Visit http://www.mathsmadeeasy.co.uk/ for more fantastic resources.

AQA, Edexcel, OCR

A Level

A Level Mathematics

Know and use exact values of sin and cos, tan and multiples thereof (Answers)

Name:

Mathsmadeeasy.co.uk

Total Marks:

Know and use exact values of sin and cos for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \pi$ and multiples thereof- Answers

AQA, Edexcel, OCR

1) **Evaluate the following expression.**

$$\boldsymbol{cos45^{\circ} cos30^{\circ} + sin45^{\circ} sin30^{\circ}}$$

We can evaluate this by using the following values of sine and cosine.

[1 mark]

$sin30° = \frac{1}{2}$

$cos30° = \frac{\sqrt{3}}{2}$

$sin45° = \frac{1}{\sqrt{2}}$

$cos45° = \frac{1}{\sqrt{2}}$

Now,

$cos45^{\circ} cos30^{\circ} + sin45^{\circ} sin30^{\circ} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{\sqrt{2}}\right)\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right]$

[1 mark]

$cos45^{\circ} cos30^{\circ} + sin45^{\circ} sin30^{\circ} = \left(\frac{1}{\sqrt{2}}\right)\left[\frac{\sqrt{3}+1}{2}\right] = \frac{\sqrt{3}+1}{2\sqrt{2}}$

2) **If $\boldsymbol{cos\frac{\pi}{6} sin\frac{\pi}{3} tan\frac{\pi}{6} = \frac{1}{4x}}$, then what is the value of x?**

We can find the value of x by using the following values of sine, cosine and tangent.

[1 mark]

$$sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Now,

$$cos\frac{\pi}{6} sin\frac{\pi}{3} tan\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{4x}$$

[1 mark]

$$\frac{\sqrt{3}}{4} = \frac{1}{4x}$$

Multiplying both sides by 4

$$\sqrt{3} = \frac{1}{x}$$

$$x = \frac{1}{\sqrt{3}}$$

3) **If $\boldsymbol{sin^2\frac{\pi}{6} + 1 = x + cos^2\frac{\pi}{3}}$, then what is the value of x?**

We can find the value of x by using the following values of sine and cosine.

[1 mark]

$$sin\frac{\pi}{6} = \frac{1}{2}$$

$$cos\frac{\pi}{3} = \frac{1}{2}$$

$$sin^2\frac{\pi}{6} + 1 = x + cos^2\frac{\pi}{3}$$

$$\left(\frac{1}{2}\right)^2 + 1 = x + \left(\frac{1}{2}\right)^2$$

$$\frac{1}{4} + 1 = x + \frac{1}{4}$$

Subtracting $\frac{1}{4}$ from both sides, we get

[1 mark]

$$x = 1$$

4) **If $\alpha + \beta + \gamma = 180°$, then what is the value of $sin\left(\frac{\alpha}{2} + \frac{\beta}{2}\right)$?**

We can find the value of x by using the following relation.

$$\alpha + \beta = 180° - \gamma$$

[1 mark]

$\text{Sin}\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \sin\left(\frac{\alpha+\beta}{2}\right) = \sin\left(\frac{180°-\gamma}{2}\right) = \sin\left(\frac{180°}{2} - \frac{\gamma}{2}\right)$

$i.e. \sin\left(90° - \frac{\gamma}{2}\right) = \sin\left(90° + \left(-\frac{\gamma}{2}\right)\right) = -\sin\left(-\frac{\gamma}{2}\right)$

Since $\sin(-\theta) = -sin\theta$

[1 mark]

Therefore

$-\sin\left(-\frac{\gamma}{2}\right) = -\left(-sin\frac{\gamma}{2}\right) = sin\frac{\gamma}{2}$

5) **What is the solution of $tan\theta + \sqrt{3} = 0$ in $\left[0.\frac{\pi}{2}\right]$?**

[1 mark]

We simplify the given equation.

