GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP3

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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FP3 Paper B – Marking Guide

1. $\ln(1+ax)\times(1+bx)^{-1} = (ax - \frac{1}{2}a^2x^2 + \ldots)(1 - bx + \ldots)$ B1

$= ax - abx^2 - \frac{1}{2}a^2x^2 + \ldots$ M1 A1

$\therefore ax + (-ab - \frac{1}{2}a^2)x^2 = 3x + \frac{3}{2}x^2$

$\therefore a = 3$ and $-ab - \frac{1}{2}a^2 = \frac{3}{2}$ M1

giving $-3b - \frac{9}{2} = \frac{3}{2}$ so $a = 3$, $b = -2$ A1 **(5)**

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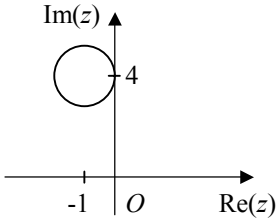
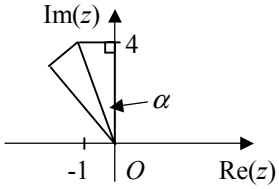
2. *(a)*



circle centre $-1 + 4i$, radius 1 B2

(b)



$\tan\alpha = \frac{1}{4}$, $\alpha = 14.036...°$ M1

max. arg $z = 90° + 2\alpha = 118.1°$ (1dp) M1 A1 **(5)**

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3. *(a)* $\cosh ix = \frac{1}{2}(e^{ix} + e^{-ix}) = \frac{1}{2}[\cos x + i\sin x + \cos(-x) + i\sin(-x)]$ M1

$= \frac{1}{2}[\cos x + i\sin x + \cos x - i\sin x] = \cos x$ A1

(b) $\cosh ix = e^{ix}$ $\therefore \cos x = \cos x + i\sin x$ M1

$\therefore \sin x = 0$ giving $x = 0, \pi$ M1 A1 **(5)**

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4. assume true for $n = k$ and $n = k + 1$ $\therefore u_k = 2^k$, $u_{k+1} = 2^{k+1}$ M1

$\therefore u_{k+2} = 5(2^{k+1}) - 6(2^k)$ M1

$= 10(2^k) - 6(2^k) = 4(2^k) = 2^{k+2}$ M1 A1

$\therefore$ true for $n = k + 2$ if true for $n = k$ and $n = k + 1$

if $n = 1$, $u_1 = 2^1 = 2$; if $n = 2$, $u_2 = 2^2 = 4$ $\therefore$ true for $n = 1$ and $n = 2$ B1

$\therefore$ by induction true for integer n, $n \geq 1$ A1 **(6)**

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5. *(a)* $\lambda = -1,\ \begin{vmatrix} 2 & 2 & -1 \\ 0 & 2 & -4 \\ x & 3 & 0 \end{vmatrix} = 0$ M1

$\therefore\ 2(0 + 12) - 2(0 + 4x) - 1(0 - 2x) = 0$ M1

$24 - 8x + 2x = 0$ so $x = 4$ A1

(b) $\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & -4 \\ 4 & 3 & -1-\lambda \end{vmatrix} = 0$ M1

$(1 - \lambda)[(1 - \lambda)(-1 - \lambda) + 12] - 2(0 + 16) - 1[0 - 4(1 - \lambda)] = 0$ A1

$-(1 + \lambda)(1 - \lambda)^2 + 12 - 12\lambda - 32 + 4 - 4\lambda = 0$ M1

$-(1 + \lambda)(1 - \lambda)^2 - 16\lambda - 16 = 0$

$(1 + \lambda)(1 - \lambda)^2 + 16(1 + \lambda) = 0$

$(1 + \lambda)[(1 - \lambda)^2 + 16] = 0$ A1

$\lambda = -1$; or $(1 - \lambda)^2 = -16$, not poss. for real λ M1

$\therefore\ \lambda = -1$ is only real eigenvalue A1

(c) $\begin{pmatrix} 2 & 2 & -1 \\ 0 & 2 & -4 \\ 4 & 3 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$2x + 2y - z = 0$ $\quad \therefore 2x + 4z - z = 0$

$2y - 4z = 0$ $\quad 2x + 3z = 0$ $\quad \therefore$ eigenvector $k\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ M1 A1 **(11)**

$\therefore\ y = 2z$ $\quad \therefore 2x = -3z$

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6. *(a)* $\frac{y_1 - y_0}{0.1} \approx x_0 y_0\ \ \therefore\ y_1 \approx 0.1x_0y_0 + y_0$ M1 A1

