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AQA, Edexcel, OCR

A Level

A Level Mathematics

Understand and use proportional relationships and their graphs (Answers)

Name:

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Total Marks:

B7- Proportional relationships and their graphs- Answers

AQA, Edexcel, OCR

1) **For each of the tables, state the form of the relationship as a formula, complete the missing values and match the table to the one-word description of the type of relationship.**

[1 mark for each table completely correctly completed- 4 in total]

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

t	c
12	-2.8
13	-2.2
14	-1.6
15	-1
16	-0.4
17	0.2
18	0.8

n	p
5	50
6	62
7	98
8	128
9	182
10	200
11	142

a	b
1	3
2	17
3	1
4	3
5	9
6	2
7	$\in \mathbb{Z}$

[1 mark for each correctly matched table]

Quadratic	**Linear**	**Quadratic**	**None**

[1 mark for each correctly identified formula]

$\boldsymbol{y = x^2}$ $\boldsymbol{y = 0.6x - 10}$ $\boldsymbol{p = 2(n^2)}$ **NA**

2) **i) Demonstrate graphically Poiseuille's Law, which is defined as:**

$$\boldsymbol{Q = \frac{\pi P\, r^4}{8\eta l}}$$

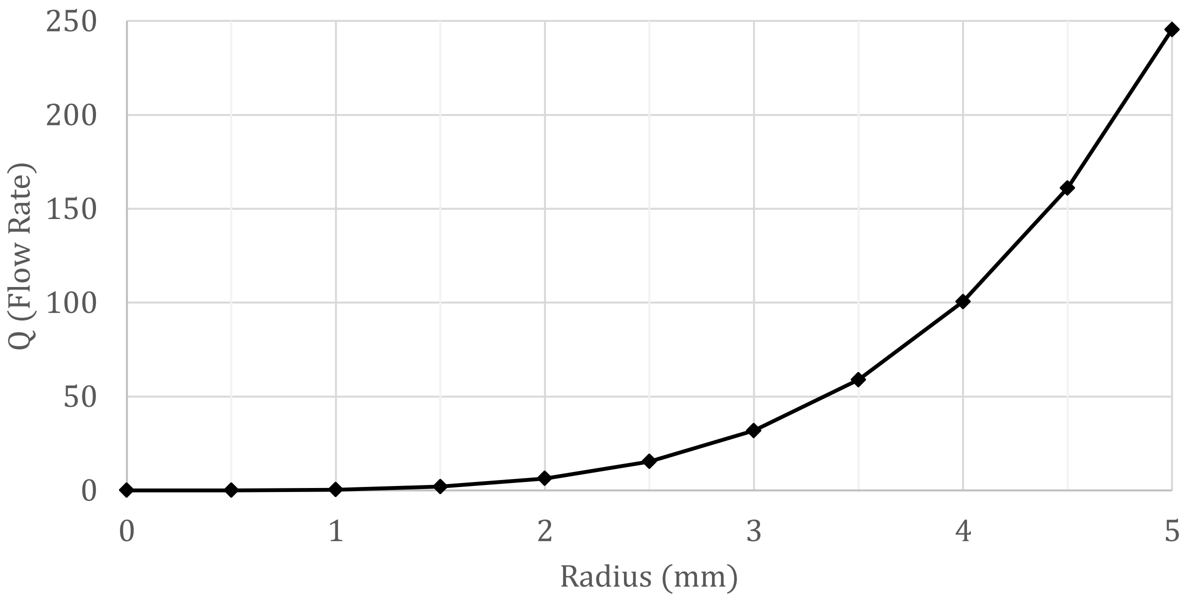
where Q is Flow Rate, P is Pressure, r is radius, η is Fluid Viscosity and l is length of tubing. You may assume that all parameters are fixed, except for radius, r.

[1 mark – drawing graph axes is KS3]

As only the radius is variable, this graph can be plotted as Q against r. A sensible x-axis would be the first few natural numbers.

[1 mark- fixing all variables correctly and drawing exponential]

The graph below has $P, \eta,$ and $l = 1$.



ii) Then use it to explain why Arteriosclerosis (the thickening of the artery walls) causes health complications.

[1 mark]

Problem solving. If the wall thickness increases, the cross-sectional area decreases. Just a half mm change in radius from 5mm to 4.5mm sees the flow rate decrease by almost 100.

[1 mark]

This then restricts the amount of blood flowing through the cardiovascular system.

3) **i) A virus has broken out. Each day that passes, t, the number of people infected, P, doubles. Show that this can be modelled as follows:**

$$\boldsymbol{P = Be^{kt}}$$

where B and k are constants.

