AQA

A Level

A Level Maths

AQA Core Maths C4 June 2011 Model Solutions

Name:

M M E

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Total Marks:

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AQA June 11 C4

1a. $f(x) = 4x^3 - 13x + 6$

$f(-2) = 4(-2)^3 - 13(-2) + 6$

$= 0 \quad \therefore \quad (x+2)$ is a factor

1b. $f(3/2) = 4(3/2)^3 - 13(3/2) + 6$

$= 0 \quad \therefore \quad (2x-3)$ is a factor

1c. $f(x) = 4x^3 - 13x + 6 = (x+2)(2x-3)(ax+b)$

x^3 terms. $4x^3 = x.\,2x.\,ax$

$4x^3 = 2ax^3 \Rightarrow a = 2$

constants : $6 = 2.(-3).b$

$6 = -6b \Rightarrow b = -1$

$$\frac{2x^2+x-6}{f(x)} = \frac{\cancel{(2x-3)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(2x-3)}(2x-1)} = \frac{1}{2x-1}$$

2ai. $P = \cancel{A^t}\ Ak^t$

when $t = 0$, $P = 80$

$80 = A\cancel{k^0} \Rightarrow A = 80$

2aii. $P = 80k^t$

when $t = 25$, $P = 2000$

$2000 = 80k^{25}$

$k^{25} = 25 \Rightarrow k = \sqrt[25]{25} = 1.137411$ (6dp)

16b.

$$P = 80k^t$$

$$100{,}000 = 80\left(\sqrt[25]{25}\right)^t$$

$$1250 = 1.137411^t$$

$$\ln 1250 = t \ln 1.137411$$

$$t = \frac{\ln 1250}{\ln 1.137411}$$

$$= 55.3834\ldots$$

so 56 years

3ai.

$$(1-x)^{1/3} \approx 1 + \left(\tfrac{1}{3}\right)(-x) + \frac{\left(\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)}{2}(-x)^2 + \ldots$$

$$= 1 - \frac{1}{3}x - \frac{1}{9}x^2 + \ldots$$

3aii.

$$(125 - 27x)^{1/3} = \left[125\left(1 - \tfrac{27}{125}x\right)\right]^{1/3} = 125^{1/3}\left(1 - \tfrac{27}{125}x\right)^{1/3}$$

$$= 5\left(1 - \frac{1}{3}\left(\frac{27}{125}x\right) - \frac{1}{9}\left(\frac{27}{125}x\right)^2 + \ldots\right)$$

$$= 5 - \frac{9}{25}x - \frac{81}{3125}x^2 + \ldots$$

3b.

$$125 - 27x = 119$$

$$27x = 6$$

$$x = 2/9$$

so $\sqrt[3]{119} = (125 - 27x)^{1/3}$ when $x = 2/9$

$$= 5 - \frac{9}{25}\left(\frac{2}{9}\right) - \frac{81}{3125}\left(\frac{2}{9}\right)^2 + \ldots$$

$$= 4.91872 \quad (5\text{dp})$$

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4ai.

$x = 3\cos 2\theta$ $\qquad$ $y = 2\cos\theta$

$\frac{dx}{d\theta} = -6\sin 2\theta$ $\qquad$ $\frac{dy}{d\theta} = -2\sin\theta$

$\frac{dy}{dx} = \frac{-2\sin\theta}{-6\sin 2\theta} = \frac{1}{3}\frac{\sin\theta}{\sin 2\theta}$

$= \frac{1}{3}\cdot\frac{\cancel{\sin\theta}}{2\cancel{\sin\theta}\cos\theta}$ $\qquad$ $(\sin 2\theta \equiv 2\sin\theta\cos\theta)$

$= \frac{1}{6\cos\theta}$

4aii.

$\theta = \pi/3$; $x = 3\cos 2\pi/3$ $\qquad$ $y = 2\cos\pi/3$

$= -\frac{3}{2}$ $\qquad$ $= 1$

$(-3/2, 1)$

$\frac{dy}{dx} = \frac{1}{6\cos\pi/3} = \frac{1}{3}$

$\therefore$ grad of normal $= -3$ $\qquad$ (since $\perp$)

$y - 1 = \frac{1}{3}\left(x + \frac{3}{2}\right)$

4b.