$tan\theta + \sqrt{3} = 0$

Adding $-\sqrt{3}$ on both sides we get

$tan\theta + 0 = -\sqrt{3}$

$tan\theta = -\sqrt{3} < 0$

We know that

[1 mark]

From 0 to $\frac{\pi}{2}$ $tan\theta > 0$

Hence,

In $\left[0, \frac{\pi}{2}\right]$ there is no solution of $tan\theta + \sqrt{3} = 0$

6) **What is the smallest positive angle for which $2sin^2\theta + \sqrt{3}cos\theta + 1 = 0$?**

[1 mark]

We can find the smallest positive angle θ by solving the given equation and using the trigonometric identity

i.e. $sin^2\theta + cos^2\theta = 1$

Now,

$2sin^2\theta + \sqrt{3}cos\theta + 1 = 0$

$2(1 - cos^2\theta) + \sqrt{3}cos\theta + 1 = 0$

$2 - 2cos^2\theta + \sqrt{3}\cos\theta + 1 = 0$

$3 = 2cos^2\theta - \sqrt{3}\cos\theta$ or $2cos^2\theta - \sqrt{3}\cos\theta - 3 = 0$

[1 mark]

This is quadratic equation in $cos\theta$

Using quadratic formula i.e.

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ we get

$cos\theta = \frac{-(-\sqrt{3})\pm\sqrt{(-\sqrt{3})^2-4(2)(-3)}}{2(2)}$

$cos\theta = \frac{\sqrt{3}\pm\sqrt{3+24}}{4} = \frac{\sqrt{3}\pm\sqrt{3(1+8)}}{4} = \frac{\sqrt{3}\pm\sqrt{3(9)}}{4} = \frac{\sqrt{3}\pm3\sqrt{3}}{4}$

[1 mark]

Now

$cos\theta = \frac{\sqrt{3}+3\sqrt{3}}{4}$ or $cos\theta = \frac{\sqrt{3}-3\sqrt{3}}{4}$

$cos\theta = \frac{4\sqrt{3}}{4} = 1.73\ (Impossible)$ or $cos\theta = \frac{\sqrt{3}-3\sqrt{3}}{4} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$

$cos\theta = -\frac{\sqrt{3}}{2} < 0$ is in 2nd quadrant with $\theta = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

[1 mark]

$Required\ Angle = \frac{5\pi}{2}$

7) **What is the general solution of the trigonometric equation $\boldsymbol{tan\theta = cot\alpha}$?**

[1 mark]

We know that

$tan\theta = \frac{sin\theta}{cos\theta}$ & $cot\alpha = \frac{cos\alpha}{sin\alpha}$

Now,

$\frac{sin\theta}{cos\theta} = \frac{cos\alpha}{sin\alpha}$

$sin\theta sin\alpha = cos\theta cos\alpha$

$or\ \ cos\theta cos\alpha - \ sin\theta sin\alpha = 0$

[1 mark]

Using Fundamental law of trigonometry

$\cos(\theta + \alpha) = cos\theta cos\alpha - \ sin\theta sin\alpha$

$\cos(\theta + \alpha) = 0$

[1 mark]

The general solution of this equation is

$\theta + \alpha = n\pi + \frac{\pi}{2}$

Or simply

$\theta = n\pi + \frac{\pi}{2} - \alpha$

8) **What is the number of solutions of $\boldsymbol{tan^3\theta = 0}$ in the interval $\boldsymbol{\left[\pi, \frac{3\pi}{2}\right]}$?**

Here,

We can write $tan^3\theta$ as $(tan\theta)^3$

[1 mark]

Since $tan^3\theta = 0$

Therefore

$(tan\theta)^3 = 0$

[1 mark]

This implies $tan\theta = 0$

$tan\theta$ is equal to zero when

$\theta = 0, \pi, 2\pi, 3\pi, \ldots..$

Since only π lies in the interval $\left[\pi, \frac{3\pi}{2}\right]$

Hence $tan^3\theta = 0$ has only one solution in this interval.

