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OCR

A Level

# A Level Maths

## OCR Core Maths C4 June 2015 Model Solutions

Name:

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Total Marks:

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OCR Jun 15 C1+

1i. $\frac{2}{3-x} + \frac{3}{1+x} = \frac{2(1+x) + 3(3-x)}{(3-x)(1+x)}$

$= \frac{11-x}{(3-x)(1+x)}$

1ii. $\frac{11-x}{(3-x)(1+x)} \times \frac{(x+11)(x-3)}{(11+x)(11-x)} = \frac{x-3}{(3-x)(1+x)} = -\frac{1}{1+x}$

2. A (1, 1, 3) B (5, 9, -5) C (6, 5, -4)

$\vec{AB} = \begin{pmatrix} 4 \\ 8 \\ -8 \end{pmatrix}$ $\vec{AP} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix}$, $\vec{PB} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

so P (4, 7, -3) $\vec{CP} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$

$\vec{AB} \cdot \vec{CP} = 4(-2) + 8(2) - 8(1) = 0 \Rightarrow$ ⊥

2ii. $A = \frac{1}{2}|AB| \times |CP|$

$= \frac{1}{2}\left(4^2 + 8^2 + (-8)^2\right)^{1/2} \times \left[(-2)^2 + 2^2 + 1^2\right]^{1/2}$

$= 18$

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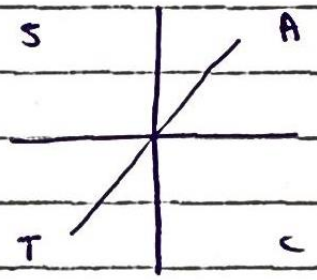
3. $y = e^{2x}\cos x$

$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x$

$e^{2x}\sin x = 2e^{2x}\cos x$

$\sin x = 2\cos x \qquad (e^{2x} > 0)$

$\tan x = 2$




$x = 1.11, -2.03$

$(1.11, 4.09), (-2.03, 0.00765)$

ii. $(8 - 9x)^{2/3} = 8^{2/3}\left(1 - \frac{9}{8}x\right)^{2/3}$

$= 4\left[1 + \left(\frac{2}{3}\right)\left(-\frac{9}{8}x\right) + \frac{\left(\frac{2}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{9}{8}x\right)^2}{2!} + \ldots\right.$

$= 4 - 3x - \frac{9x^2}{16}$

iii. $\left|-\frac{9}{8}x\right| < 1$

$-\frac{8}{9} < x < \frac{8}{9}$

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5. $\int e^{2\sqrt{x+1}}\,dx$

$t = (x+1)^{1/2}$

$x = t^2 - 1$

$dx = 2t\,dt$

$\int 2t\,e^{2t}\,dt$

$u = 2t \qquad v' = e^{2t}$

$u' = 2 \qquad v = \frac{1}{2}e^{2t}$

$te^{2t} - \int e^{2t}\,dt$

$= te^{2t} - \frac{1}{2}e^{2t}$

$= (x+1)^{1/2} e^{2(x+1)^{1/2}} - \frac{1}{2}e^{2(x+1)^{1/2}} + C$

6i. $\frac{\cos x}{\sin x}$

$f = \cos x \qquad g = \sin x$

$f' = -\sin x \qquad g' = \cos x$

$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= -\frac{(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= -\frac{1}{\sin^2 x}$

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6ii. $$\int_{\pi/6}^{\pi/4} \frac{(1+\cos 2x)^{1/2}}{\sin x \sin 2x}\, dx$$

$$\sin 2x \equiv 2\sin x \cos x$$
$$\cos 2x \equiv 2\cos^2 x - 1$$

$$\int \frac{(2\cos^2 x)^{1/2}}{2\sin^2 x \cos x}\, dx$$

$$= \int \frac{\sqrt{2}.\cos x}{2\sin^2 x \cos x}\, dx$$

$$= -\frac{\sqrt{2}}{2}\int_{\pi/6}^{\pi/4} -\operatorname{cosec}^2 x\, dx$$

$$= -\frac{\sqrt{2}}{2}\int_{\pi/6}^{\pi/4} \operatorname{cosec}^2 x\, dx$$

$$= -\frac{\sqrt{2}}{2}\Big[\cot x\Big]_{\pi/6}^{\pi/4}$$

$$= -\frac{\sqrt{2}}{2}\left(1 - \sqrt{3}\right)$$

$$= -\frac{\sqrt{2}}{2} + \frac{\sqrt{3}.\sqrt{2}}{2}$$

$$= \frac{1}{2}\left(\sqrt{6} - \sqrt{2}\right)$$

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7. $(x+y)^2 = xy^2$

$$2(x+y) + 2(x+y)y' = y^2 + 2xyy'$$

$$y'(2(x+y) - 2xy) = y^2 - 2(x+y)$$

$$\frac{dy}{dx} = \frac{y^2 - 2(x+y)}{2(x+y) - 2xy}$$

$x = 1 \quad y = ?$

$$(1+y)^2 = y^2$$
$$y^2 + 2y + 1 = y^2$$
$$2y = -1$$
$$y = -1/2$$

$$\frac{dy}{dx} = \frac{(-1/2)^2 - 2(1 - 1/2)}{2(1-1/2) - 2(1(-1/2))}$$

$$= -\frac{3}{8}$$

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8i. $\frac{dP}{dt} = \frac{100}{P}$

