MEI

A Level

A Level Mathematics

C2 Sequences and Series

Name:

MME
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Total Marks: /73

1. Consider an arithmetic sequence with k^{th} term given by $a_k = a + (k-1)d$. Prove that

$$S = \sum_{k=1}^{n} a_k = \frac{1}{2}n(2a + (n-1)d).$$

[8]

Hint: this is a proof that you may have seen in class. To begin, write out the sum term by term:

$$S = a + (a+d) + (a+2d) + (a+3d) + \cdots + (a+(n-2)d + (a+(n-1)d).$$

Now compute $S+S$ using the above by adding the first term to the last term, the second term to the second-to-last term, the third term to the third-to-last term and so on. One the left hand side you have $2s$, but what do you have on the right hand side? Can you make any simplifications by collecting like terms and rearranging?

2. Consider the sequence defined recursively by:

$$u_{n+2} = 4u_{n+1} - u_n, \qquad n \geq 1,$$

where,

$$u_1 = 1, \quad u_2 = 2.$$

(a) Calculate u_3 and u_4. [3]

(b) Calculate $\sum_{n=1}^{5} u_n$. [3]

3. Consider the sequence defined recursively by:

$$u_{n+2} = 2u_{n+1} - u_n, \qquad n \geq 1,$$

where,

$$u_1 = 5, \quad u_2 = 7.$$

(a) Calculate u_3, u_4 and u_5. [3]

(b) Calculate $\sum_{n=1}^{5} u_n$. [2]

(c) Write u_n in the form $u_n = a + bn$ for some coeficients a, b to be determined. [4]

(d) Calculate $\sum_{n=1}^{100} u_n$. [5]

4. The k^{th} term of an arithmetic sequence is given by $a_k = 2 + 4(k-1)$.

   (a) Write down the value of the first term a_1. [1]

   (b) Calculate the value of the second term a_2. [2]

   (c) What is the common difference d in this arithmetic sequence?. [1]

   (d) Calculate the value of $\sum_{k=1}^{2} a_k$. [1]

   (e) Evaluate $\sum_{k=1}^{50} a_k$. [3]

5. An arithmetic sequence has fifth term $a_5 = 17$ and eigth term $a_8 = 26$.

   (a) Find the common difference d and the first term a. [2]

   (b) Evaluate $\sum_{k=1}^{30} a_k$. [3]

6. Consider the sequence with n^{th} term given by $u_n = 2n + 1$.

   (a) Calculate the first four terms u_1, u_2, u_3 and u_4. [2]

   (b) What type of sequence is this? [2]

   (c) Evaluate $\sum_{n=1}^{20} u_n$. [3]

7. Which of the following series are convergent? Which are divergent? *You are not required to evaluate the convergent series.*

   (a) $\sum_{n=0}^{\infty} 1$. [1]

   (b) $\sum_{n=0}^{\infty} 2n$. [1]

   (c) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$. [1]

   (d) $\sum_{n=0}^{\infty} 20 \left(\frac{1}{2}\right)^n$. [1]

   (e) $\sum_{n=0}^{\infty} 2^n$. [1]

   (f) $\sum_{n=0}^{\infty} r^n$, What condition do we need to impose for it to be convergent? [2]

   (g) $\sum_{n=0}^{\infty} (2r)^n$, What condition do we need to impose for it to be convergent? [2]

8. Consider the geometric sequence with general term given by $a_k = \left(\frac{1}{2}\right)^k$:

   (a) Calculate the value of a_3. [1]

   (b) Calculate the value of $\sum_{k=0}^{20} a_k$. [3]

   (c) Evaluate $\sum_{k=0}^{\infty} a_k$. [3]

9. Evaluate $\sum_{k=1}^{\infty} 4\left(\frac{1}{6}\right)^k$. [4]

10. Consider a geometric sequence with k^{th} term $a_k = ar^k$ such that:

$$a_1 = 1,$$
$$\sum_{k=0}^{\infty} ar^k = \frac{9}{2}.$$

   (a) Find the two sets of possible values for a and r.

   (Hint: form two equations in a and r using the information given in the question) [5]