

Please write clearly in block capitals.

Centre number

Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

H

Higher Tier

Paper 1 Non-Calculator

Thursday 25 May 2017

Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- mathematical instruments.

You must **not** use a calculator.



Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice

- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2-3	
4-5	
6-7	
8-9	
10-11	
12-13	
14-15	
16-17	
18-19	
20-21	
22-23	
24-25	
TOTAL	



Answer all questions in the spaces provided

- 1 Simplify $2^5 \times 2^3$
Circle your answer.

[1 mark]

4^8

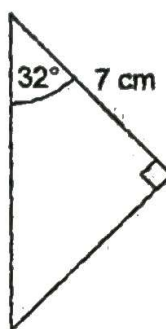
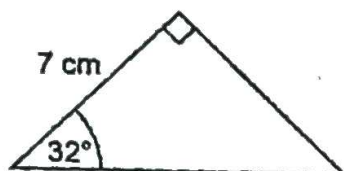
2^8

2^{15}

4^{15}

$2^{3+5} = 2^8$

2

Not drawn
accurately

Circle the reason why these triangles are congruent.

[1 mark]

SSS

SAS

ASA

RHS

- 3 Which of these is a geometric progression?
Circle your answer.

[1 mark]

2, 4, 6, 8, 10

2, 3, 5, 8, 12

2, 6, 18, 54, 162

2, 6, 10, 14, 18

$\times 3 \longrightarrow$



4 $a : b = 4 : 3$

Circle the correct statement.

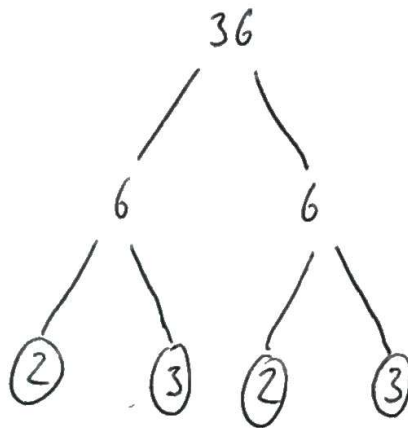
[1 mark]

 b is $\frac{4}{7}$ of a b is $\frac{3}{7}$ of a b is $\frac{4}{3}$ of a b is $\frac{3}{4}$ of a

$$a = 4, b = 3 = \frac{3}{4} \text{ OF } 4$$

5 Write 36 as a product of prime factors.
Give your answer in index form.

[3 marks]



$$36 = 2 \times 2 \times 3 \times 3$$

$$= 2^2 \times 3^2$$

Answer $2^2 \times 3^2$

Turn over for the next question

Turn over ►



6

The table shows information about the times for 10 people to complete a task.

Time, t (minutes)	Frequency
$0 < t < 20$	1
$20 < t < 40$	6
$40 < t < 60$	3

These statements are about the mean and range of the actual times.

Tick the correct box for each statement.

[4 marks]

	True	False
The mean could be less than 20 minutes	<input type="checkbox"/>	<input checked="" type="checkbox"/>
The mean could be more than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>
The mean could be less than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>
The range could be more than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>
The range could be less than 40 minutes	<input checked="" type="checkbox"/>	<input type="checkbox"/>
The range could be more than 60 minutes	<input type="checkbox"/>	<input checked="" type="checkbox"/>

$$\text{MAXIMUM MEAN} = \frac{(1 \times 20) + (6 \times 40) + (3 \times 60)}{10} = \frac{20 + 240 + 180}{10} = \frac{440}{10} = 44$$

$$\text{MINIMUM MEAN} = \frac{(1 \times 1) + (6 \times 21) + (3 \times 41)}{10} = \frac{1 + 126 + 123}{10} = \frac{250}{10} = 25 = 44$$

$$\text{MAXIMUM RANGE} = 60 - 1 = 59$$

$$\text{MINIMUM RANGE} = 41 - 20 = 21$$



7 $\frac{3}{5}$ of a number is 162

Work out the number.

$$162 \times 5 = \frac{162 \times 10}{2} \quad [2 \text{ marks}]$$

$$= \frac{1620}{2} = 810$$

$$\frac{3}{5} \times x = 162$$

$$3x = 162 \times 5 = 810$$

$$x = \frac{810}{3} = 270$$

$$\begin{array}{r} 270 \\ 3 \overline{) 810} \end{array}$$

Answer 270

8 $x \text{ km/h} = y \text{ mph}$

Use $8 \text{ km/h} = 5 \text{ mph}$ to write a formula for y in terms of x .

