AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Exponentials and Natural Logarithms (Answers)

Name:

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Total Marks: /29

C3 - Exponentials and Natural Logarithms (Answers)
MEI, OCR, AQA, Edexcel

1. Sketch the following functions, clearly indicating and points of intersection with the axis:

(a) [2]

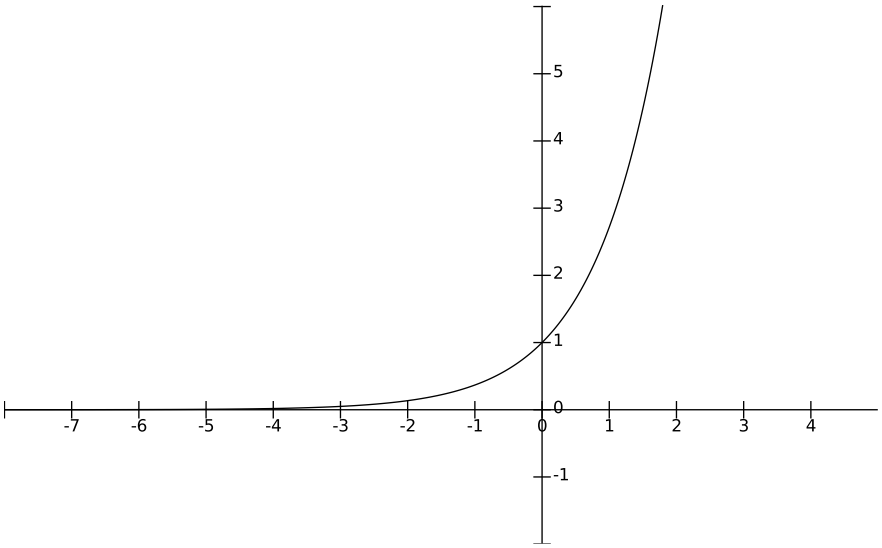

Figure 1: $y = e^x$.

(b) [2]

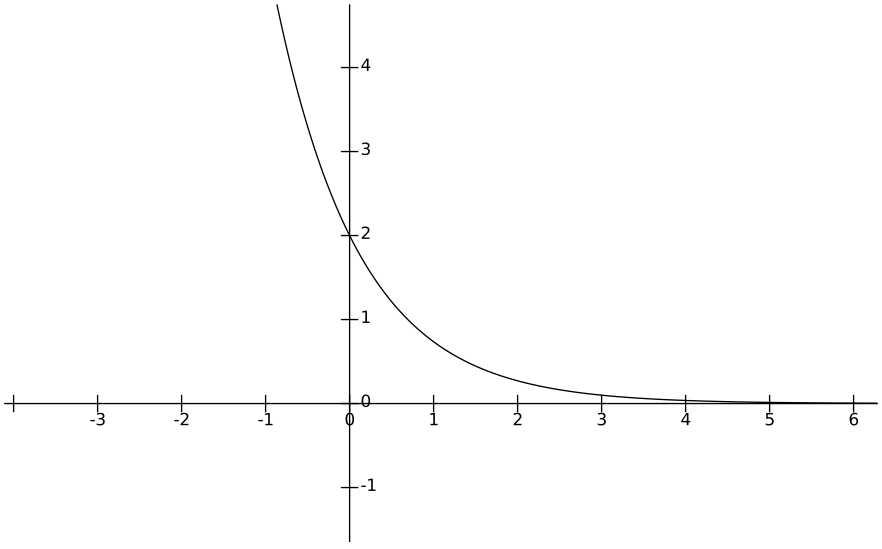

Figure 2: $y = 2e^{-x}$.

(c) [2]

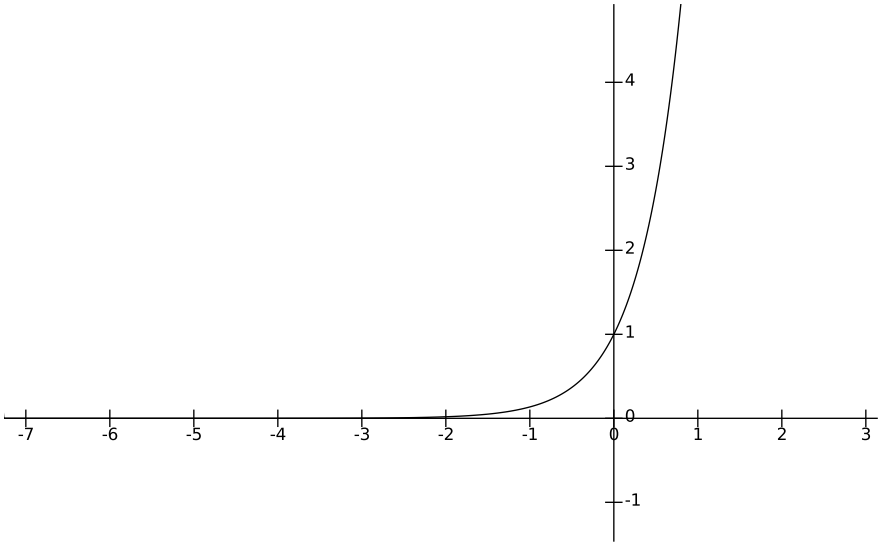

Figure 3: $y = e^{2x}$.