[1 mark]

The rate of infection is proportional to the number of people who are already infected or:

$$\frac{dP}{dt} \propto P$$

$$\frac{dp}{dt} = kp$$

Separation of variables gives

$$\frac{dp}{P} = k\,dt$$

$$\int \frac{dp}{P} = \int k\,dt$$

[1 mark]

$$lnP = kt + c$$

$$P = e^{kt+c}$$

$$P = e^{kt}e^{c} \qquad (1)$$

$$P = Be^{kt}$$

ii) Initially 4 people are known to be infected. State the value of B.

[1 mark]

P=4 and t= 0, inserting these into (1) gives

$$4 = Be^{0k}$$

$$4 = B \times 1$$

$$B = 4$$

iii) After 20 days, 218 people are infected. Work out the value of k (to 2dp), thus generating a solution for the modelling of this virus.

[1 mark]

P=218, t= 20, B= 4 inserting these into (1) gives:

$$218 = 4e^{20k}$$

$$\frac{218}{4} = e^{20k}$$

$$\ln\left(\frac{218}{4}\right) = 20k$$

$$3.998200702 = 20k$$

$$k = 0.199910035$$

$$k = 0.2\ (to\ 2dp)$$

iv) Dr Lewis says that the constants B, k and t must all be positive integer values. Construct an argument in favour of, or rejecting her assertion.

[1 mark]

We already know t values are positive integers- fine.

B is also a positive integer.

[1 mark]

If k was a positive integer of value 1, after 21 days over 5 billion people would be infected. After 22 days, over 14 billion people would be infected. Realistically a virus that would infect the world's population twice over in less than a month is unlikely.

If k was 2, the 14 billion values would be achieved in 11 days. Therefore, we can reject her assertion.

v) Suggest an amendment that could be made to this model to make it more realistic.

[1 mark]

Any suggestion which considers the population size, herd immunity, people recovering, dying – or any form of limitation on the model so that it cannot grow forever.

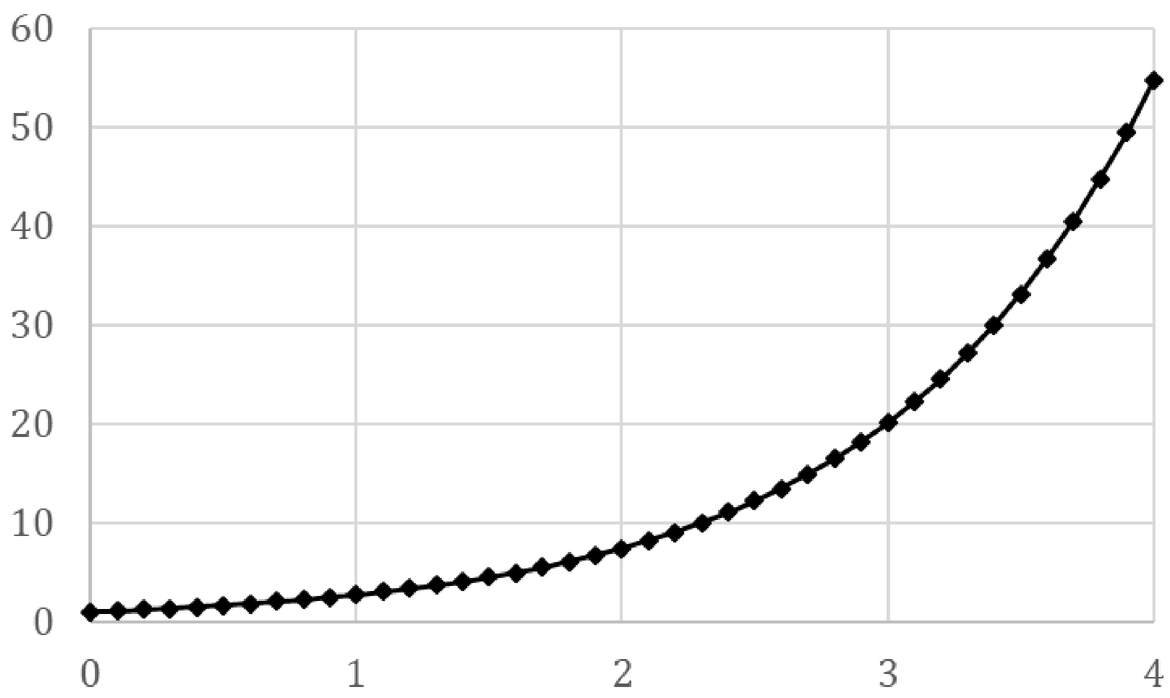
4) **Sketch the graph of the function that has the derivative:**

$$\frac{dy}{dx} = ky$$

No marks will be awarded for working out.

[1 mark]

Any natural exponential graph will be sufficient.



