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A24 RECOGNISE AND USE SEQUENCES OF TRIANGULAR, SQUARE AND CUBE NUMBERS, SIMPLE ARITHMETIC PROGRESSIONS, FIBONACCI TYPE SEQUENCES, QUADRATIC SEQUENCES, AND SIMPLE GEOMETRIC PROGRESSIONS (r^n WHERE n IS AN INTEGER, AND r IS A RATIONAL NUMBER > 0 **OR A SURD**) **AND OTHER SEQUENCES (higher tier)**

GEOMETRIC PROGRESSIONS

In mathematics, a **geometric progression**, also known as a **geometric sequence**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio.

Consider the sequence

$$2 \qquad 8 \qquad 32 \qquad 128 \qquad 512$$

Each term in the sequence is 4 times the previous term.

Consider the sequence

$$1 \qquad -3 \qquad 9 \qquad -27 \qquad 81$$

Each term in the sequence is −3 times the previous term.

Sequences such as these are called **geometric progressions**.

Let us write down a general geometric progression, using algebra. We shall take ***a*** to be the first term. The multiplying factor is known as the common ratio and denoted by ***r***.

With this notation, the general geometric progression can be expressed as

$$a \qquad a \times r \qquad a \times r \times r \qquad a \times r \times r \times r$$

Hence,

$$a \qquad ar \qquad ar^2 \qquad ar^3$$

EXAMPLE 1

Which of these sequences is a geometric progression?

(a)	1	2	3	4	(b)	1	2	4	7
(c)	1	2	4	8	(d)	1	2	3	5

(a) 1 2 3 4 No ← Adding 1 not multiplying by 1

+1 +1 +1

(b) 1 2 4 7 No ← Not multiplying by the same number each time

+1 +2 +3

(c) 1 2 4 8 Yes ✓ ← Multiplying by 2 each time

×2 ×2 ×2

(d) 1 2 3 5 No ← Not multiplying by the same number each time

1 + 2 2 + 3

EXAMPLE 2

Find the next two terms of the following geometric sequences.

(a) $1 \quad -4 \quad 16 \quad -64$ (b) $\sqrt{2} \quad 2 \quad 2\sqrt{2} \quad 4$

(a) $1 \quad -4 \quad 16 \quad -64$ ← Multiplier =any term ÷ term before eg $16 \div -4 = -4$

$\times -4 \quad \times -4 \quad \times -4$

Next term $= -64 \times -4 = 256$ ← 5th term = 4th term × (−4)

Next term $= 256 \times -4 = -1024$ ← 6th term = 5th term × (−4)

(b) $\sqrt{2} \quad 2 \quad 2\sqrt{2} \quad 4$ ← Multiplying by $\sqrt{2}$ each time e.g. $2\sqrt{2} \div \sqrt{2} = 2$

$\times\sqrt{2} \quad \times\sqrt{2} \quad \times\sqrt{2}$

Next term $= 4 \times \sqrt{2} = 4\sqrt{2}$ ← 5th term = 4th term × $\sqrt{2}$

Next term $= 4\sqrt{2} \times \sqrt{2} = 8$ ← $4\sqrt{2} \times \sqrt{2} = 4 \times (\sqrt{2} \times \sqrt{2}) = 4 \times 2 = 8$

EXERCISE 1:

1. Find the next four terms of the following geometric sequences.

(a)	2	6	18	(b)	3	12	48
(c)	5	25	125	(d)	8	12	18
(e)	3	7.5	18.75	(f)	100	50	25
(g)	–6	12	–24	(h)	–8	2	0.5
(i)	9	–3	1	(j)	20	4	0.8

2. Find the next three terms of the following geometric sequences.
Leave your answers in surd form.

(a)	2	$2\sqrt{3}$	6	(b)	$\sqrt{5}$	5	$5\sqrt{5}$
(c)	5	$5\sqrt{7}$	35	(d)	8	$4\sqrt{2}$	4
(e)	3	$\sqrt{3}$	1	(f)	$3\sqrt{2}$	6	$6\sqrt{2}$

REAL LIFE PROBLEMS INVOLVING GEOMETRIC SEQUENCES

The concept of geometric progressions can be applied to real life situations.

EXAMPLE 3

Stocks of a company are initially issued at the price of £18.
The value of the stock grows by 20% every year.

(a) Show that the value of a stock follows a geometric sequence.

(b) Calculate the value of the stock ten years after the initial public offering.

(a) It is a geometric sequence as multiplied by 1.20 each year ← Multiplier = 100% + 20% = 120% = 1.20

(b) nth term = $18 \times (1.20)^n$ ← Starts at £18 and multiplied by 1.20 each year

10th year = $18 \times (1.20)^{10} = 111.4512...$ ← 10 years so $n = 10$

New price = £111.45 ← Put your answer in correct money form to 2 d.p.