(d) [2]

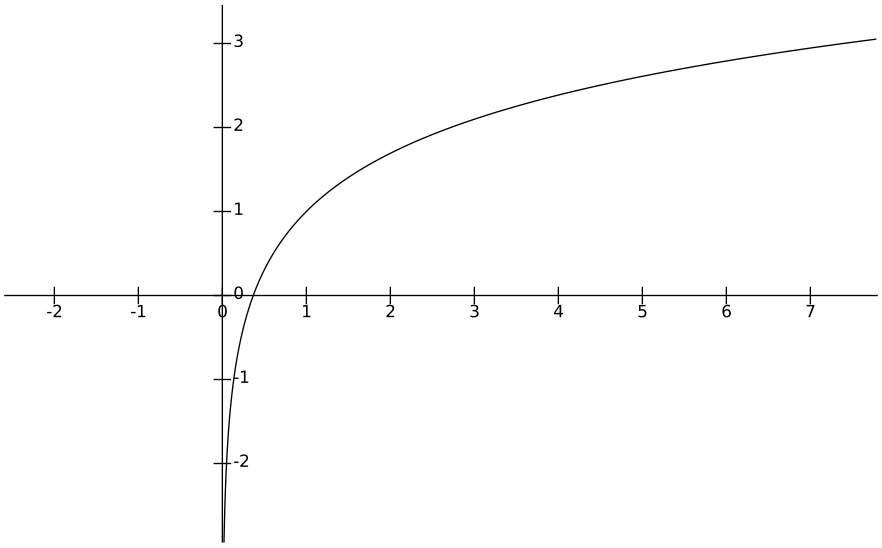

Figure 4: $y = \ln x + 1$.

(e) [2]

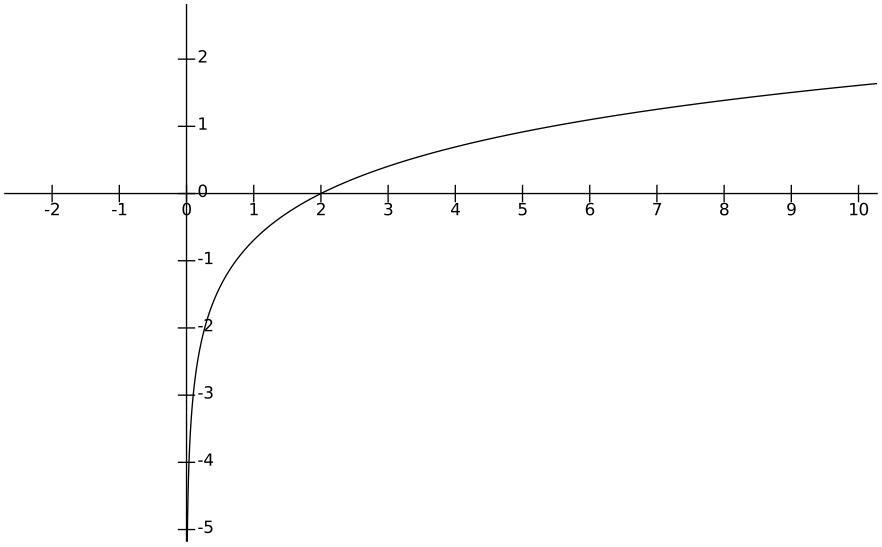

Figure 5: $y = \ln\left(\frac{1}{2}x\right)$.

2. Solve the following equations. Give your answers to two decimal places when necessary:

   (a) $x = \frac{\ln 3}{2} = 0.55$ (2 decimal places). [2]

   (b) $x = 0$. [2]

   (c) $x = \pm 1$. [2]

   (d) $x = \ln 2$ and $x = \ln 3$. [3]

   (e) $x = 0$ and $x = \ln 3 = 1.10$ (2 decimal places). [3]

3. Imagine that you put £100 into a savings account that pays fixed $\beta\%$ interest annually. After t years the balance of the account B is given by:

$$B = 100e^{t \ln 1.02}.$$

(a) Substituting $t = 3$ into the formula we get $B = 100e^{3 \ln 1.02} = 106.1208$ Thus the value after three years is £106.12. [2]

(b) We need to solve the inequality $B \geq 130$:

$$\begin{aligned} B &\geq 130 \\ \implies 100e^{t \ln 1.02} &\geq 130 \\ \implies e^{t \ln 1.02} &\geq 1.3 \\ \implies e^{\ln(1.02)^t} &\geq 1.3 \\ \implies (1.02)^t &\geq 1.3 \\ \implies t \ln 1.02 &\geq \ln 1.3 \\ \implies t &\geq \frac{\ln 1.3}{\ln 1.02} = 13.24896\cdots \end{aligned}$$

And so $t \geq 13.25$ years. Therefore the balance reaches £130 after roughly 13 years and 3 months. [4]

(c)

$$\begin{aligned} B &= 100e^{t \ln 1.02} \\ &= 100e^{\ln(1.02)^t} \\ &= 100(1.02)^t. \end{aligned}$$

And this is the form we require, with $a = 100$ and $k = 1.02$.

(d) The above formula is $B = 100(1.02)^t$. This is just a compound interest formula to mark a deposit of 100 units and an interest rate of 2%. Thus, the answer is 2% interest. [1]