AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Trigonometry (Answers)

Name:

MME

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Total Marks: /59

1. Consider the well-known trigonometric identity:

$$\sin^2 x + \cos^2 x = 1.$$

   (a) We simply divide both sides of the equation by $\cos^2 x$:

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.$$

   Hence:

$$\tan^2 x + 1 = \sec^2 x,$$

   as required. [2]

   (b) This time we divide both sides by $\sin^2 x$ to get:

$$1 + \cot^2 x = \mathrm{cosec}^2 x.$$

   [2]

2. Simplify the following trig expressions:

   (a) $\sin(x + y)$. [2]

   (b) $\cos(2x)$. [2]

   (c) $\sin(2x)$. [2]

   (d) $\tan^2 x$. [2]

   (e) $4\sin^2(2x)$. [2]

   (f)

$$\begin{aligned} \sin x \cos(2x) + 2\sin x \cos^2 x &= \sin x \cos(2x) + 2\sin x \cos x \times \cos x \\ &= \sin x \cos(2x) + \sin(2x) \cos x \\ &= \sin(2x + x) \\ &= \sin(3x). \end{aligned}$$

   [3]

(g)

$$\begin{aligned}\cos^4 x - \frac{1}{2}\sin^2(2x) + \sin^4 x &= cos^4 x - \frac{1}{2}\left(2\sin x \cos x\right)^2 + \sin^4 x \\ &= \cos^4 x - 2\sin^2 x \cos^2 x + \sin^4 x \\ &= \left(\cos^2 x - \sin^2 x\right)^2 \\ &= (\cos(2x))^2 \\ &= \cos^2(2x).\end{aligned}$$

[3]

3. Write the following expressions in the form $R\sin(x+\alpha)$:

(a) $4\sin\left(x+\frac{\pi}{3}\right)$. [3]

(b) $3\sin\left(x+\frac{\pi}{4}\right)$. [3]

Turn over

4. The positive double angle formulas for sine and cosine are given by:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B,$$
$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

(a) Using the identities above:

$$\begin{aligned}\tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} \\ &= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \\ &= \frac{\frac{\sin A}{\cos A} \cos B + \frac{\cos A}{\cos A} \sin B}{\frac{\cos A}{\cos A} \cos B - \frac{\sin A}{\cos A} \sin B} \\ &= \frac{\tan A \cos B + \sin B}{\cos B - \tan A \sin B} \\ &= \frac{\tan A \frac{\cos B}{\cos B} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\cos B} - \tan A \frac{\sin B}{\cos B}} \\ &= \frac{\tan A + \tan B}{1 - \tan A \tan B},\end{aligned}$$

as required. [4]

(b)

$$\tan(2A) = \tan(A+A).$$

All we need to do is replace B with A in the above formula. This yields:

$$\begin{aligned}\tan(2A) &= \tan(A+A) \\ &= \frac{\tan A + \tan A}{1 - \tan A \tan A} \\ &= \frac{2\tan A}{1 - \tan^2 A}.\end{aligned}$$

[2]

5. Solve the following trigonometric equations for x values in the range $-\pi \leq x \leq \pi$:

(a) $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$. [3]

(b) Same answer as part a) since $\sec^2 x = 1 + \tan^2 x$. [3]

(c) $x = -\frac{11\pi}{12}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{11\pi}{12}$. [3]

Turn over

(d)

$$\begin{aligned}\frac{1}{2} &= \cos^4 x - 2\sin^2 x\cos^2 x + \sin^4 x\\ &= \left(\cos^2 x - \sin^2 x\right)\\ &= \cos^2(2x).\end{aligned}$$

Thus we need to solve:

$$\cos(2x) = \pm\frac{1}{\sqrt{2}}.$$

The solutions are $x = -\frac{7\pi}{8}, -\frac{5\pi}{8}, -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$. [4]

(e) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos\left(x + \frac{\pi}{3}\right) = 1$.

The only solution in the range is $x = -\frac{\pi}{3}$. [4]

(f)

$$\begin{aligned}1 &= \frac{1}{1-\tan x} - \frac{1}{1+\tan x}\\ &= \frac{2\tan x}{1-\tan^2 x}\\ &= \tan(2x).\end{aligned}$$

Thus we need to solve $\tan(2x) = 1$. The solutions are $x = -\frac{7\pi}{8}, -\frac{3\pi}{8}, \frac{\pi}{8}, \frac{5\pi}{8}$. [4]

6. Consider the function $f(x) = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$:

(a) Write $f(x) = \cos\left(x + \frac{\pi}{3}\right)$. [3]

(b) [3]

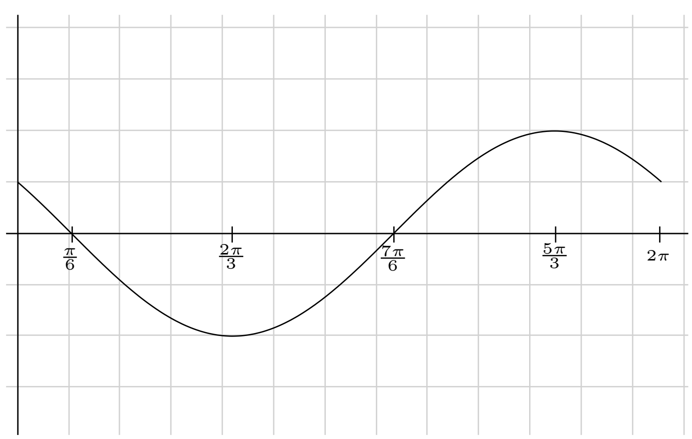

Figure 1: $y = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$.