5) **The number of bacteria growing in a petri dish is proportional to the number of days it has been allowed to grow for. The model is as follows:**

$$\boldsymbol{N = e^{0.1t}}$$

What will be the rate of growth on the 100th day.

[1 mark]

$$N = e^{0.1t}$$

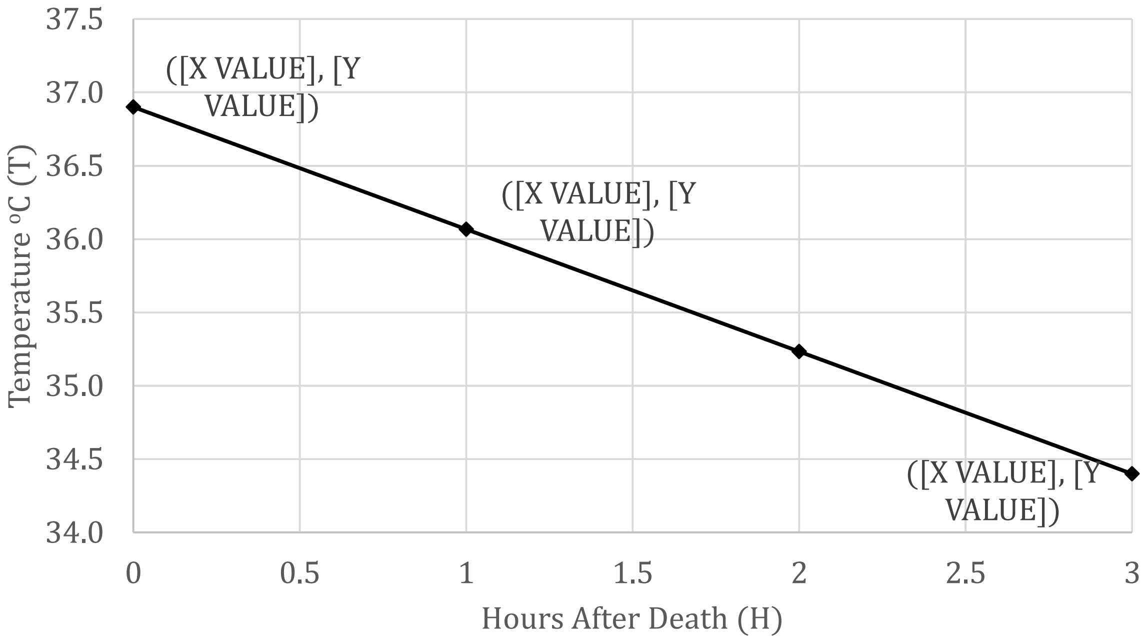
$$\frac{dN}{dt} = 0.1e^{0.1t}$$

t=100

$$\frac{dN}{dt} = 0.1e^{10}$$

$$\frac{dN}{dt} = 2202.646579$$

6) **The Glaister equation is a simple estimate of the hours elapsed since death based on the body temperature. You have only a small amount of the graph, to create a model then, extrapolate from this to work out body temperature after:**



Firstly, a model needs to be established. In this instance, a linear model of the form

$$y = mx + c$$

where m and c are constants can be found. The intercept, c is found at 36.9.

The gradient can be obtained by

$$\frac{y_1 - y_0}{x_1 - x_0}$$

i.e. the difference in y values divided by difference in x values. The gradient is

$$m = -\frac{5}{6}$$

[2 marks for correct model, 1 extra mark for correctly rearranged]

The model is, therefore,

$$T = -\frac{5}{6}H + 36.9$$

where T is temperature and H is time after death in in hours. Rearranging gives a model where H is the subject.

$$T - 36.9 = -\frac{5}{6}H$$

$$-\frac{6}{5}(T - 36.9) = H \qquad (1)$$

$$1.2(36.9 - T) = H$$

i) 20 ^{o}C

[1 mark]

Inserting $T = 20$ into (1) gives, H=20.28, 0.28 of an hour in minutes is 16.8, or 17 to the nearest minute. The answer is 16 hours and 17 minutes.

ii) 10 ^{o}C

[1 mark]

Inserting $T = 10$ into (1) gives, H=30.28, 0.28 of an hour in minutes is 16.8, or 17 to the nearest minute. The answer is 30 hours and 17 minutes.

iii) By example, show the problem with such a simplistic model.

