

AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C3 Integration

Name:

M M E

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Total Marks: /44

1. Calculate the following integrals. Remember to include a constant of integration where necessary:

(a) $\int 2x \, dx.$ [1]

(b) $\int \sin x \, dx.$ [1]

(c) $\int \frac{1}{x} \, dx.$ [1]

(d) $\int_0^2 1 \, dx.$ [1]

2. Calculate the following integrals by using integration by substitution:

(a) $\int xe^{x^2} \, dx.$ [2]

(b) $\int x^2 \sin(x^3) \, dx.$ [2]

(c) $\int (x+1)e^{(x+1)^2} \, dx.$ [3]

(d) $\int \tan x \, dx.$ [3]

(e) $\int \sin x \cos x \, dx.$ [3]

(f) $\int \frac{\ln x}{x} \, dx.$ [3]

3. *Challenge:* Using the fact that $\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$, evaluate the following integral:

$$\int_{-\infty}^{\infty} e^{(2-x)(2+x)} \, dx,$$

Hint: you do not need to evaluate the integral by substitution, by parts or any other means. A simple bit of algebraic manipulation will yield the answer. [4]

4. Calculate the following integrals by using integration by parts:

(a) $\int x \sin x \, dx.$ [3]

(b) $\int x \cos x \, dx.$ [3]

(c) $\int x^2 \sin x \, dx.$ [3]

(d) $\int \ln x \, dx.$ (Hint: $\int \ln x \, dx = \int 1 \times \ln x \, dx.$) [3]

(e) $\int x^3 \ln x \, dx.$ [3]

5. *Challenge:* By using the technique of integration by parts, evaluate the following integral:

$$I = \int \sin(2x) \sin(x) \, dx.$$

[5]

6. Consider the function $y = x \sin x$ sketched below:

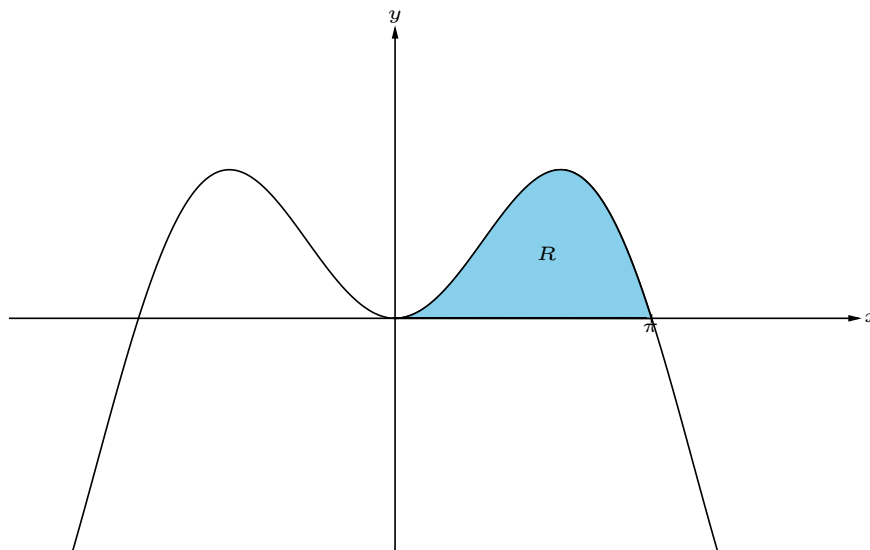


Figure 1: The graph of $y = x \sin x$.

(a) Calculate the area of the shaded region R . [3]