OCR 06 Algebra (Higher)

1. Simplify $5x^2 \times 4x^{-5}$.

2. Simplify $8y^{\frac{1}{2}} \div 2y^{\frac{3}{2}}$.

3. A function is given by $y = 5x - 3$. Write an expression for the inverse of this function.

4. A value, x, is input into this function.

$$x \rightarrow \boxed{\times 2} \rightarrow \boxed{-1} \rightarrow y$$

The output, y, is then input into this function.

$$y \rightarrow \boxed{+4} \rightarrow \boxed{\times 3} \rightarrow z$$

Complete the function below.

$$x \rightarrow \boxed{.....} \rightarrow \boxed{.....} \rightarrow z$$

5. Find the value of x in the following.

$$3(x-4) = 2(8-2x).$$

6. Write $x^2 + 8x + 5$ in the form $(x+a)^2 + b$.

7. Simplify $\dfrac{x^2 - 3x - 18}{x^2 + 4x + 3}$.

8. Rearrange the formula $y = \dfrac{x+3}{x-2}$ to make x the subject.

9. Factorise $3x^2 + 7x - 20$.

10. Find the formula for the nth term of the quadratic sequence 5, 12, 25, 44, 69, ….

11. Kasim adds together two fractions. His working is shown below.

$$\frac{2}{x+1} + \frac{1}{x-2} = \frac{2+1}{x+1+x-2} = \frac{3}{2x-1}$$

Identify Kasim's error and calculate the correct answer.

12. By completing the square, show that the solutions to the equation $x^2 - 4x - 7 = 0$ are $x = 2 - \sqrt{11}$ and $x = 2 + \sqrt{11}$.

13. Prove algebraically that the difference between the squares of two consecutive **odd** integers is even.

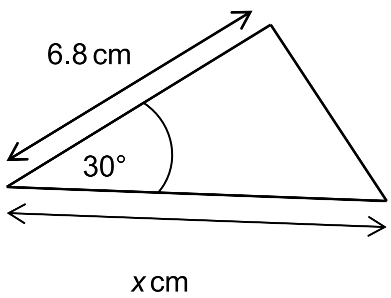
14. A formula for the *n*th term of the sequence 6, 11, 20, 33,… is $2n^2 - n + 5$. Write down a formula for the *n*th term of the sequence 10, 15, 24, 37,... and use it to show that 240 is a term of this sequence.

15. The equation $x^3 + x - 85 = 0$ has a solution between 4 and 5.
Show that this solution is 4.3 to one decimal place.

16. List the integer values that satisfy both of the inequalities $2y - 3 > -5$ and $3y + 1 < 13$.

17. A Fibonacci type sequence is given by $x_{n+2} = x_n + x_{n+1}$.
$x_1 = a$ and $x_2 = b$. Write expressions for the third and fifth terms. If the third term is 11 and the fifth term is 39, find the values of *a* and *b*.

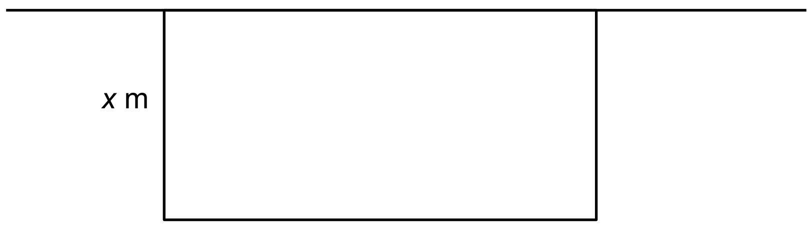
18. The area of the triangle below is 14.28 cm^2. Find the value of *x*.



Not to scale

19. A straight line has equation $y = 2x - 1$. A circle has equation $x^2 + y^2 = 9$. Find the coordinates of the points of intersection of the circle and the straight line. Give your answers correct to 3 significant figures.

20. Sarah wants to make a rectangular pen for her rabbits. One side of the pen will be a brick wall and she has 16 m of wire fencing for the other three sides. Find possible values for *x* if the area of the pen is at least 24 m^2.



Answers

1. $20x^{-3}$

2. $4y^{-1}$

3. $y = \dfrac{x+3}{5}$

4. $x \rightarrow \boxed{\times 6} \rightarrow \boxed{+9} \rightarrow z$

5. $$\begin{aligned} 3(x-4) &= 2(8-2x) \\ 3x-12 &= 16-4x \\ 7x-12 &= 16 \\ 7x &= 28 \\ x &= 4 \end{aligned}$$

6. Complete the square:
$x^2+8x+5=(x+4)^2-16+5=(x+4)^2-11$

7. $\dfrac{x^2-3x-18}{x^2+4x+3}=\dfrac{(x-6)\cancel{(x+3)}}{(x+1)\cancel{(x+3)}}=\dfrac{x-6}{x+1}$