$\int_{-\pi/4}^{\pi/4} \sin^2 x \, dx$ $\qquad$ $\sin^2 x \equiv \frac{1}{2}(1 - \cos 2x)$

$= \frac{1}{2}\int_{-\pi/4}^{\pi/4} 1 - \cos 2x \, dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_{-\pi/4}^{\pi/4}$

$= \frac{1}{2}\left(\left(\pi/4 - \frac{1}{2}\sin\pi/2\right) - \left(-\pi/4 - \frac{1}{2}\sin(-\pi/2)\right)\right)$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2} + \frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{4} - \frac{1}{2}$

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5a. A $(5, 1, -2)$ B $(4, -1, 3)$

$l_1 : \underline{r} = \begin{pmatrix} -8 \\ 5 \\ -6 \end{pmatrix} + m \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$

$l_2 : \underline{r} = \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$

5bi. $l_1 = l_2$:

$-8 + 5m = 5 - \lambda$ $\quad \underline{i}$

$5 = 1 - 2\lambda$ $\quad \underline{j}$

$-6 - 2m = -2 + 5\lambda$ $\quad \underline{k}$

from $\underline{j}$ $\quad 5 = 1 - 2\lambda$

$-2\lambda = 4 \Rightarrow \lambda = -2$

'sub $\lambda = 2$ into $\underline{i}$' : $\quad -8 + 5m = 5 + 2$

$5m = 15 \Rightarrow m = 3$

'verify in $\underline{k}$' : $\quad -6 - 2(3) = -2 + 5(-2)$

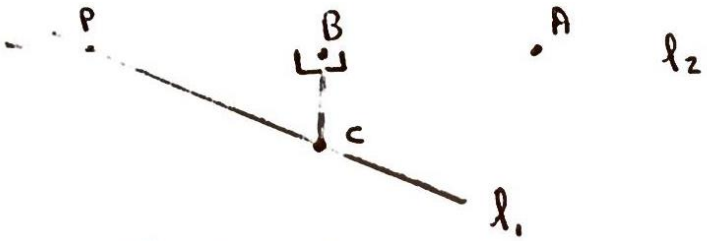
$-12 = -12$ ✓

sub m or λ into l_1 or l_2

$l_1 \quad \underline{r} = \begin{pmatrix} -8 \\ 5 \\ -6 \end{pmatrix} + 3 \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$

$= (7, 5, -12)$

5bii.



so $\overrightarrow{BC} \perp \overrightarrow{AB}$

so $\overrightarrow{BC} \cdot \overrightarrow{AB} = 0$

C lies on l_1 so $\overrightarrow{OC} = \begin{pmatrix} -8+5m \\ 5 \\ -6-2m \end{pmatrix}$ for some m

$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} -8+5m \\ 5 \\ -6-2m \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -12+5m \\ 6 \\ -9-2m \end{pmatrix}$

$\overrightarrow{BC} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} -12+5m \\ 6 \\ -9-2m \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix} = 0$

$-1(-12+5m) - 2(6) + 5(-9-2m) = 0$

$12 - 5m - 12 - 45 - 10m = 0$

$15m = -45$

$m = -3$

$\therefore \overrightarrow{OC} = \begin{pmatrix} -8+5(-3) \\ 5 \\ -6-2(-3) \end{pmatrix} = \begin{pmatrix} -23 \\ 5 \\ 0 \end{pmatrix}$

so C at $(-23, 5, 0)$

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6a. $2y + e^{2x}y^2 = x^2 + c$

when $x = 1$, $y = e^{-1}$

$2e^{-1} + e^2 . e^{-2} = 1 + c$

$2e^{-1} + 1 = 1 + c \quad \Rightarrow \quad c = 2e^{-1} \quad \left(= \frac{2}{e}\right)$

6b. $2y + e^{2x}y^2 = x^2 + 2e^{-1}$

$2\frac{dy}{dx} + 2y^2e^{2x} + 2ye^{2x}\frac{dy}{dx} = 2x$

$\frac{dy}{dx}\left(2 + 2ye^{2x}\right) = 2x - 2y^2e^{2x}$

$\frac{dy}{dx} = \frac{2x - 2y^2e^{2x}}{2 + 2ye^{2x}} \quad (\div 2)$

$= \frac{x - y^2e^{2x}}{1 + ye^{2x}}$

6c. if $(1, \frac{1}{e})$ is a stat pt. when $x = 1$, $y = \frac{1}{e}$, $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{1 - e^{-2}.e^2}{1 + e^{-1}e^2}$

$= \frac{0}{1+e}$

$= 0$

$\therefore \quad (1, \frac{1}{e})$ is a stat. point.

