^{2}+4\right)-1=(3 x-1)^{2}+4$
$3 x^{2}+12-1=9 x^{2}-6 x+1+4$
$3 x^{2}+11=9 x^{2}-6 x+5$
$6 x^{2}-6 x-6=0$
$x^{2}-x-1=0$
6. $x=\frac{3}{2}$
7.
(a) $g\left(\frac{1}{1-x}\right)=\frac{1}{1-\frac{1}{1-x}}=\frac{1}{\frac{1-x-1}{1-x}}=\frac{1-x}{-x}=\frac{x-1}{x}$
(b) $-\frac{1}{4}$
8. $x=1$ and $x=8$

## Exercise 3

1. 

(a) $\frac{x+1}{3}$
(b) $\frac{x-3}{2}$
(c) $\frac{1-x}{2}$
(d) $\pm \sqrt{x-5}$
(e) $\frac{x+6}{24}$
(f) $4-x$
(g) $\pm \sqrt{\frac{x+2}{3}}$
(h) $\frac{2-x}{2}$
(i) $\frac{2-x}{x}$
(j) $\frac{2 x+1}{x-1}$
2.
(a) $\frac{x+3}{7}$
(b) $x=0.5$
3.
(a) $\frac{8}{x}-2$
(b) $x=-4$ and $x=2$
4. $-\frac{13}{3}$
5.
(a) $\frac{5}{2}$
(b) $f\left(\frac{3 x}{1-x}\right)=\frac{\frac{3 x}{1-x}}{\frac{3 x}{1-x}+3}=\frac{\frac{3 x}{1-x}}{\frac{3 x+3(1-x)}{1-x}}=\frac{\frac{3 x}{1-x}}{\frac{3 x+3-3 x}{1-x}}=\frac{3 x}{3}=x$
6. $\pm \sqrt{\frac{x-5}{3}}$

## Mixed Exercise

1. 

(a) 8
(b) 29
(c) 2
2.
(a) (i) 5
(ii) 0
(b) $\frac{x+1}{2}$
(c) $\frac{3}{x}$
3.
(a) $\frac{x+1}{4}$
(b) $k=\frac{13}{16}$
4.
(a) 18
(b) $5(x-1)$
(c) $\mathrm{ff}(x)=3(3(x-4)-4)=3(3 x-12-4)=3(3 x-16)=9 x-48$
5. $x=6$ and $x=7$
6.
(a) -11
(b) $a=4$
(c) 4
(d) $(x-1)^{2}$
7.
(a) $\frac{1}{5}$
(b) $x^{2}+2$
8.
(a) $6 x-13$
(b) $\frac{x-2}{3}$
(c) $x=-19$
9.
(a) $\frac{1}{5}$
(b) $a=7$
(c) $\frac{2 x+7}{x+3}$
10.
(a) $x^{2}-3$
(b) $x+3$
(c) $x=-2$ and $x=3$
11.
(a) 49
(b) $x=6$
(c) (i) $\frac{x}{x-1}$
(ii) $x$
12.
(a) 0.8
(b) $\frac{1}{x}$
13.
(a) 1
(b) $2 x+6$
(c) $\sqrt{\frac{x-14}{2}}$
14. (a) $\frac{x+2}{3}$
(b) $\frac{10}{3 x}$
15. (a) $x=4$
(b) $\frac{1}{x-1}$
16. (a) $-\frac{1}{3}$
(b) $\frac{x+4}{2 x+3}$
(c) $\frac{5 x}{2 x-1}$

