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AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C2 Calculus

Name:

MME

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Total Marks: /138

C2 - Calculus
MEI, OCR, AQA, Edexcel

1. For each of the following functions calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$:

   (a) $y = x$. [2]

   (b) $y = x^{\frac{1}{3}}$. [2]

   (c) $y = \frac{4}{3}x^3$. [2]

   (d) $y = 5x^4 + 3x + 20$. [3]

   (e) $y = x(x-1)$. [3]

   (f) $3x^2 + 2y = 108$. [3]

   (g) $y = 2x(x-3)(x-5)$. [3]

   (h) $y = \frac{x^2+3x+2}{x}$. [3]

   (i) $y = \frac{3x^3+6\sqrt{x}+3}{3x^{\frac{1}{4}}}$. [3]

   (j) $xy - 2y - 2x^3 + 4x^2 = 0$ (for $x \neq 2$). [4]

2. Find the gradients of the following functions at the speficied points:

   (a) $y = 2x^2$ at $x = 3$. [2]

   (b) $y = 3x^2 - \frac{2}{3}x + 1$ at $x = 0$. [3]

   (c) $xy - y - 2x^2 + 2x = 0$ at $x = 2$. [4]

3. Consider the function $f(x) = x^2 - 2x + 4$:

   (a) By finding $f'(x)$ show that $f(x)$ has a stationary point at $(1, 3)$. [5]

   (b) Determine the nature of the stationary point. [2]

   (c) By writing $f(x)$ in the form $f(x) = (x+a)^2 + b$, verify that $f(x)$ has a stationary point at $(1, 3)$. [2]

   (d) Calculate the gradient of $f(x)$ at $x = 4$. [2]

   (e) Hence, or otherwise show that the equation of the tangent line to $f(x)$ at $x = 4$ is $g(x) = 6(x-2)$, where $g(x)$ denotes the function of the tangent line. [5]

4. Consider the function $f(x) = \frac{2}{3}x^3 + bx^2 + 2x + 3$, where b is some undetermined coefficient:

   (a) Find $f'(x)$ and $f''(x)$. [4]

   (b) You are given that $f(x)$ has a stationary point at $x = 2$. Use this information to find b. [3]

   (c) Find the *coordinates* of the other stationary point. [2]

   (d) Determine the nature of both stationary points. [3]

5. Integrate the following functions. *Remember to include a constant of integration*:

   (a) $\frac{dy}{dx} = 1$. [2]

   (b) $\frac{dy}{dx} = 2x^{\frac{1}{3}}$. [2]

   (c) $\frac{dy}{dx} = \frac{3}{4}x^3$. [2]

   (d) $\frac{dy}{dx} = x^4 + 3x + 8$. [3]

   (e) $\frac{dy}{dx} = x(x - 1)$. [3]

   (f) $5x^2 + 2\frac{dy}{dx} = 10$. [3]

   (g) $\frac{dy}{dx} = 2x(x - 3)(x - 5)$. [3]

6. Consider the derivative $f'(x) = x + 3$. Find $f(x)$ using the fact that the point $(0, 1)$ lies on the curve. [4]

7. Consider the function $f'(x) = 16x^3 + 9x^2 + \frac{1}{2}$. You are given that $f(1) = -\frac{5}{2}$. Find $f(x)$. [5]

8. Consider the second derivative $f''(x) = 6x + 4$ of some cubic function $f(x)$.

   (a) Find $f'(x)$. [2]

   (b) You are given that $f(0) = 10$ and $f(1) = 13$, find $f(x)$. [4]

   (c) Find all the stationay points of $f(x)$ and determine their nature. [5]