8. $$\begin{aligned} y &= \frac{x+3}{x-2} \\ y(x-2) &= x+3 \\ xy-2y &= x+3 \\ xy-x &= 2y+3 \\ x(y-1) &= 2y+3 \\ x &= \frac{2y+3}{y-1} \end{aligned}$$

9. $(3x-5)(x+4)$

10. $u_n = 3n^2-2n+4$

11. Kasim has just added the numerators and added the denominators of the fractions. He should have written each fraction over a common denominator and then just added the numerators.
$\dfrac{2}{x+1}+\dfrac{1}{x-2}=\dfrac{2(x-2)+1(x+1)}{(x+1)(x-2)}=\dfrac{2x-4+x+1}{(x+1)(x-2)}=\dfrac{3x-3}{(x+1)(x-2)}$

12. $x^2 - 4x - 7 = 0$

$$(x-2)^2 - 4 - 7 = 0$$
$$(x-2)^2 - 11 = 0$$
$$(x-2)^2 = 11$$
$$x - 2 = \pm\sqrt{11}$$
$$x = 2 + \sqrt{11} \text{ or } x = 2 - \sqrt{11}$$

13. Let the two odd numbers be $(2x+1)$ and $(2x+3)$

$$(2x+1)^2 = 4x^2 + 4x + 1$$
$$(2x+3)^2 = 4x^2 + 12x + 9$$

Subtracting gives

$$(2x+3)^2 - (2x+1)^2 = 4x^2 + 12x + 9 - \left(4x^2 + 4x + 1\right)$$
$$= 4x^2 + 12x + 9 - 4x^2 - 4x - 1$$
$$= 8x + 8$$
$$= 8(x+1)$$

If x is an integer, $x+1$ is an integer so $8(x+1)$ is even.

14. Each term is 4 larger than the corresponding term in the first sequence so the formula for the nth term is $2n^2 - n + 9$

$$2n^2 - n + 9 = 240$$
$$2n^2 - n - 231 = 0$$
$$(2n+21)(n-11) = 0$$
$$n = -10.5 \text{ or } n = 11$$

n can only be a positive integer so $n = 11$ and therefore 240 is a term in the sequence.

15. Try $x = 4.25$, $x^3 + x - 85 = -3.98...$

Try $x = 4.35$, $x^3 + x - 85 = 1.66...$

As there is a sign change between $x = 4.25$ and $x = 4.35$, 4.3 is a solution to one decimal place.

16. $2y - 3 > -5$

$$2y > -2$$
$$y > -1$$

$$3y + 1 < 13$$
$$3y < 12$$
$$y < 4$$

The integers which satisfy both inequalities are in the interval $-1 < y < 4$ so 0, 1, 2 and 3.

17. The first term is a; the second term is b; the third term is $a + b$; (the fourth term is $a + 2b$); the fifth term is $2a + 3b$.
Form two simultaneous equations:
$2a+3b=39 \quad \rightarrow 2a+3b=39$
$a+b=11 \quad \times 3 \rightarrow 3a+3b=33$
Substituting gives $a=-6$
Substituting gives $b=17$

18. Area of triangle $=\frac{1}{2}ab\sin C$

$\frac{1}{2}\times x\times 6.8\times \sin 30=14.28$

$x=\frac{14.28\times 2}{6.8\times \sin 30}=8.4$ cm

19. Substitute $y=2x-1$ into $x^2+y^2=9$

$x^2+(2x-1)^2=9$
$x^2+4x^2-4x+1=9$
$5x^2-4x-8=0$

Using the quadratic formula: $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{4\pm\sqrt{4^2-4\times 5\times -8}}{2\times 5}=1.7266...$ or -0.9266…i.e. 1.73 or -0.927 (to 3 significant figures)

Substitute into $y=2x-1$
If $x=1.73$, $y=2.46$
If $x=-0.927$, $y=-2.85$

So the coordinates of the points of intersection are (1.73, 2.45) and (–0.927, –2.85).

20. Let the width be x
$x(16-2x)\geq 24$
$-2x^2+16x-24\geq 0$
$(x-2)(6-x)\geq 0$

$2\leq x\leq 6$ (in metres)

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AO1	1	Simplify an expression involving products of powers			
AO1	2	Simplify an expression involving quotients of powers			
AO1	3	Write an expression for an inverse function			
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AO1	6	Complete the square			
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AO1	8	Rearrange a formula where the subject appears twice			
AO1	9	Factorise a quadratic expression where $a \neq 0$ or 1			
AO1	10	Find a formula for the *n*th term of a quadratic sequence			
AO2	11	Add two algebraic fractions			
AO2	12	Solve a quadratic equation by completing the square			
AO2	13	Use algebra to construct a proof			
AO2	14	Use a formula for the *n*th term of a quadratic sequence			
AO2	15	Find an approximate solution to a cubic equation			
AO3	16	Solve two linear inequalities			
AO3	17	Set up and solve two simultaneous equations to solve a problem involving sequences			
AO3	18	Substitute values into a formula and solve to find an unknown			
AO3	19	Solve linear and quadratic simultaneous equations and use the quadratic formula			
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