

N15 USE INEQUALITY NOTATION TO SPECIFY SIMPLE ERROR INTERVALS DUE TO TRUNCATION OR ROUNDING (foundation and higher tier)

You should be able to find least and greatest value of a number that has been rounded to the nearest whole number.

e.g. 10 km measured to the nearest km lies between 9.5 km and 10.5 km.

e.g. The volume of a cylinder is 24 cm^3 to the nearest cm^3 .

The least possible volume is 23.5 cm^3 and the greatest possible volume is 24.5 cm^3 .

Error Intervals.

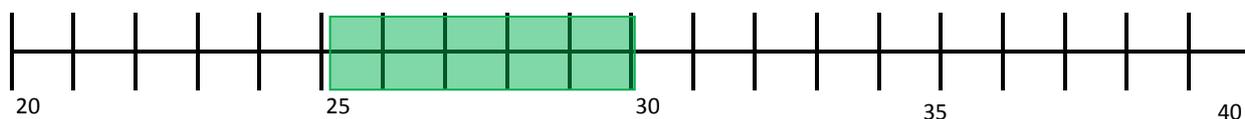
Once you have found the least possible value and the greatest possible value you can write down the **error interval**.

For a value x , the **error interval** is: least possible value $\leq x <$ greatest possible value

EXAMPLE 1

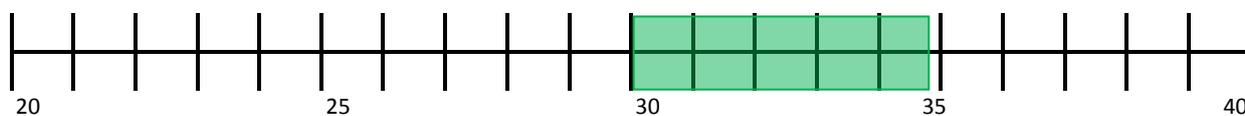
A fence is 30 m long to the nearest 10 m.
Write down the error interval for the length, l .

Here we have to think of all the numbers that can be rounded up to 30.
A number line can be useful to observe these numbers.



The least possible value of l is 25 m

← To the nearest 10 so between 20 and 30



The greatest possible value of l is 35 m

← To the nearest 10 so between 30 and 40

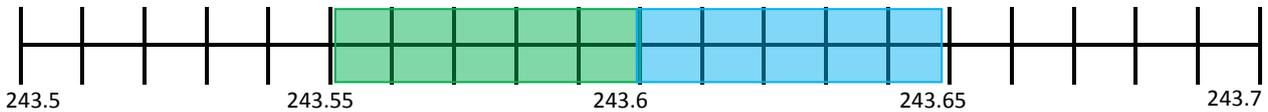
The error interval for the length is written as

$$25 \text{ m} \leq l < 35 \text{ m}$$

NOTE: We do not write the error interval as $25 \text{ m} \leq l < 34.9999999999\dots \text{ m}$ as we want to know the greatest possible value that l is less than (and not equal to).
For our error interval we have $l < 35$ and not $l \leq 35$ as l cannot equal 35

EXAMPLE 2

The width, w , of a stool is 243.6 mm correct to one decimal place.
Write down the error interval for the width of the stool.



The least possible value of w is 243.55 mm

← To 1 d.p. so between 243.5 and 243.6

The greatest possible value of w is 243.65 mm

← To 1 d.p. so between 243.6 and 243.7

The error interval for the width is written as

$$243.55 \text{ mm} \leq w < 243.65 \text{ mm}$$

EXERCISE 1:

- The weight of a bag of potatoes is 35 kg, correct to the nearest kg.
 - Write down the smallest possible weight of the bag of potatoes.
 - Write down the largest possible weight of the bag of potatoes.
- The length of a line is 73 centimetres, correct to the nearest centimetre.
 - Write down the **least** possible length of the line.
 - Write down the **greatest** possible length of the line.
- The weight, w , of a suitcase is 14 kg, correct to the nearest kilogram.
Write down an inequality to show the error interval for w .
- Katy drove for 238 miles, correct to the nearest mile.
 - Write down an error interval for the number of miles (m) driven.
Katy used 27.3 litres of petrol, to the nearest tenth of a litre.
 - Write down an error interval for the number of litres (l) used.
- The length, l , of a piece of wood is 123 mm, correct to the nearest mm.
Write down an inequality to show the error interval for l .