9) **Find the value of $\boldsymbol{sin50^{\circ} - sin70^{\circ} + sin10^{\circ}}$?**

[1 mark]

We can find the value of this expression by using the following formula

$$sin(\alpha \pm \beta) = sin(\alpha)cos(\beta) \pm cos(\alpha)sin(\beta)$$

[1 mark]

$Here\ \ sin50^{\circ} = sin(60^{\circ} - 10^{\circ})$

$= sin60^{\circ}\cos10^{\circ} - \cos60^{\circ}\sin10^{\circ} = \frac{\sqrt{3}}{2}\cos10^{\circ} - \frac{1}{2}\sin10^{\circ} \ldots\ldots\ldots (a)$

$sin70^{\circ} = sin(60^{\circ} + 10^{\circ})$

$= sin60^{\circ}\cos10^{\circ} + \cos60^{\circ}\sin10^{\circ} = \frac{\sqrt{3}}{2}\cos10^{\circ} + \frac{1}{2}\sin10^{\circ} \ldots\ldots\ldots (b)$

[1 mark]

Now,

$sin50^{\circ} - sin70^{\circ} + sin10^{\circ} = (a) - (b) + sin10^{\circ}$

$= \frac{\sqrt{3}}{2}\cos10^{\circ} - \frac{1}{2}\sin10^{\circ} - \frac{\sqrt{3}}{2}\cos10^{\circ} - \frac{1}{2}\sin10^{\circ} + sin10^{\circ} = -\left(\frac{1}{2} + \frac{1}{2}\right) sin10^{\circ} + sin10^{\circ}$

$= -sin10^{\circ} + sin10^{\circ} = 0$ Answer.

10) **If $\boldsymbol{sin(\alpha - \beta) = -\frac{1}{2}}$ and $\boldsymbol{cos(\alpha + \beta) = \frac{1}{2}}$ then find the values of $\boldsymbol{\alpha}$ & $\boldsymbol{\beta}$?**

[1 mark]

We can find the value of this expression by using the fact that cosine has positive value in 4th quadrant while sine has negative value.

[1 mark]

Therefore,

$\alpha - \beta = -\frac{\pi}{6}$

And

$\alpha + \beta = \frac{\pi}{3}$

Adding both equations we get,

$2\alpha + \beta - \beta = -\frac{\pi}{6} + \frac{\pi}{3}$

$2\alpha = -\frac{\pi}{6} + \frac{2\pi}{6} = \frac{\pi}{6}$

$\alpha = \frac{\pi}{12}$

[1 mark]

And therefore

$\beta = \frac{\pi}{3} - \frac{\pi}{12} = \frac{4\pi}{12} - \frac{\pi}{12} = \frac{3\pi}{12}$

11) **If $\boldsymbol{cot\alpha cot\beta = 2}$ *then what is the value of* $\frac{cos(\alpha+\beta)}{cos(\alpha-\beta)}$?**

[1 mark]

Here

$$cot\alpha = \frac{cos\alpha}{sin\alpha} \text{ and } cot\beta = \frac{cos\beta}{sin\beta}$$

[1 mark]

Now,

$cot\alpha\ cot\beta = 2$

$\frac{cos\alpha\ cos\beta}{sin\alpha\ sin\beta} = 2$

i.e.

$cos\alpha\ cos\beta = 2sin\alpha\ sin\beta$ **(1)**

[1 mark]

Now

$\frac{cos(\alpha+\beta)}{cos(\alpha-\beta)} = \frac{cos\alpha\ cos\beta - sin\alpha\ sin\beta}{cos\alpha\ cos\beta + sin\alpha\ sin\beta}$

[1 mark]

Using (1),

$\frac{cos(\alpha+\beta)}{cos(\alpha-\beta)} = \frac{2\ sin\alpha\ sin\beta - sin\alpha\ sin\beta}{2\ sin\alpha\ sin\beta + sin\alpha\ sin\beta} = \frac{sin\alpha\ sin\beta}{3sin\alpha\ sin\beta} = \frac{1}{3}$

12) **If $\boldsymbol{cos\theta + sec\theta = 2}$ then what is the value of $\boldsymbol{cos^2\theta + sec^2\theta}$?**

We can find the value of this expression by taking square of given equation. i.e.