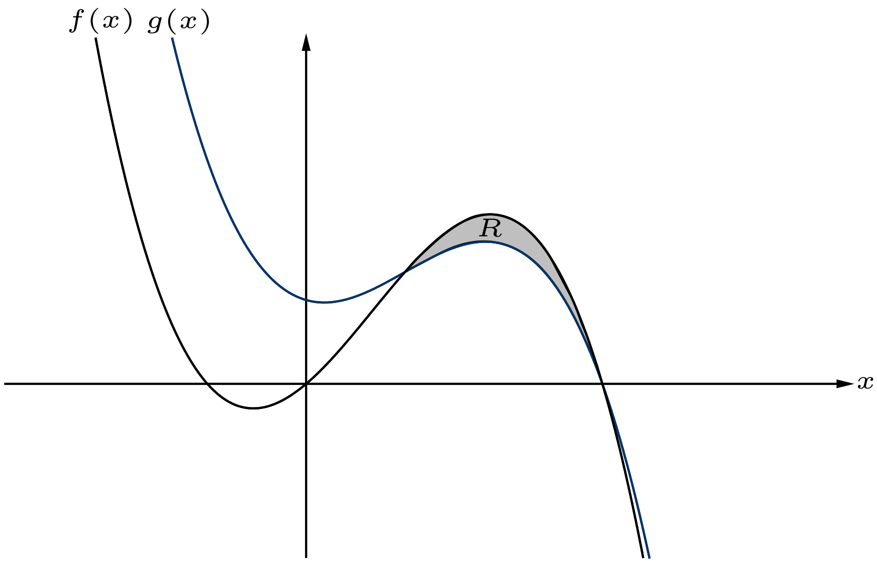
9. Consider the quadratic function $f(x) = 3x^2 + 2x + 4$.

   (a) Calculate $\int_{-1}^{2} f(x)\, dx$. [4]

   (b) What does the quantity found in part (a) represent? [2]

10. The gradient function of a curve is $\frac{dy}{dx} = 4x - \frac{1}{x^2}$. Find the equation of the curve using the fact that the curve passes through the point $(1, 4)$. [4]

11. Consider the functions $f(x) = -x^3 + 2x^2 + 3x$ and $g(x) = -x^3 + 3x^2 - x + 3$ sketched below.



(a) Find $f'(x)$ and hence show that $f(x)$ has turning points at when $x = \frac{2}{3} \pm \frac{\sqrt{13}}{3}$. [5]

(b) Find the points where $f(x)$ and $g(x)$ intersect. [4]

(c) Evaluate $\int_1^3 -x^3 + 2x^2 + 3x \, dx$. [3]

(d) Calculate the area under $g(x)$ between $x = 1$ and $x = 3$. [3]

(e) Using your answers to parts b) and c), calculate the area of the shaded region R. [2]

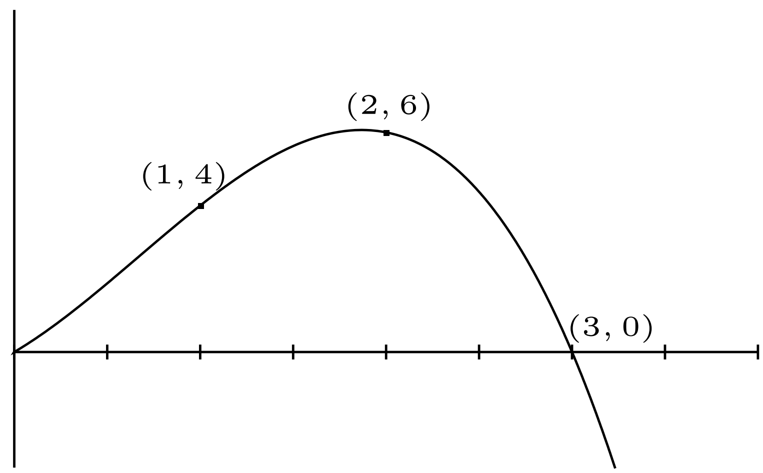
12. Consider the function $f(x) = 2x + 1$. By differentiating *from first principles* show that $f'(x) = 2$.

Hint: Calculate the following limit:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

[4]

13. Consider the curve plotted below.



(a) Use the trapezium rule with three stips to estimate the area of the region bounded by the curve and the axes. [4]