6. Sandeep takes 35 seconds, to the nearest second, to run a race.
Write down an error interval for the time. t seconds, taken to run the race.
7. The length L of a piece of string is 16 cm, correct to the nearest cm.
Write down an error interval for L .
8. The number x , rounded to one decimal place, is 5.7
(a) Write down the error interval for x .
The number y , rounded to two decimal places, is 7.36.
(b) Write down the error interval for y .
The number z , rounded to two significant figures, is 5800.
(c) Write down the error interval for z .
9. The perimeter of a square is 48 cm to the nearest centimetre.
Work out the error interval of the length, l , of a side of this square.
10. A pencil is 12 cm long to the nearest centimetre.
Tom says that this pencil will always fit in a pencil case that is 12.3 cm.
Is Tom correct? Explain your answer.
11. A number x has three non-zero digits after the decimal point.
When x is rounded to two decimal places it is 2.75
When x is rounded to one decimal place it is 2.8
Work out the range of values for x .
12. James was timed at 60.74 seconds, to the nearest hundredth of a second, for driving one circuit of a racing track.
(a) State the lower bound of the time.
(b) Hence, write down the error interval for the time, t seconds.
13. Sam measured the distance between two points as 18.7 cm to the nearest mm.
Write down the error interval for the distance, d cm.
14. Fifty identical books are stacked in one pile. The height of each book is 7 cm, to the nearest cm.
Work out the maximum height of the pile.

15. A small van can take a load of up to 1500 kg.

This van has to take the following to a market:

50 large bags of potatoes, each bag weighing 25kg to the nearest kg,

240 boxes of oranges, each box weighing 3000 g to the nearest 10 g.

Could this load be over the limit?

You must show all your working.

Truncation

Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point without rounding.

EXAMPLE 3

Truncate 3.14159265 to 4 decimal places.

3.1415|9265

Answer is 3.1415

← First draw a vertical line after 4 d.p.

← Remove all the digits after the vertical line

NOTE: 3.14159265 rounded to 4 decimal places is 3.1416

EXAMPLE 4

Truncate 782.9 to the whole number.

782.|9

Answer is 782

← First draw a vertical line by the decimal point

← You do not need to put in the decimal point

NOTE: 782.9 rounded to the nearest whole number is 783

EXAMPLE 5

A length, l , is 97.62 truncated to two decimal places.
Write down the error interval for the length l .

The least possible value is 97.62

← It has already been truncated so it cannot be less than 97.62

The greatest possible value is 97.63

← It has been truncated so the 2nd decimal place must be less than 3

The error interval is $97.62 \leq l < 97.63$

← Remember to use $\dots \leq l < \dots$

Problems with upper and lower boundary

You also need to consider more complex problems using greatest and least values.

EXAMPLE 6

Ravina measured the length and the width of her rectangular garden.

She measured the length to be 15 m correct to the nearest metre and the width to be 9.28 m correct to the nearest centimetre.

- Write down the error interval for the length, l metres.
- Write down the error interval for the width, w metres.
- Hence, or otherwise work out the error interval for the perimeter (P metres) of the rectangle.

- (a) The least possible value is 14.5 m
The greatest possible value is 15.5 m
The error interval is $14.5 \text{ m} \leq l < 15.5 \text{ m}$

To nearest metre so least is between 14 and 15
and greatest is between 15 and 16

- (b) The least possible value is 9.275 m
The greatest possible value is 9.285 m
The error interval is $9.275 \text{ m} \leq w < 9.285 \text{ m}$

To nearest cm (i.e. to 2 decimal places)
so least is between 9.27 and 9.28
and greatest is between 9.28 and 9.29

- (c) Least possible perimeter
 $= 14.5 + 14.5 + 9.275 + 9.275$
 $= 47.55 \text{ m}$

Add the 4 sides using least length and least width

- Greatest possible perimeter
 $= 15.5 + 15.5 + 9.285 + 9.285$
 $= 49.57 \text{ m}$

Add the 4 sides using greatest length
and greatest width

error interval for the perimeter is $47.55 \text{ m} \leq P < 49.57 \text{ m}$

EXERCISE 2:

1. Truncate 4.87635
 - (a) to 2 decimal places
 - (b) to 4 decimal places
 - (c) to the whole number
2. The length, l cm, of cable is 74.3 m truncated to 1 decimal place.
Write down the error interval for the length l .
3. The length, L cm, of a line is measured as 13 cm truncated to the whole number of centimetres.
Write down the error interval for the length.
4. Jim truncates the number x to one decimal place.
The result is 7.2
Write down the error interval for x .

EXERCISE 3:

1. Steve measures the length (l) and the width (w) of a rectangle.
He measures the length to be 640 mm correct to the nearest 10 mm.
 - (a) Write down the error interval for the length.
Steve also measures the width to be 400 mm correct to the nearest 100 mm.
 - (b) Write down the error interval for the width.
 - (c) Hence, or otherwise work out the error interval for the perimeter, P , of the rectangle.
2. A field is in the shape of a rectangle.
The length of the field is 340 m, to the nearest metre.
The width of the field is 117 m, to the nearest metre.
Work out the error interval for the perimeter, p , of the field.
3. Mia's weight is 57 kg, correct to the nearest kilogram.
 - (a) Write down
 - (i) her least possible weight (w kg),
 - (ii) the error interval of her weight.Alice's weight is 62 kg, correct to the nearest kilogram.
 - (b) Work out the greatest possible difference between Alice's weight and Mia's weight.

