AQA, Edexcel, OCR

A Level

A Level Mathematics

Understand and use double angle formulae

Name:

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Total Marks:

C5- Understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$; understand geometrical proofs of these formulae- Answers

AQA, Edexcel, OCR

1) **For the following questions α, β and δ are all acute angles.**

$\text{Sin}(\alpha) = \frac{3}{5}$ **$\cos(\beta) = \frac{2}{3}$** **$\tan(\delta) = \frac{1}{4}$**

The answers for the following questions are applications of the following formula.

$$sin(A \pm B) = sinAcosB \pm sinBcosA \quad (1)$$

$$cos(A \pm B) = cosAcosB \mp sinAsinB \quad (2)$$

$$tan(A \pm B) = \frac{tanA \pm tanB}{1 \mp tanAtanB} \quad (3)$$

You also need to recall $cosec(x) = \frac{1}{\sin x}$; $\sec(x) = \frac{1}{\cos(x)}$; the double angle formulas are just applications of (1), (2) and (3), where B is replaced by another A.

[1 mark for each correct answer- 8 max]
Find **exact** values for:

(a) **sin(α + β)** $\frac{6+4\sqrt{5}}{15}$

(b) **sin(α – β)** $\frac{6-4\sqrt{5}}{15}$

(c) **cos(α + β)** $\frac{8-3\sqrt{5}}{15}$

(d) **cos(α + δ)** $\frac{13\sqrt{17}}{85}$

(e) **cos(β - δ)** $\frac{\sqrt{85}+8\sqrt{17}}{51}$

(f) **tan(α - β)** $\frac{6-4\sqrt{5}}{8+3\sqrt{5}}$

(g) **tan(α + δ)** $\frac{6+4\sqrt{5}}{8-3\sqrt{5}}$

(h) **tan(β + δ)** $\frac{36+34\sqrt{5}}{59}$

[1 mark for each correct answer- 8 max]
Find **exact** values for:

(i) **sin(2α)** $\frac{24}{25}$

(j) **cos(2α)** $\frac{7}{25}$

(k) **tan(2α)** $\frac{24}{7}$

(l) **sin(2β)** $\frac{4\sqrt{5}}{9}$

(m) **cos(2β)** $\frac{-1}{9}$

(n) **tan(2β)** $-4\sqrt{5}$

(o) **sec(2δ)** $\frac{17}{15}$

(p) **cosec(2δ)** $\frac{17}{8}$

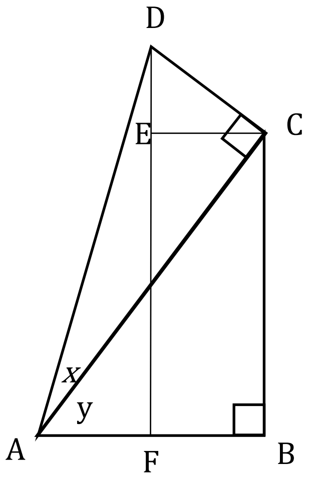
2) **Demonstrate geometric proof of the double angle formula for sine and cosine.**

For sine, we know the double angle formula is

$$\sin(x+y) = sinxcosy + cosxsiny$$

[1 mark for drawing]

We can demonstrate this geometrically by stacking two right-angle triangles on top of each other. The two triangles Triangle ACD, where AD is length 1 and Triangle ABC are shown below. There is also a triangle AFD and the side FD is oppposite angles x and y. If we



establish the length of FD we can prove the formula.

At the moment we can say

$$\sin(x+y) = DF$$

Writing DF as DE and EF gives (1)

1

$$\sin(x+y) = DE + EF$$

and writing EF as CB (same length as BCEF) is rectangle.

$$\sin(x+y) = DE + CB$$

[1 mark]

Establish some of the unknown lengths of the sides of the polygon.

$$\sin(x) = \frac{opp}{hyp} = \frac{CD}{1}$$

$$\Rightarrow CD = \sin(x)$$

And similarly, we know that AC can be written as

$$\cos(x) = \frac{adj}{hyp} = \frac{AC}{1}$$ (2)

$$\Rightarrow AC = cosx$$

[1 mark]

We can now use this to establish length CB

$$\sin(y) = \frac{opp}{hyp} = \frac{CB}{\cos(x)}$$

$$\Rightarrow CB = \cos(x)\sin(y)$$

Angle BCE we know is the same size as angle y (CE is parallel to AB – alternate angles). Therefore we know

$$\angle ECD = 90 - y$$

$$\angle CED = 90$$

$$\Rightarrow \angle EDC = y$$

as the angles in the triangle CDE must add up to 180. (3)

$$\cos(y) = \frac{adj}{hyp} = \frac{DE}{CD} = \frac{DE}{\sin(x)}$$

$$\Rightarrow DE = \sin(x)\cos(y)$$

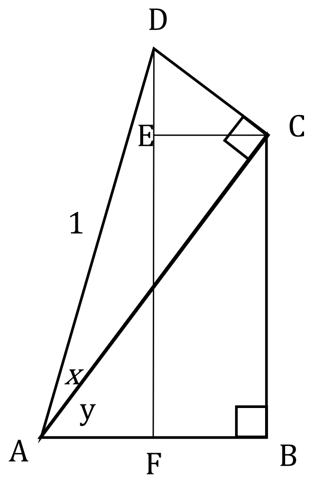
[1 mark]

