

AQA, Edexcel, OCR, MEI

A Level

A Level Mathematics

C4 Trigonometry

Name:

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Total Marks: /59

1. Consider the well-known trigonometric identity:

$$\sin^2 x + \cos^2 x = 1.$$

(a) By manipulating the above identity, show that $\tan^2 x + 1 = \sec^2 x$. [2]

(b) Using the same technique used in part a) come up with a similar identity involving $\cot x$ and $\operatorname{cosec} x$. [2]

2. Simplify the following trig expressions:

(a) $\sin x \cos y + \cos x \sin y$. [2]

(b) $1 - 2 \sin^2 x$. [2]

(c) $2 \sin x \cos x$. [2]

(d) $\frac{1}{\cos^2 x} - 1$. [2]

(e) $16 \sin^2 x \cos^2 x$. [2]

(f) $\sin x \cos(2x) + 2 \sin x \cos^2 x$. [3]

(g) $\cos^4 x - \frac{1}{2} \sin^2(2x) + \sin^4 x$. [3]

3. Write the following expressions in the form $R \sin(x + \alpha)$:

(a) $2 \sin x + 2\sqrt{3} \cos x$. [3]

(b) $\frac{3}{\sqrt{2}}(\sin x + \cos x)$. [3]

4. The positive double angle formulas for sine and cosine are given by:

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B, \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B.\end{aligned}$$

(a) Using the identities above, prove that:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

[4]

(b) Hence show that:

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}.$$

[2]

5. Solve the following trigonometric equations for x values in the range $-\pi \leq x \leq \pi$:

(a) $\tan^2 x = 1.$ [3]

(b) $\sec^2 x = 2.$ [3]

(c) $\cos(2x) = \frac{\sqrt{3}}{2}.$ [3]

(d) $\cos^4 x - 2 \sin^2 x \cos^2 x + \sin^4 x = \frac{1}{2}.$ [4]

(e) $\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x = 1.$ [4]

(f) $\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} = 1.$ [4]

6. Consider the function $f(x) = \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x$:

(a) Write $f(x)$ in the form $f(x) = R \cos(x + \alpha)$, where R and α are constants to be determined. [3]

(b) Sketch the graph of $f(x)$ in the range $0 \leq x \leq 2\pi$. [3]