AQA, Edexcel, OCR

A Level

A Level Mathematics

C1 Differentiation (Answers)

Name:

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Total Marks: /65

C1 - Differentiation (ANSWERS)
OCR, AQA, Edexcel

1. For each of the following functions calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$:

   (a) $\frac{dy}{dx} = 1, \qquad \frac{d^2y}{dx^2} = 0.$ [2]

   (b) $\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}, \qquad \frac{d^2y}{dx^2} = -\frac{2}{9}x^{-\frac{5}{3}}.$ [2]

   (c) $\frac{dy}{dx} = 4x^2, \qquad \frac{d^2y}{dx^2} = 8x.$ [2]

   (d) $\frac{dy}{dx} = 20x^3 + 3, \qquad \frac{d^2y}{dx^2} = 60x^2.$ [3]

   (e) $\frac{dy}{dx} = 2x - 1, \qquad \frac{d^2y}{dx^2} = 2.$ [3]

   (f) $\frac{dy}{dx} = -3x, \qquad \frac{d^2y}{dx^2} = -3.$ [3]

   (g) $\frac{dy}{dx} = 6x^2 - 32x + 30, \qquad \frac{d^2y}{dx^2} = 12x - 32.$ [3]

   (h) $\frac{dy}{dx} = 1 - \frac{2}{x^2}, \qquad \frac{d^2y}{dx^2} = \frac{4}{x^3}.$ [3]

   (i) $\frac{dy}{dx} = \frac{1}{4}\left(11x^{\frac{7}{4}} + 2x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right), \qquad \frac{d^2y}{dx^2} = \frac{1}{16}\left(77x^{\frac{3}{4}} - 6x^{-\frac{7}{4}} + 5x^{-\frac{9}{4}}\right).$ [3]

   (j) $\frac{dy}{dx} = 4x, \qquad \frac{d^2y}{dx^2} = 4.$ [4]

2. Find the gradients of the following functions at the speficied points:

   (a) 12. [2]

   (b) $-\frac{2}{3}$. [3]

   (c) 2. [4]

3. Consider the function $f(x) = x^2 - 2x + 4$:

   (a) First we verify that the point $(1, 3)$ lies on the curve which indeed it does. Nwo $f'(x) = 2x - 2$ and $f'(1) = 0$ and so $(1, 3)$ is a stationary point. [5]

   (b) $f''(x) = 2 > 0$ therefore the stationary point is a minimum. [2]

   (c) Completing the square yields $f(x) = (x-1)^2+3$ and so $(1, 3)$ is the minimum point of the quadratic; it is a stationary point. [2]

   (d) $f'(4) = 6$ therefore the gradient is 6. [2]

   (e) When $x = 4$, $f(4) = 12$, so the point $(4, 12)$ lies on the curve. Now we use coordinate geometry on the point $(4, 12)$ using a gradient of 6 to obtain the tangent line $g(x) = 6(x - 2)$ as required. [5]

4. Consider the function $f(x) = \frac{2}{3}x^3 + bx^2 + 2x + 3$, where b is some undetermined coefficient:

   (a) $f'(x) = 2x^2 + 2bx + 2$ and $f''(x) = 4x + 2b$. [4]

   (b) $f(x)$ has a stationary point at $x = 2$ therefore we know that $f'(2) = 0$, where $f'(2) = 0 = 10 + 4b$. Rearranging gives $b = -\frac{5}{2}$. [3]

   (c) We need to solve $f'(x) = 0$. Thus we need to solve the quadratic $2x^2 - 5x + 2 = 0$. The solutions are $x = 2$ and $x = \frac{1}{2}$. And so the other stationary point has coordinates $\left(\frac{1}{2}, \frac{83}{24}\right)$. [2]

   (d) $f''(2) = 3 > 0$ and $f''(\frac{1}{2}) = -3 < 0$, therefore the point at $x = 2$ is a minimum and the point at $x = \frac{1}{2}$ is a maximum point. [3]