Inserting (2) and (3) into (1) gives

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

For cosine, we know the double angle formula is

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$.



$$\cos(x+y) = \frac{adh}{hyp} = \frac{AF}{1} \qquad (4)$$

$$\cos(x+y) = AF$$

$$\cos(x+y) = AF = AB - FB \qquad (5)$$

[1 mark]

Using equation number (2) we can establish that

$$\cos(y) = \frac{adj}{hyp} = \frac{AB}{AC} = \frac{AB}{\cos(x)}$$

$$\Rightarrow AB = \cos(x)\cos(y)$$

[1 mark]

To obtain the length of FB, we can obtain the length of EC, and they are the same because BCEF is a rectangle. (6)

$$\angle EDC = y$$

$$DC = \sin(x)$$

which we previously showed.

$$\sin(y) = \frac{opp}{hyp} = \frac{EC}{\sin(x)}$$

$$\Rightarrow EC = \sin(x)\sin(y) = FB$$

[1 mark]

Putting (5) and (6) into (4) gives

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

3) **State the formula for $\sin(A+B)$, $\cos(A+B)$ and use these to write the formula for $\tan(A+B)$.**

$$sin(A+B) = sinAcosB + sinBcosA$$

$$cos(A+B) = cosAcosB - sinAsinB$$

We know that $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and that the same relationship is true for the double angle/additional formula.

[1 mark]
Thus, we can write

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$\tan(A+B) = \frac{sinAcosB + sinBcosA}{cosAcosB - sinAsinB}$$

[1 mark]
If we divide each term by $cosAcosB$ we get the following

$$\tan(A+b) = \frac{\frac{sinAcosB}{cosAcosB} + \frac{sinBcosA}{cosAcosB}}{\frac{cosAcosB}{cosAcosB} - \frac{sinAsinB}{cosAcosB}}$$

Cancelling out the $cosB$ left- hand part of the numerator allows us to write it as

$$\tan(A+b) = \frac{\frac{sinA}{cosA} + \frac{sinBcosA}{cosAcosB}}{\frac{cosAcosB}{cosAcosB} - \frac{sinAsinB}{cosAcosB}}$$

Cancelling out the $cosA$ right- hand part of the numerator allows us to write it as

$$\tan(A+b) = \frac{\frac{sinA}{cosA} + \frac{sinB}{cosB}}{\frac{cosAcosB}{cosAcosB} - \frac{sinAsinB}{cosAcosB}}$$

Cancelling out the left-hand part of the numerator allows us to write it as

$$\tan(A+b) = \frac{\frac{sinA}{cosA} + \frac{sinB}{cosB}}{1 - \frac{sinAsinB}{cosAcosB}}$$

[1 mark]
Now we have many terms with a sin numerator and cos denominator, meaning we can replace them with tan.

$$\tan(A+B) = \frac{tanA + tanB}{1 - tanAtanB}$$

4) **Demonstrate using your knowledge of trigonometric identities that the following is true:**

$$\boldsymbol{cos\ 2A = 1 - 2\ sin2A}$$

We know that

$$cos(A+B) = cosAcosB - sinAsinB$$

[1 mark]

so we can write that

$$\cos(2A) = cosAcosA - sinAsinA$$
$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \textbf{(1)}$$

[1 mark]

We also know that

$$1 = \sin^2 a + \cos^2 a$$

Rearranging this gives

$$\cos^2 a = 1 - \sin^2 a \qquad \textbf{(2)}$$

[1 mark]

Inserting (2) into (1) gives

$$\cos(2A) = (1 - \sin^2 a) - \sin^2 a$$

$$\cos(2A) = 1 - 2\sin^2 a$$

5) **Show** $\boldsymbol{\cos(3x) = 4\cos^3(x) - 3\cos(x)}$

$$\cos(3x) = \cos(3x + x)$$
$$= \cos(2x)\cos x - \sin(2x)\sin x$$

[1 mark]

Inserting double angle formulas for cosine and sine gives

$$\cos(3x) = (\cos^2 x - \sin^2 x)\, cosx - 2\sin^2 x\, cosx$$

[1 mark]

Replacing $\sin^2 x$ with $1 - \cos^2 x$

$$\cos(3x) = (\cos^2 x - (1 - \cos^2 x)cosx - 2(1 - \cos^2 x)cosx$$

$$\cos(3x) = \cos^3 x - cosx + \cos^3 x - 2cosx + 2\cos^3 x$$

$$\cos(3x) = 4\cos^3 x - 3cosx$$

6) **Simplify the following**

$$\frac{\boldsymbol{cos}(\mathbf{2}\boldsymbol{x})}{\boldsymbol{sin}(\boldsymbol{x}) + \boldsymbol{cos}(\boldsymbol{x})}$$

Using the double angle formula for $\cos(2x)$ allows us to write

$$\frac{\cos^2(x) - \sin^2(x)}{\sin(x) + \cos(x)}$$

[1 mark]

Spotting that the numerator is the difference of two squares, means we can rewrite it as

$$\frac{(\cos(x) - \sin(x))(\cos(x) + \sin(x))}{\sin(x) + \cos(x)}$$

[1 mark]

And as the bracket on the right of the numerator is the same as the denominator they will cancel to give 1, leaving us with:

$$\cos(x) - \sin(x)$$