$(cos\theta + sec\theta)^2 = 2^2$

We know that $(a+b)^2 = a^2 + b^2 + 2ab$

[1 mark]

Therefore

$cos^2\theta + sec^2\theta + 2cos\theta sec\theta = 4$

$cos^2\theta + sec^2\theta + 2cos\theta\left(\frac{1}{cos\theta}\right) = 4$

$cos^2\theta + sec^2\theta + 2(1) = 4$

[1 mark]

This implies $cos^2\theta + sec^2\theta = 4 - 2 = 2$

13) If $\alpha + \beta = 90^\circ$ and $\alpha - \beta = 30^\circ$ then what will be the value of $sin3\alpha$?

[1 mark]

Solving the given equations, we get value of α then further we find the value of $sin3\alpha$

Adding both sides of given equations, we get

$2\alpha + \beta - \beta = 90^\circ + 30^\circ$

$2\alpha = 120^\circ$

$\alpha = \frac{120^\circ}{2} = 60^\circ$

Now,

$sin3\alpha = sin3(60^\circ) = sin180^\circ = 0$

14) If $\sqrt{\frac{1+sin\alpha}{1-sin\alpha}} = 4$ then what is the value of $\frac{sin\frac{\alpha}{2} + cos\frac{\alpha}{2}}{sin\frac{\alpha}{2} - cos\frac{\alpha}{2}}$?

[1 mark]

We use following identities to find the value of required expression.

$sin2\theta = 2sin\theta cos\theta$ and $sin^2\theta + cos^2\theta = 1$

[1 mark]

Now by putting $\theta = \frac{\alpha}{2}$ in above equations

$\sqrt{\frac{1+sin\alpha}{1-sin\alpha}} = 4$ becomes $\sqrt{\frac{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} + 2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} - 2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}} = 4$

[1 mark]

Here,

$sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} \pm 2sin\frac{\alpha}{2}cos\frac{\alpha}{2} = \left(sin\frac{\alpha}{2}\right)^2 + \left(cos\frac{\alpha}{2}\right)^2 \pm 2\left(sin\frac{\alpha}{2}\right)\left(cos\frac{\alpha}{2}\right)$

$= \left(\sin\frac{\alpha}{2} \pm cos\frac{\alpha}{2}\right)^2$

[1 mark]

Now

$$\sqrt{\frac{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} + 2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}{sin^2\frac{\alpha}{2} + cos^2\frac{\alpha}{2} - 2sin\frac{\alpha}{2}cos\frac{\alpha}{2}}} = \sqrt{\frac{\left(\sin\frac{\alpha}{2} + cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2} - cos\frac{\alpha}{2}\right)^2}} = \sqrt{\left(\frac{\sin\frac{\alpha}{2} + cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - cos\frac{\alpha}{2}}\right)^2} = 4$$

$$\frac{\sin\frac{\alpha}{2} + cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2} - cos\frac{\alpha}{2}} = 4$$

15) **Find the value of $\boldsymbol{cos\frac{\pi}{12}}$?**

We know that

$\pi\, rad. = 180^{\circ}$

[1 mark]

Therefore

$\frac{\pi}{12} = \frac{180^{\circ}}{12} = 15^{\circ}$

Now

$cos\frac{\pi}{12} = cos15^{\circ} = cos(45^{\circ} - 30^{\circ})$

Using Fundamental law of Trigonometry

[1 mark]

$cos\frac{\pi}{12} = cos45^{\circ}cos30^{\circ} + sin45^{\circ}sin30^{\circ}$

$cos\frac{\pi}{12} = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \left(\frac{1}{\sqrt{2}}\right)\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \left(\frac{1}{\sqrt{2}}\right)\left[\frac{\sqrt{3}+1}{2}\right] = \frac{\sqrt{3}+1}{2\sqrt{2}}$