$x_0 = 0.2, x_1 = 0.3, y_0 = 1\ \ \therefore\ \ y_1 \approx 1.02$ M1 A1

$y_2 \approx 0.1x_1y_1 + y_1$ M1

$x_1 = 0.3, x_2 = 0.4, y_1 = 1.02\ \ \therefore\ \ y_2 \approx 1.0506$ A1

(b) $\int_1^y \frac{1}{y}\ \mathrm{d}y = \int_{0.2}^{0.4} x\ \mathrm{d}x$ M1

$[\ \ln|y|\ ]_1^y = [\ \frac{1}{2}x^2\ ]_{0.2}^{0.4}$ A1

$\ln|y| - \ln 1 = 0.08 - 0.02$ giving $y = \mathrm{e}^{0.06}$ M1 A1

(c) % error $= \frac{\mathrm{e}^{0.06} - 1.0506}{\mathrm{e}^{0.06}} \times 100\% = 1.1\%$ (1dp) M1 A1 **(12)**

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7. *(a)* $z^n + \frac{1}{z^n} = \cos n\theta + \text{i}\sin n\theta + \cos(-n\theta) + \text{i}\sin(-n\theta)$ M1

$= \cos n\theta + \text{i}\sin n\theta + \cos n\theta - \text{i}\sin n\theta = 2\cos n\theta$ A1

$z^n - \frac{1}{z^n} = \cos n\theta + \text{i}\sin n\theta - \cos n\theta + \text{i}\sin n\theta = 2\text{i}\sin n\theta$ A1

(b) $(z + \frac{1}{z})^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4}$ M1

$(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$ M1 A1

$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ A1

$(z - \frac{1}{z})^4 = z^4 - 4z^2 + 6 - \frac{4}{z^2} + \frac{1}{z^4}$ M1

$(2\text{i}\sin\theta)^4 = 2\cos 4\theta - 8\cos 2\theta + 6$

$16\sin^4\theta = 2\cos 4\theta - 8\cos 2\theta + 6$ A1

$\therefore\ 16(\cos^4\theta + \sin^4\theta) = 4\cos 4\theta + 12$ M1

$\cos^4\theta + \sin^4\theta = \frac{1}{4}\cos 4\theta + \frac{3}{4}$ so $A = \frac{1}{4}$, $B = \frac{3}{4}$ A1

(c) $\text{I} = \int_0^{\frac{\pi}{8}} \frac{1}{4}\cos 4\theta + \frac{3}{4}\ \text{d}\theta$

$= [\frac{1}{16}\sin 4\theta + \frac{3}{4}\theta]_0^{\frac{\pi}{8}}$ M1 A1

$= \frac{1}{16} + \frac{3}{32}\pi$ A1 **(14)**

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8. *(a)* $\overrightarrow{AB} = -5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\overrightarrow{AC} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ M1 A1

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 1 & -3 \\ -2 & 3 & 4 \end{vmatrix}$$

$= \mathbf{i}(4 + 9) - \mathbf{j}(-20 - 6) + \mathbf{k}(-15 + 2) = 13\mathbf{i} + 26\mathbf{j} - 13\mathbf{k}$ M1 A2

(b) $\overrightarrow{AD} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ B1

volume $= \frac{1}{6}\left|(13\mathbf{i} + 26\mathbf{j} - 13\mathbf{k}).(-4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})\right|$ M1

$= \frac{13}{6}\left|-4 - 8 - 6\right| = 39$ units^3 A1

(c) $\mathbf{n} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ B1

$\mathbf{r}.\mathbf{n} = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}).(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 3 - 2 - 2 = -1$ M1

$\therefore$ eqn. is $\mathbf{r}.(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -1$ A1

(d) line through DE has eqn. $\mathbf{r} = -\mathbf{i} - 5\mathbf{j} + 8\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ M1 A1

at intersection $[(-1 + \lambda)\mathbf{i} + (-5 + 2\lambda)\mathbf{j} + (8 - \lambda)\mathbf{k}].(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = -1$ M1

$-1 + \lambda - 10 + 4\lambda - 8 + \lambda = -1$ giving $\lambda = 3$ M1 A1

$\therefore$ E is (2, 1, 5) A1 **(17)**

Total **(75)**

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Performance Record – FP3 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	Maclaurin series	complex loci	complex nos.	proof by induction	matrices, eigenvals.	step-by-step soln. of diff. eqn.	complex nos., De Moivre's	vectors	
Marks	5	5	5	6	11	12	14	17	75
Student									

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