Edexcel

A Level

A Level Mathematics

C1 Sequences and Series

(Answers)

Name:

MME

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Total Marks: /28

1. Consider an arithmetic sequence with k^{th} term given by $a_k = a + (k-1)d$. Prove that

$$S = \sum_{k=1}^{n} a_k = \frac{1}{2}n(2a + (n-1)d).$$

To begin, write out the sum term by term:

$$S = a + (a+d) + (a+2d) + (a+3d) + \cdots + (a+(n-2)d + (a+(n-1)d).$$

Now compute $S + S$ using the above by adding the first term to the last term, the second term to the second-to-last term, the third term to the third-to-last term and so on. To make this easier to visualise we may write:

$$\begin{aligned} S &= a \qquad\qquad\qquad + (a+d) \qquad\quad + \cdots + (a+(n-2)d + (a+(n-1)d). && (1)\\ S &= (a+(n-1)d) + (a+(n-2)d) + \cdots + (a+d) \qquad\quad + a. && (2) \end{aligned}$$

We now add both lines together to obtain:

$$2S = (2a + (n-1)d) + (2a + (n-1)d) + \cdots + (2a + (n-1)d) + (2a + (n-1)d),$$

Where we note that we simply have n lots of $(2a + (n-1)d)$ on the right. And so we write:

$$2S = n(2a + (n-1)d),$$

which when rearranged yields:

$$S = \frac{1}{2}n(2a + (n-1)d),$$

as required. [8]

2. Consider the sequence defined recursively by:

$$u_{n+2} = 3u_{n+1} - u_n, \qquad n \geq 1,$$

where,

$$u_1 = 1, \quad u_2 = 3.$$

(a) $u_3 = 8$ and $u_4 = 21$. [3]

(b) We first calculate $u_5 = 55$, then $\sum_{n=1}^{5} u_n = 1 + 3 + 8 + 21 + 55 = 88$. [3]

3. Consider the sequence defined recursively by:

$$u_{n+2} = 2u_{n+1} - u_n, \qquad n \geq 1,$$

where,

$$u_1 = 5, \quad u_2 = 7.$$

(a) $u_3 = 9, u_4 = 11$ and $u_5 = 13$. [3]

(b) $\sum_{n=1}^{5} u_n = 5 + 7 + 9 + 11 + 13 = 45$. [2]

(c) We find the n^{th} term of the sequence to be $u_n = 2n + 3$. [4]

(d) This is a hard question. To calculate this sum we adopt the method used in question 1). Let $S = \sum_{n=1}^{100} u_n$. We write this out term by term as:

$$S = 5 + 7 + 9 + 11 + \cdots + 201 + 203. \tag{3}$$

Now we rewrite S again but this time order the terms from largest to smallest rather than smallest to largest:

$$S = 203 + 201 + 199 + 197 + \cdots + 7 + 5. \tag{4}$$

We now add these two lines together term by term (in other words we do (3)+(4)) to get:

$$2S = 208 + 208 + 208 + 208 + \cdots + 208 + 208.$$

Now there are 100 terms on the right hand side and so we have:

$$2S = 100 \times 208,$$

which we rearrange to get:

$$S = 50 \times 208 = 10400.$$

Therefore we get the answer:

$$S = \sum_{n=1}^{100} u_n = 10400.$$

[5]