16) **Simplify the expression $\boldsymbol{sin(\alpha - \beta) + 2cos\alpha sin\beta}$?**

[1 mark]

We can simplify this expression by using the following formula

$$sin(\alpha - \beta) = sin(\alpha)cos(\beta) - cos(\alpha)sin(\beta)$$

Now

$sin(\alpha - \beta) + 2cos\alpha sin\beta = sin(\alpha)cos(\beta) - cos(\alpha)sin(\beta) + 2cos(\alpha)sin(\beta)$

[1 mark]

Which is equal to

$sin(\alpha - \beta) + 2cos\alpha sin\beta = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$

Also we know that $sin(\alpha + \beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$

Therefore,

$sin(\alpha - \beta) + 2cos\alpha sin\beta = sin(\alpha + \beta)$

17) **What is the reference angle of $\boldsymbol{cos\theta = -\frac{1}{2}}$?**

[1 mark]

Here $cos\theta = -\frac{1}{2}$ is negative.

And $cos\theta$ is negative in 2nd and 3rd quadrant with standard angles of 120° and 240°

[1 mark]

And the corresponding reference angle is given by

$180^{\circ} - 120^{\circ} = 240^{\circ} - 180^{\circ} = 60^{\circ}$ Answer

18) **What is the solution of $\sqrt{3}csc\theta + 2 = 0 \ \ in \ [0, 2\pi]$?**

$\sqrt{3}csc\theta + 2 = 0$

Subtracting 2 from both sides

$\sqrt{3}csc\theta + 2 - 2 = 0 - 2$

$\sqrt{3}csc\theta = -2$

$csc\theta = -\frac{2}{\sqrt{3}}$

[1 mark]

This means

$sin\theta = -\frac{\sqrt{3}}{2}$

And $sin\theta$ is negative in 3rd and 4th quadrant and equals to $-\frac{\sqrt{3}}{2}$ with reference angle $\frac{\pi}{3}$

[1 mark]

Now

$\theta = \pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{3\pi}{3} - \frac{\pi}{3}, \frac{6\pi}{3} - \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

19) **What is the solution of $sin\theta = -\frac{1}{2} \ \ in \ [0, 2\pi]$?**

[1 mark]

We know that $sin\theta$ is negative in 3rd and 4th quadrant and equals to $\frac{1}{2}$ with reference angle $\frac{\pi}{6}$

[1 mark]

Now

$\theta = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\theta = \frac{6\pi}{6} - \frac{\pi}{6}, \frac{12\pi}{6} - \frac{\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

20) **What is the solution of $sec^2\theta = 2\ \ in\ [\pi, 2\pi]$?**

We know that

$sec\theta = \frac{1}{cos\theta}$

Now

$sec^2\theta = \left(\frac{1}{cos\theta}\right)^2 = 2$

$\frac{1}{cos^2\theta} = 2$

This means

$cos^2\theta = \frac{1}{2}$

[1 mark]

Taking square root on both sides

$cos\theta = \frac{1}{\sqrt{2}}$

We know that $cos\theta$ is positive in 1st and 4th quadrant and equals to $\frac{1}{\sqrt{2}}$ with reference angle $\frac{\pi}{4}$

[1 mark]

Now,

$\theta = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{8\pi}{4} - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

21) **What is the solution set of $\frac{tan3x - tan2x}{1 + tan3xtan2x} = 1$?**

We can use following formula to find its solution

$tan(\alpha - \beta) = \frac{tan\alpha - tan\beta}{1 + tan\alpha tan\beta}$

$Here\ \alpha = 3x\ \ and\ \ \beta = 2x$

[1 mark]

Now,

$\frac{tan\alpha - tan\beta}{1 + tan\alpha tan\beta} = \frac{tan3x - tan2x}{1 + tan3xtan2x} = 1$

Means

$\tan(3x - 2x) = 1$

$\tan x = 1$

[1 mark]

Hence Solution Set $= \left\{n\pi + \frac{\pi}{4}; n = 1,2,3, \dots\right\}$

22) Find the most general value of θ which satisfies both equations $\sin\theta = -\frac{1}{2}$ & $\tan\theta = \frac{1}{\sqrt{3}}$.