[1 mark]

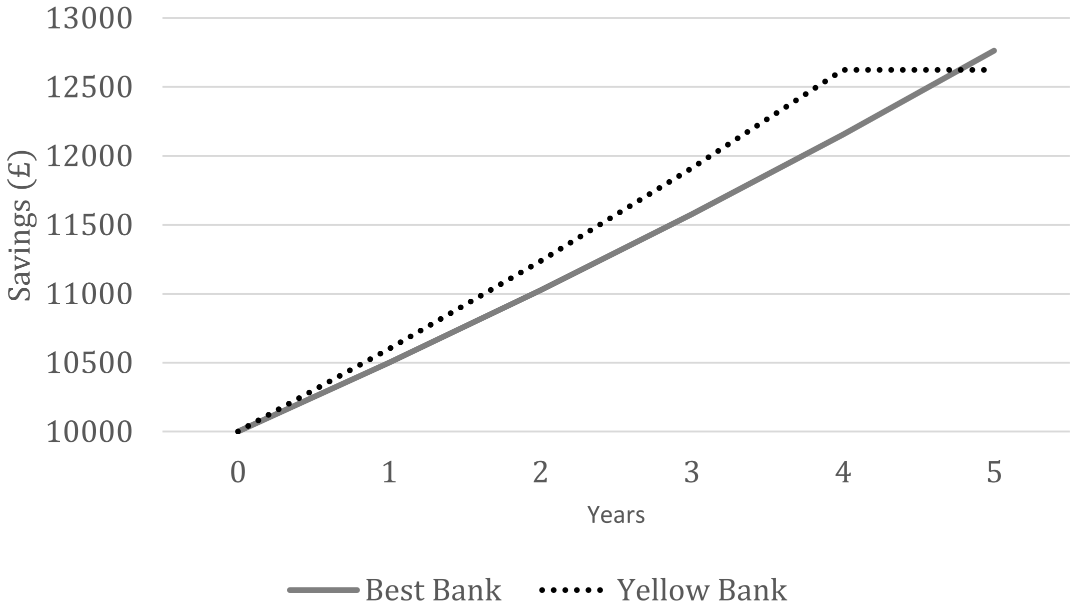
After 44.28 hours, the body temperature would be 0 ^{o}C, frozen, which in most places would be highly unlikely. Most locations have a temperature above freezing and so the objects within these locations would not be frozen either. This model does not consider room temperature or layers of insulation on the body.

7) **You want to save £10,000 in a high interest account for five years. Best Bank offer you 5 years at 5% compound interest, paid at the end of each year. Yellow Bank offer you 4 years at 6% compound interest paid at the end of each year, then 0% for the final year. Sketch your savings projection for both banks to help you decide which option will give you the best return.**

Here, the key is compound interest. This is the type of interest that considers the previous year's value. The multiplier for Best Bank is 1.05 and Yellow Bank is 1.06.

Although the Yellow Bank has the best rate, as it stops paying out after four years, Best Bank is the option that gives the best return.

[3 marks: 1 mark-Best Bank line; 1 mark- Yellow Bank line; 1 mark- correct conclusion]



Years	**0**	**1**	**2**	**3**	**4**	**5**
Best Bank (£)	10000	10500	11025	11576	12155	12762.82
Yellow Bank (£)	10000	10600	11236	11910	12624	12624.77

8) **Find the general solution for the following:**

To find the general solution, requires separation of variables. A common technique when establishing a model from a given rate of change.

$$\boldsymbol{a)\ \frac{dy}{dx} = x^2y^2}$$

[1 mark]

$$\frac{dy}{y^2} = x^2dx$$

$$\int \frac{dy}{y^2} = \int x^2dx$$

$$-\frac{1}{y} = \frac{x^3}{2} + c$$

$$-\frac{2}{y} = x^3 + d$$

[1 mark]

$$y = -\frac{2}{x^3 + d}$$

$$\boldsymbol{b)\frac{dk}{dm} = ke^m}$$

[1 mark]

$$\frac{dk}{k} = e^m dm$$

$$\int \frac{dk}{k} = \int e^m dm$$

$$ln(k) = e^m + c$$

[1 mark]

$$k = e^{e^m + c}$$

$$k = Ae^{e^m}$$

$$\boldsymbol{c)\frac{dy}{dx} = ysinx}$$

[1 mark]

$$\frac{dy}{y} = sinx\ dx$$

$$\int \frac{dy}{y} = \int sinx\ dx$$

$$ln(y) = -cosx + c$$

[1 mark]

$$y = e^{-cosx+c}$$

$$y = Ae^{-cosx}$$

9) **The height a ball reaches decreases by 10% every time it bounces off the floor. The ball initially starts 2m above the ground. Using a graph, or other method, determine how high the ball would be on the fourth bounce.**

[1 mark]

The height can be established by using the formula

$$H = IP^b$$

where H is height a ball reaches, I is the initial height and b is the number of bounces.

Therefore, H in this instance is

$$H = 2 \times 0.9^4$$

$$H = 1.3122$$

$$H = 1.31m$$

[1 mark]

Calculating it by iteration would give the following table

Bounce	0	1	2	3	4
Height (m)	2	1.8	1.62	1.46	1.31