OCR

A Level

# A Level Maths

## OCR Core Maths C4 January 2013 Model Solutions

Name:

M M E
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Total Marks:

OCR Jan 13 C4

1. $\int x\cos 3x \quad dx$ $\qquad u : x \qquad v' , \cos 3x$

$u' : 1 \qquad v : \frac{1}{3}\sin 3x$

$= \frac{x}{3}\sin 3x - \int \frac{1}{3}\sin 3x \quad dx$

$: \frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x + c$

2. $(9-16x)^{3/2} = 9^{3/2}(1-{}^{16}/_{9}x)^{3/2}$

$27\left[1 + \frac{3}{2}\left(\frac{-16}{9}x\right) + \frac{(3/2)(1/2)}{2}\left(\frac{-16}{9}x\right)^2 + \ldots\right.$

$= 27\left(1 - \frac{8x}{3} + \frac{32}{27}x^2 + \ldots\right.$

$: 27 - 72x + 32x^2$

valid for $\left|-\frac{16}{9}x\right| < 1 \qquad |x| < \frac{9}{16}$

3. $xy^2 = x^2 + 1 \quad ; \quad y^2 : \frac{x^2+1}{x}$

$y^2 + 2yxy' = 2x \quad \Rightarrow \quad y' : \frac{2x - y^2}{2xy}$

at $y' = 0 \quad , \qquad 2x = \frac{x^2+1}{x}$

$x^2 = 1 \quad \Rightarrow \quad x = \pm 1$

$(1, \sqrt{2}) \quad , \quad (-1, \sqrt{2})$

4i. ① $1+2\lambda = 6+\mu$ ; $2\lambda = \mu+5$ ④

② $2+\lambda = 8+4\mu$ ; $\lambda = 4\mu+6$ $\times 2$

③ $3\lambda = 1-5\mu$ ; $2\lambda = 8\mu+12$ ⑤

⑤ - ④ $0 = 7\mu + 7 \Rightarrow \mu = -1$

sub in ④ $2\lambda = -1+5 \Rightarrow \lambda = 2$

VERIFY in ③ : $3(2) = 1-5(-1)$

$6 = 6 \Rightarrow$ lines meet

P at $(5, 4, 6)$

4ii.

$$\frac{\begin{pmatrix}2\\1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\4\\-5\end{pmatrix}}{\sqrt{2^2+1^2+3^2}\cdot\sqrt{1^2+4^2+5^2}} = \cos\theta$$

$$\frac{-9}{\sqrt{14}\cdot\sqrt{42}} = \cos\theta$$

$$\theta = 68.2^\circ$$

$$\begin{array}{cc} S & C \\ -C & -S \end{array}$$

5i. $x = 2 + 3\sin\theta$ $\qquad$ $y = 1 - 2\cos\theta$

$$\frac{dx}{d\theta} = 3\cos\theta \qquad \frac{dy}{d\theta} = 2\sin\theta$$

$$\frac{dy}{dx} = \frac{d\theta}{dx} \times \frac{dy}{d\theta} = \frac{2\sin\theta}{3\cos\theta} = \frac{2}{3}\tan\theta$$

$$\frac{1}{2} = \frac{2}{3}\tan\theta$$

$$\tan\theta = \frac{3}{4}$$

$$\theta = 0.6435 \qquad \left(\frac{19}{5}, -\frac{3}{5}\right)$$

5ii. $x = 2 + 3\sin\theta$ $\qquad$ $y = 1 - 2\cos\theta$

$$\sin^2\theta = \left(\frac{x-2}{3}\right)^2 \qquad \cos^2\theta = \left(\frac{1-y}{2}\right)^2$$

$$1 = \frac{(x-2)^2}{9} + \frac{(1-y)^2}{4}$$

$$4x^2 + 9y^2 - 16x - 18y - 11 = 0$$

6. $$\int_0^{1/2} \frac{4x-1}{(2x+1)^5}\,dx$$

$u = 2x+1$

$x = \frac{u-1}{2}$

$dx = \frac{1}{2}\,du$

$$\frac{1}{2}\int_1^2 \frac{2u-3}{u^5}\,du$$

| $x$ | $1/2$ | $0$ |
|---|---|---|
| $u$ | $2$ | $1$ |

$2u = 4x+2$

$2u-3 = 4x-1$

$$= \frac{1}{2}\int_1^2 2u^{-4} - 3u^{-5}\,du$$

$$= \frac{1}{2}\left[-\frac{2}{3}u^{-3} + \frac{3}{4}u^{-4}\right]_1^2$$

$F[2] = -7/192$ $\qquad$ $F[1] = 1/12$

$$\int = \frac{1}{2}\left(-\frac{7}{192} - \frac{1}{12}\right) = -\frac{23}{384}$$