[1 mark]

We know that $sin\theta$ is negative in third quadrant while $tan\theta$ is positive.

And also at $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$sin\frac{7\pi}{6} = -\frac{1}{2}$

And

$tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

[1 mark]

Hence $\theta = 2n\pi + \frac{7\pi}{6}$ is the most general value.

23) What is the solution of $(2cosx - 1)(3 + 2cosx) = 0$ in the interval $0 \leq x \leq 2\pi$?

[1 mark]

Firstly, we simplify the given equation.

$(2cosx - 1)(3 + 2cosx) = 0$

$6cosx + 4cos^2x - 3 - 2cosx = 0$

$4cos^2x + 4cosx - 3 = 0$

Here $a = 4 , b = 4 \ and \ c = -3$

[1 mark]

Using quadratic formula for $cosx$

$cosx = \frac{-4\pm\sqrt{4^2-4(4)(-3)}}{2(4)} = \frac{-4\pm\sqrt{16+48}}{8} = \frac{-4\pm\sqrt{64}}{8} = \frac{-4\pm 8}{8}$

$cosx = \frac{-4+8}{8} \ or \ cosx = \frac{-4-8}{8}$

$cosx = \frac{4}{8} = \frac{1}{2} \ or \ cosx = \frac{-12}{8} = -\frac{3}{2} = -1.5 > 0 \ (Impossible)$

[1 mark]

Hence only $cosx = \frac{1}{2}$ is valid

Hence $x = \frac{\pi}{3} \ and \ x = 2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

24) What is the number of roots of quadratic equation $8sec^2\theta - 6sec\theta + 1 = 0$?

Using quadratic formula, we first solve it.

Here $a = 8$, $b = -6$ and $c = 1$

[1 mark]

Now,

$$sec\theta = \frac{-(-6)\pm\sqrt{(-6)^2-4(8)(1)}}{2(8)} = \frac{6\pm\sqrt{36-32}}{2(8)} = \frac{6\pm\sqrt{4}}{2(8)} = \frac{6\pm2}{16}$$

$$sec\theta = \frac{6+2}{16} \ or \ sec\theta = \frac{6-2}{16}$$

$$sec\theta = \frac{8}{16} = \frac{1}{2} \ or \ sec\theta = \frac{6-2}{16} = \frac{4}{16} = \frac{1}{4}$$

$sec\theta = \frac{1}{2}\ or\ \frac{1}{4}$ means $cos\theta = 2\ or\ 4$ which is impossible.

[1 mark]

Hence there are no roots of this equation.

25) What is the most general solution of $tan\theta = -1$ *and* $cos\theta = \frac{1}{\sqrt{2}}$?

[1 mark]

We know that $cos\theta$ is positive in 4th quadrant while $tan\theta$ is negative.

And also at $\theta = -\frac{\pi}{4}$

$$\tan\left(-\frac{\pi}{4}\right) = -1$$

And

$$\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

[1 mark]

Hence the most general solution is given by

$$\theta = n\pi + 2\pi - \frac{\pi}{4}$$

$$\theta = n\pi + \frac{7\pi}{4}$$

26) What is the number of solutions of $sin^2\theta = \frac{1}{2}$ in the interval $\left[0, \frac{3\pi}{2}\right]$?

Given that

$sin^2\theta = \frac{1}{2}$

[1 mark]

Taking square root on both sides, we get

$sin\theta = \pm\sqrt{\frac{1}{2}}$

$= sin\theta = \pm\frac{1}{\sqrt{2}}$

$sin\theta$ is positive and equals to $\frac{1}{\sqrt{2}}$ in 1st and 2nd quadrant when $\theta = \frac{\pi}{4}\ and\ \frac{3\pi}{4}$.