EXAMPLE 4

A liquid is kept in a barrel.
At the start of a year the barrel is filled with 200 litres of the liquid.

Due to evaporation, at the end of every year the amount of liquid in the barrel is reduced by 12% of its volume at the start of the year.

(a) Calculate the amount of liquid in the barrel at the end of the first year.

(b) Show that the amount of liquid in the barrel at the end of seven years is approximately 81.7 litres.

(a) This is a percentage **decrease** so multiplied by 0.88 each year ← Multiplier = 100% − 12% = 88% = 0.88

End of first year = $200 \times 0.88 = 176$ litres ← Starts at 200 and multiplied by 0.88 each year

(b) End of 7th year $= 200 \times (0.88)^7$ ← 7 years so $n = 7$

$= 81.735....$ ← Show extra decimal places for rounding

$= 81.7$

EXAMPLE 5

A tennis ball is dropped from a height of 20 feet.
After the ball hits the floor, it rebounds to 85% of its previous height.

(a) How high will the ball rebound after its third bounce?

(b) Write down a general formula for this sequence.

(a) 85% = 0.85 ← Multiplier is 85%

After first bounce = $20 \times 0.85 = 17$ ← Starts at 20 and multiplier is 0.85

After second bounce = $17 \times 0.85 = 14.45$ ← $20 \times 0.85 \times 0.85 = 20 \times 0.85^2 = 14.45$

After third bounce = $14.45 \times 0.85 = 12.2825$ ← $20 \times 0.85 \times 0.85 \times 0.85 = 20 \times 0.85^3$ $= 12.285$

(b) nth term = $20 \times (0.85)^n$

EXERCISE 2:

1. A hot tub in a hotel suite is not hot enough.
   The hotel tells you that they will increase the temperature by 5% each hour.

   If the current temperature of the hot tub is 60°F, what will be the temperature of the hot tub after 10 hours? Give your answer correct to 3 significant figures.

2. A culture of bacteria doubles every 3 hours.

   If there are 100 bacteria at the beginning, how many bacteria will there be after 24 hours?

3. A mine worker discovers an ore sample containing 600 mg of radioactive material.
   It is discovered that the radioactive material has a half life of 1 day.

   Find the amount of radioactive material in the sample at the end of the 8th day.

4. A fisherman caught 350 kg of fishes on Monday.
   From Monday to Friday, the amount of fishes, he caught increased by 10% per day.

   What is the total amount of fishes did the fisherman catch in the first five days?
   Give your answer as an integer.

5. A city has a population of 30 000 people in the year 2000.

   If the population increases 15% per year, what will the population be in 2010?

6. At the age of 16, Tara began receiving yearly payments from a trust fund.
   On each succeeding birthday, she received twice as much as in the preceding year.

   If she had received a total of £148 617 by age 21, how much did she receive at age 16?

7. Suppose you deposit £1 into your piggy bank on the first day of December and, on each day of December after that, you deposit twice as much as on the previous day.

   How much will you have in the bank after the last deposit?

8. Amanda decides that starting in January she will deposit £100 into her bank account at the start of each month. Her account earns 0.25% interest per month.

   Interest is calculated at the end of each month.

   (a) In the middle of February, how much money is in Amanda’s account?

   (b) In the middle of March, how much money is in Amanda’s account?

   (c) In the middle of the nth month (where January is the 1st month, February is the 2nd month, etc.), how much money is in Amanda's account? Give your answer in closed form.

   (d) In the middle of December, how much money is in Amanda's account?

Give all your answers to 2 decimal places.

ANSWERS

Exercise 1

1. (a) 54, 162, 486, 1458 (b) 192, 768, 3072, 12288
(c) 625, 3125, 15625, 78125 (d) 27, 40.5, 60.75, 91.125
(e) 46.875, 117.1875 ,292.96875, 732.421875
(f) 12.5, 6.25, 3.125, 1.5625 (g) –48, 96, –192, 384
(h) $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ (i) $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}$
(j) $\frac{4}{25}, \frac{4}{125}, \frac{4}{625}, \frac{4}{3125}$

2. (a) $6\sqrt{3}$, 18, $18\sqrt{3}$ (b) 25, $25\sqrt{5}$, 125
(c) $35\sqrt{7}$, 245, $245\sqrt{7}$ (d) $2\sqrt{2}$, 2, $\sqrt{2}$
(e) $\frac{\sqrt{3}}{3}, \frac{1}{3}, \frac{\sqrt{3}}{9}$ (f) 12, $12\sqrt{2}$, 24

Exercise 2

1. 97.7°F

2. 25 600

3. 2.34375

4. 2138

5. 121 367

6. £2359

7. 2 147 483 648

8. (a) £100.25 (b) £100.50
(c) 100×1.0025^n (d) £102.78