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7a. $\frac{dA}{dt} = -k$

7bi. when $t = 0$ $r = 60$, $A = 4\pi(60)^2 = 14{,}400\pi$

$t = 9$ $r = 30$, $A = 4\pi(30)^2 = 3600\pi$

$\frac{dA}{dt} = -k$

$\int dA = \int -k\, dt$

$A = -kt + c$

when $t = 0$, $A = 14{,}400\pi$

$14{,}400\pi = c$

$A = -kt + 14{,}400\pi$

when $t = 9$, $A = 3600\pi$

$3600\pi = -9k + 14{,}400\pi$

$9k = 10{,}800\pi$

$k = 1200\pi$

$A = -1200\pi t + 14{,}400\pi$

7bii. for snowball to melt completely $A = 0$

$0 = -1200\pi t + 14{,}400\pi$

$1200\pi t = 14{,}400\pi$

$t = \frac{14{,}400\pi}{1200\pi}$

$t = 12$ days

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8a.

$$\frac{1}{(3-2x)(1-x)^2} = \frac{A}{3-2x} + \frac{B}{1-x} + \frac{C}{(1-x)^2}$$

$$1 = A(1-x)^2 + B(3-2x)(1-x) + C(3-2x)$$

$x = 1$; $1 = C$

$x = 3/2$; $1 = \frac{1}{4}A \Rightarrow A = 4$

$x = 0$; $1 = A + 3B + 3C$

$1 = 4 + 3B + 3$

$-6 = 3B \Rightarrow B = -2$

$$\frac{1}{(3-2x)(1-x)^2} = \frac{4}{3-2x} - \frac{2}{1-x} + \frac{1}{(1-x)^2}$$

8b.

$$\frac{dy}{dx} = \frac{2\sqrt{y}}{(3-2x)(1-x)^2}$$

$$\frac{1}{2\sqrt{y}}\,dy = \frac{1}{(3-2x)(1-x)^2}\,dx$$

$$\frac{1}{2}\int y^{-1/2}\,dy = \int \frac{4}{3-2x} - \frac{2}{1-x} + \underbrace{\frac{1}{(1-x)^2}}\,dx$$

$$\int \frac{1}{(1-x)^2}\,dx = \int (1-x)^{-2}\,dx$$

$$\frac{d}{dx}\,k(1-x)^{-1} = (1-x)^{-2}$$

$$-k(-1)(1-x)^{-2} = (1-x)^{-2}$$

$$k = 1$$

so $\int (1-x)^{-2}\,dx = (1-x)^{-1} + c$

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$$\frac{1}{2}\int y^{-1/2}\,dy = \int \frac{4}{3-2x} - \frac{2}{1-x}\,dx + \frac{1}{1-x} + c$$

$$y^{1/2} = -2\ln|3-2x| + 2\ln|1-x| + \frac{1}{1-x} + c$$

when $x=0$, $y=0$

$$0 = -2\ln 3 + 2\ln 1 + 1 + c$$

$$2\ln 3 - 1 = c$$

$$y^{1/2} = -2\ln|3-2x| + 2\ln|1-x| + \underbrace{\frac{1}{1-x}} + 2\ln 3 \underbrace{-1}$$

consider $\frac{1}{1-x} - 1$

$$= \frac{1}{1-x} - \frac{1-x}{1-x} = \frac{1-(1-x)}{1-x} = \frac{x}{1-x}$$

$$y^{1/2} = 2\ln|1-x| - 2\ln|3-2x| + 2\ln 3 + \frac{x}{1-x}$$

$$y^{1/2} = 2\ln\left(\frac{1-x}{3-2x}\right) + 2\ln 3 + \frac{x}{1-x}$$

$$y^{1/2} = 2\ln\left(\frac{3-3x}{3-2x}\right) + \frac{x}{1-x}$$

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