While in 3rd quadrant its value is $-\frac{1}{\sqrt{2}}$ at $\frac{5\pi}{4}$

[1 mark]

Hence there are three solutions of $sin^2\theta = \frac{1}{2}$ in the interval $\left[0, \frac{3\pi}{2}\right]$

27) What is the most general solution of $sin\alpha + cos\alpha = \sqrt{2}sin\theta$?

Given that $sin\alpha + cos\alpha = \sqrt{2}sin\theta$

[1 mark]

Dividing both sides by $\sqrt{2}$, we get

$\frac{sin\alpha+cos\alpha}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}sin\theta$

$\left(\frac{1}{\sqrt{2}}\right)sin\alpha + \left(\frac{1}{\sqrt{2}}\right)cos\alpha = sin\theta$

We know that $sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}\ and\ cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

[1 mark]

Therefore,

$sin\frac{\pi}{4}sin\alpha + \cos\frac{\pi}{4}cos\alpha = sin\theta$

$= sin\left(\frac{\pi}{4} + \alpha\right) = sin\theta$

This means

$\frac{\pi}{4} + \alpha = 2n\pi + \theta$

$\alpha = 2n\pi - \frac{\pi}{4} + \theta$

28) **Find the most general value of $\boldsymbol{\theta}$ which satisfies the equations $\boldsymbol{cos\theta = -\frac{1}{\sqrt{2}}}$ & $\boldsymbol{tan\theta = 1}$.**

We know that $cos\theta$ is negative in third quadrant while $tan\theta$ is positive.

[1 mark]

And also at $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$

And

$tan\frac{5\pi}{4} = 1$

[1 mark]

Hence $2n\pi + \frac{5\pi}{4}$ is the general solution of both.

29) **What is the most general solution of $\boldsymbol{sin\theta + \sqrt{3}cos\theta = 2}$?**

Given that $sin\theta + \sqrt{3}cos\theta = 2$

[1 mark]

Dividing both sides by 2 , we get

$\frac{sin\theta+\sqrt{3}cos\theta}{2} = \frac{2}{2}$

$\left(\frac{1}{2}\right) sin\theta + \left(\frac{\sqrt{3}}{2}\right) cos\theta = 1$

[1 mark]

We know that $sin\frac{\pi}{3} = \sqrt{3}\ and\ cos\frac{\pi}{3} = \frac{1}{2}$

[1 mark]

Therefore,

$\left(\frac{1}{2}\right) sin\theta + \left(\frac{\sqrt{3}}{2}\right) cos\theta = \cos\frac{\pi}{3}\ sin\theta + sin\frac{\pi}{3}\ cos\theta$

$= sin\left(\frac{\pi}{3} + \theta\right) = 1$

[1 mark]

This means $\frac{\pi}{3} + \theta = 2n\pi \pm \frac{\pi}{2}$

$\theta = 2n\pi \pm \left(\frac{\pi}{2} - \frac{\pi}{3}\right)$

30) For what value of θ the equation is true $cot\theta = sin2\theta$ in the interval $[0, 2\pi]$?

To find the value of θ following equations are helpful.

[1 mark]

$cot\theta = \frac{cos\theta}{sin\theta}$

$sin2\theta = 2sin\theta cos\theta$

Both are equal. Therefore,

$\frac{cos\theta}{sin\theta} = 2sin\theta cos\theta$

[1 mark]

Multiplying both sides by $sin\theta$

$cos\theta = 2sin^2\theta cos\theta$

$2sin^2\theta cos\theta - cos\theta = 0$

$cos\theta[2sin^2\theta - 1] = 0$

[1 mark]

$cos\theta = 0 \quad or \quad 2sin^2\theta - 1 = 0$

$cos\theta = 0 \quad or \quad 2sin^2\theta = 1$

$cos\theta = 0 \quad or \quad sin^2\theta = \frac{1}{2}$

$cos\theta = 0 \quad or \quad sin\theta = \frac{1}{\sqrt{2}}$

[1 mark]

$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \; or \; \theta = \frac{\pi}{4}, \frac{3\pi}{4}$