$$\therefore \int = \frac{23}{384}$$

$$\begin{matrix} S & C \\ -C & -S \end{matrix}$$

7i. $y = \ln(1+\sin x) - \ln(\cos x)$

$$\frac{\cos x}{1+\sin x} + \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x + \sin x + \sin^2 x}{(1+\sin x)\cos x}$$

$$\frac{1+\sin x}{(1+\sin x)\cos x}$$

$$= \sec x$$

7ii. $\int_0^{\pi/3} \sec x \, dx = \Big[\ln(1+\sin x) - \ln(\cos x)\Big]_0^{\pi/3}$

$F[\pi/3] = \ln(1+\sqrt{3}/2) - \ln(1/2)$

$F[0] = 0$

$\int = \ln\left(1+\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right)$

$= \ln\left(2+\sqrt{3}\right)$

8i. A(3,2,1) B(5,4,-3) C(3,17,-4) D(1,6,3)

$|AB| = \sqrt{(3-5)^2+(2-4)^2+(1--3)^2} = \sqrt{24}$

$|AD| = \sqrt{(3-1)^2+(2-6)^2+(1-3)^2} = \sqrt{24}$

8ii. Mid BD = (3,5,0) = X

$\vec{AX} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$

$\underline{r} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$

8iii.

$$\left.\begin{aligned} 3 &= 3 + 0 \\ 17 &= 2 + 3\lambda \\ -4 &= 1 - \lambda \end{aligned}\right\} \lambda = 5$$

$\therefore$ C is on the line

8iv. Kite

9i. $\frac{d\theta}{dt} = -k(\theta + 20)$

$$\int \frac{1}{\theta+20}\, d\theta = \int -k \; dt$$

$$\ln|\theta + 20| = -kt + c$$

$$\theta + 20 = Ae^{-kt}$$

$$\theta = Ae^{-kt} - 20$$

9ii. $t = 0, \quad \theta = 40, \quad \frac{d\theta}{dt} = -3$

$$-3 = -k(40 + 20) \Rightarrow k = \frac{1}{20}$$

$$\theta = Ae^{-\frac{1}{20}t} - 20$$

$t = 0, \quad \theta = 40, \Rightarrow 40 = A - 20 \Rightarrow A = 60$

$$\theta = 60e^{-\frac{1}{20}t} - 20$$

$$60e^{-1/20t} = 20$$

$$e^{-1/20t} = 1/3$$

$t = -20\ln(1/3) = 22$ minutes

9iii. k is larger.

10i.

$$\begin{array}{r} x - 1 \\ x^2 - x - 6 \overline{\big) x^3 - 2x^2 - 4x + 13} \\ \underline{x^3 - x^2 - 6x} \quad \downarrow \\ -x^2 + 2x + 13 \\ \underline{-x^2 + x + 6} \\ x + 7 \end{array}$$

10ii.

$$\int_4^6 \frac{x^3 - 2x^2 - 4x + 13}{x^2 - x - 6}\,dx = \int_4^6 x - 1 + \frac{x+7}{x^2 - x - 6}\,dx$$

$$\frac{x+7}{x^2 - x - 6} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$x + 7 = A(x+2) + B(x-3)$$

$x = 3 \Rightarrow 10 = 5A \; ; \; A = 2$

$x = -2 \Rightarrow 5 = -5B \; ; \; B = -1$

$$\int_4^6 x - 1 + \frac{2}{x-3} - \frac{1}{x+2}\,dx$$

$$= \left[ \frac{1}{2}x^2 - x + 2\ln|x-3| - \ln|x+2| \right]_4^6$$

$F[6] = 12 + 2\ln 3 - \ln 8$

$- F[4] = 4 + 2\ln 1 - \ln 6$

$8 + \ln 9 - \ln 8 + \ln 6$

$8 + \ln\left(\frac{9}{8}\right) + \ln 6$

$8 + \ln\left(\frac{27}{4}\right)$