$$\int P\,dP = \int 100\,dt$$

$$\frac{1}{2}P^2 = 100t + c$$

$t = 0$, $P = 100$ ; $\frac{1}{2}(100)^2 = c$

$\Rightarrow \quad c = 5000$

$$\frac{1}{2}P^2 = 100t + 5000$$

$$P = \sqrt{200t + 10{,}000}$$

$$P = 10\sqrt{2t + 100}$$

8ii. $t = 8$, $\Rightarrow \quad P = 107.703$, model fits well ($|D| = 0.297$)

$t = 13$, $\Rightarrow \quad P = 112.249$, model doesn't fit well ($|D| = 15.751$)

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9i. $\underline{r} = \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$, $\underline{r} = \begin{pmatrix} 4 \\ 10 \\ 19 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ \alpha \end{pmatrix}$

$3 + 2\lambda = 7 = 4 + \mu$

$\lambda = 2, \quad \mu = 3$

$19 + 3\alpha = +1$

$\Rightarrow \quad \alpha = -6$

9ii. $$\frac{\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ \alpha \end{pmatrix}}{\sqrt{2^2+1^2+1^2} \cdot \sqrt{1^2+(-1)^2+\alpha^2}} = \cos 60^\circ$$

$2 - 1 + \alpha = \frac{1}{2}\sqrt{6}\cdot\sqrt{2+\alpha^2}$

$\alpha + 1 = \frac{1}{2}\sqrt{6}\sqrt{2+\alpha^2}$

$\alpha^2 + 2\alpha + 1 = \frac{1}{4}(12 + 6\alpha^2)$

$4\alpha^2 + 8\alpha + 4 = 12 + 6\alpha^2$

$\alpha^2 - 4\alpha + 4 = 0$

$(\alpha - 2)^2 = 0$

$\alpha = 2$

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10i. $\frac{x+8}{x(x+2)} \equiv \frac{A}{x} + \frac{B}{x+2}$

$x + 8 = A(x+2) + B(x)$

$x = 0 \; ; \quad 8 = 2A, \quad A = 4$

$x = -2 \; ; \quad 6 = -2B \quad B = -3$

10ii.

$$x^2+2x \overline{)\,7x^2 + 16x + 16}$$

$7$

$7x^2 + 14x + 0$

$2x + 16$

$7 + \frac{2x+16}{x(x+2)}$

$\frac{2x+16}{x(x+2)} = 2\left(\frac{x+8}{x(x+2)}\right) = \frac{8}{x} - \frac{6}{x+2}$

$7 + \frac{8}{x} - \frac{6}{x+2}$

10iii. $x = \frac{2t}{1-t} \qquad y = 3t + \frac{4}{t}$

$x - tx - 2t = 0$

$t(x+2) = x$

$t = \frac{x}{x+2}$

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$$y = \frac{3x}{x+2} + \left(4 \div \frac{x}{x+2}\right)$$

$$= 4 \times \frac{x+2}{x}$$

$$= \frac{4x+8}{x}$$

$$y = \frac{3x}{x+2} + \frac{4x+8}{x}$$

$$= \frac{3x^2 + (x+2)(4x+8)}{x(x+2)}$$

$$= \frac{3x^2 + 4x^2 + 8x + 8x + 16}{x(x+2)}$$

$$= \frac{7x^2 + 16x + 16}{x(x+2)}$$

10 iv. $$\int_1^2 7 + \frac{8}{x} - \frac{6}{x+2}\, dx$$

$$= \left[7x + 8\ln|x| - 6\ln|x+2|\right]_1^2$$

$$F(2) = 14 + 8\ln 2 - 6\ln 4$$

$$F[1] = 7 + 0 - 6\ln 3$$

$$F[2] - F[1] = 7 + 8\ln 2 - 6\ln 4 + 6\ln 3$$

$$6\ln 4 = \ln 2^{12} \; ; \quad \ln 2^8 - \ln 2^{12} = \ln(1/16)$$

$$= -\ln 16 = -\ln 2^4 = -4\ln 2$$

$$\Rightarrow 7 - 4\ln 2 + 6\ln 3$$

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