AQA, Edexcel

A Level

A Level Physics

Oscillations (Answers)

Name:

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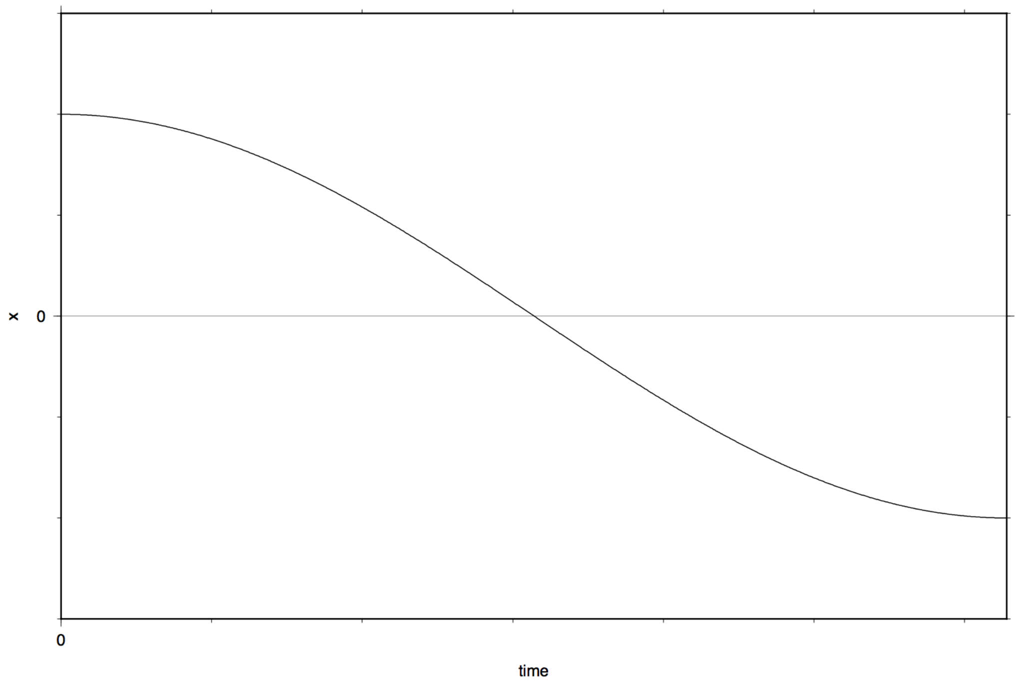
Total Marks: /30

1. The graph below shows the displacement of a spring, x, against the time since its release, t.

Total for Question 1: 6



(a) On the plot above, sketch graphs of velocity versus time and acceleration versus time. Hence conclude whether or not the spring is in simple harmonic motion. Justify your conclusion. [3]

Solution: v-t is gradient of x-t (i.e. -sin(x)); a-t is gradient of v-t (i.e. -cos(x)).
Since a is -x, this is SHM.

(b) The period of a pendulum in simple harmonic motion is independent of its amplitude. How would you verify this statement experimentally? [3]

> **Solution:** Mark the equilibrium position (the plumb-line) to give an easy reference mark.
> Release the pendulum from a variety of different points and time 5-10 oscillations.
> Divide by the number of oscillations to get the period for each release point (amplitude).

2. Geoff, a rock climber, is following his friend Marie up an overhanging cliff. Marie is attached to the rock and ensures that the rope between herself and Geoff is taut at all times. Unfortunately, Geoff falls off and subsequently swings. Unless otherwise stated, ignore air resistance.

Total for Question 2: 24

(a) Geoff passes through the equilibrium position, for the second time, after 4.5 s. At this point, he is travelling at a speed of 5 ms^{-1}. Calculate the following:

i. The frequency of his oscillations. [2]

Solution: 0.17 s^{-1}

ii. His velocity 2.25 s after he falls. [3]

Solution: $-\frac{5\sqrt{2}}{2} \approx$ -3.54 ms^{-1}

For a simple pendulum undergoing simple harmonic motion, $\omega^2 = \frac{g}{l}$, where l is the length of the pendulum, ω is the angular frequency and g is the acceleration due to gravity. For the small angles involved in simple harmonic motion, $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\sin\theta \approx \theta$.

(b) It can be shown geometrically that $\cos\omega t = \frac{l}{A}\sin\theta$, where x is Geoff's displacement, l is the rope's length, A is his initial displacement and θ is the angle between the rope and the vertical. Using this result, show that his kinetic energy is given by $\frac{1}{2}m\omega^2(A^2 - l^2\theta^2)$. [3]

Solution:
$\frac{1}{2}mv^2 = \frac{1}{2}m(\omega^2(A^2 - x^2)) = \frac{1}{2}m\omega^2(A^2 - A^2\cos^2\omega t) = \frac{1}{2}m\omega^2 A^2(1 - \cos^2\omega t)$
So, $E_k = \frac{1}{2}m\omega^2 A^2(1 - \frac{l^2}{A^2}\sin^2\theta)$.
Small angle approximation: $E_k \approx \frac{1}{2}m\omega^2 A^2(1 - \frac{l^2\theta^2}{A^2})$

(c) By considering his vertical displacement, derive an expression for his potential energy in terms of l, g, θ and his mass, m. [3]

Solution: $E_p \approx \frac{1}{2}mgl\theta^2$

(d) Hence show that energy is conserved. [3]

Solution:
Total energy is $E_k + E_p$
Since $l\omega^2 = g$, $E_p = \frac{1}{2}m\omega^2 l^2\theta^2$
Total energy $= \frac{1}{2}m\omega^2 l^2\theta^2 + \frac{1}{2}m\omega^2(A^2 - l^2\theta^2) = \frac{1}{2}m\omega^2(l^2\theta^2 + A^2 - l^2\theta^2) = \frac{1}{2}m\omega^2 A^2$
This is constant; energy is conserved.

(e) Sketch, on a single set of axes, the variation of E_{total}, E_p and E_k with θ for $-\theta_{max} \leq \theta \leq \theta_{max}$. [2]

Solution:
Total energy constant.
Kinetic energy max at $\theta = 0$ and zero at extremities.
Potential energy zero at $\theta = 0$ and maximum at extremities.
Both have quadratic form.

(f) Sketch, on a single set of axes, the displacements of a lightly damped and a critically damped simple pendulum as a function of time. In reality, after numerous oscillations, Geoff comes to rest. Is the damping in the system light, heavy or critical? [3]

Solution: Light: amplitude decays to zero after many oscillations in an exponential manner.
Very heavy: no complete oscillations occur.
In Geoff's case, light damping.

(g) In this case, the damping causes an exponential decrease in the amplitude of his displacements. Given that after 4 s the amplitude has reduced to 4.00 m, calculate the amplitude when $t = 8$ s. Use your answer to part a,ii as an initial amplitude. [2]

Solution: $= \frac{A_{4s}^2}{A_{0s}} = 3.35$ m

(h) In other systems, oscillations can be forced. Briefly explain what is meant by the term resonance and when it might occur. You may find it helpful to provide a sketch. [3]

Solution: Resonance occurs when a system is forced to vibrate at its natural frequency. In such cases, it absorbs a lot of energy, causing it to vibrate with a very large amplitude.
Sketch should show amplitude against frequency, with a peak at the natural frequency.