AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Differentiation (Answers)

Name:

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Total Marks: /48

C3 - Differentiation (Answers)
MEI, OCR, AQA, Edexcel

1. Differentiate the following functions by using the product rule:

   (a) $\frac{dy}{dx} = 2x$. [1]

   (b) $\frac{dy}{dx} = \sin x + x\cos x$. [2]

   (c) $\frac{dy}{dx} = 2x\cos x - x^2 \sin x$. [2]

   (d) $\frac{dy}{dx} = \cos^2 x - \sin^2 x \qquad (= \cos(2x))$. [2]

   (e) $\frac{dy}{dx} = e^x\left[(x+1)\sin x + x\cos x\right]$. [3]

2. Differentiate the following functions by using the quotient rule:

   (a) $\frac{dy}{dx} = \frac{x(x-2)}{(x-1)^2}$. [2]

   (b) $\frac{dy}{dx} = \frac{e^x}{x^2}(x-1)$. [2]

   (c) $\frac{dy}{dx} = \sec^2 x$. [3]

3. Differentiate the following functions by using the chain rule:

   (a) $\frac{dy}{dx} = 2\cos(2x)$. [2]

   (b) $\frac{dy}{dx} = 2(x+1)$. [2]

   (c) $\frac{dy}{dx} = 4xe^{x^2}$. [2]

   (d) $\frac{dy}{dx} = 2xe^{x^2}\cos(e^{x^2})$. [2]

   (e) $\frac{dy}{dx} = 2e^{\sin(2x)}\cos(2x)$. [2]

4. Differentiate the following functions:

   (a) $\frac{dy}{dx} = \frac{1}{x}$. [1]

   (b) $\frac{dy}{dx} = \frac{2}{x}$. [2]

   (c) $\frac{dy}{dx} = x^2 e^{2x}(2x + 3)$. [3]

5. Differentiate the following functions implicitly:

   (a) $\frac{dy}{dx} = \frac{3x^2}{2}$. [2]

   (b) $\frac{dy}{dx} = \frac{x}{y}$. [2]

   (c) $\frac{dy}{dx} = \frac{1}{2y}\left(\sin(2x) + 2x\cos(2x)\right)$. [3]

   (d) $\frac{dy}{dx} = -\frac{x}{2y}$. [2]

6. *Challange:* This is a tricky question. We start with $y = \arcsin x$. We apply the sine function to both sides to get:

$$x = \sin y. \tag{1}$$

We now differentiate with respsect to y:

$$\frac{dx}{dy} = \cos y.$$

And so we have that:

$$\frac{dy}{dx} = \frac{1}{\cos y}.$$

But this isn't the answer as the right hand side contains a y. We need to rewrite $\cos y$ in terms of x. In order to do this we use the well-known trig identity:

$$\sin^2 y + \cos^2 y = 1.$$

We rearrange this identity to get:

$$\cos y = \pm\sqrt{1 - \sin^2 y}.$$

But from equation (1) we know that $x = \sin y$. Thus, using the above we write:

$$\cos y = \pm\sqrt{1 - x^2}.$$

And so we can write

$$\frac{dy}{dx} = \frac{1}{\pm\sqrt{1 - x^2}}.$$

But which square root do we take? In order to make this decision we need to consider the range of $y = \arcsin x$. We know that the range of $y = \arcsin x$ is $[-1, 1]$. This means that y ranges between -1 and 1. But for y values in the interval $[-1, 1]$, we know that $\cos y$ is positive. Thus we must take the positive square root. Hence our final answer is:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}.$$

[6]