[2 marks]

$$x : y = 8 : 5 \Rightarrow \frac{5}{8} = \frac{y}{x} \Rightarrow \frac{5}{8}x = y$$

Answer $y = \frac{5}{8}x$

Turn over for the next question



9 (a) Density = $\frac{\text{mass}}{\text{volume}}$

The mass of solid A is 6 times the mass of solid B.

The volume of solid A is 3 times the volume of solid B.

$$\frac{6}{3} = 2$$

Complete the sentence.

[1 mark]

The density of solid A is 2 times the density of solid B.

9 (b) Average speed = $\frac{\text{distance}}{\text{time}}$

If the distance is halved and the time is doubled, what happens to the average speed?
Circle your answer.

[1 mark]

× 2

× 4

no change

÷ 2

÷ 4

$$\frac{\frac{1}{2}}{2} = \frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$



10

Solve the simultaneous equations.

$$2x + y = 18 \quad (1)$$

$$x - y = 6 \quad (2)$$

[3 marks]

$$\begin{array}{r} (1) + (2): \quad 2x + y = 18 \\ \quad \quad \quad x - y = 6 \quad + \\ \hline \quad \quad \quad 3x = 24 \end{array}$$

$$\Rightarrow x = 8$$

$$\begin{array}{l} x = 8, \text{ SUB INTO } (1): \quad 2(8) + y = 18 \\ \Rightarrow y = 18 - 16 = 2 \end{array}$$

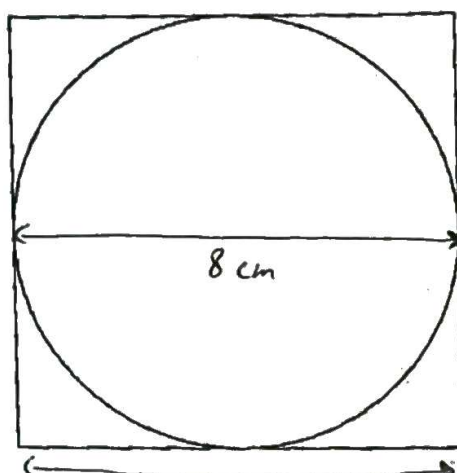
Answer $x = 8, y = 2$

Turn over for the next question



12

Here is a circle touching a square.

Not drawn
accuratelyThe area of the square is 64 cm^2 8 cm

Work out the area of the circle.

Give your answer in terms of π .

[3 marks]

$$\sqrt{64} = 8 = \text{WIDTH OF SQUARE} = \text{DIAMETER OF CIRCLE}$$

$$\text{RADIUS} = \frac{8}{2} = 4$$

$$\text{AREA} = \pi r^2 = \pi \times 4^2 = 16\pi$$

Answer 16π cm^2

Turn over for the next question

Turn over ►



11

Billy wants to buy these tickets for a show.

4 adult tickets at £15 each

2 child tickets at £10 each

A 10% booking fee is added to the ticket price.

3% is then added for paying by credit card.

Work out the total charge for these tickets when paying by credit card.

[5 marks]

$$\begin{aligned} \text{TOTAL INITIAL COST} &= (4 \times 15) + (2 \times 10) \\ &= 60 + 20 = \underline{\underline{80}} \end{aligned}$$

$$10\% \text{ OF } 80 = \frac{80}{10} = 8, \text{ THEN } 80 + 8 = \underline{\underline{88}}$$

$$1\% \text{ OF } 88 = \frac{88}{100} = 0.88,$$

$$3\% \text{ OF } 88 = 0.88 \times 3 = 2.64$$

$$\begin{array}{r} 88 \\ 3 \\ \hline 264 \end{array}$$

$$88 + 2.64 = \underline{\underline{90.64}}$$

Answer £ 90.64



- 13 Write the number six million five thousand two hundred in standard form. [2 marks]

$$\underline{\underline{6,005,200}} = 6.0052 \times 10^6$$

Answer 6.0052 × 10⁶

- 14 Solve $-3x > 6$ [1 mark]

$$-3x > 6 \Rightarrow 3x < -6$$

$$\Rightarrow x < -\frac{6}{3} = -2$$

Answer $x < -2$

- 15 $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{9}$ are four fractions.
 ✓ ✓ × ✓

How many of these fractions convert to a recurring decimal?

Circle your answer.

[1 mark]

0

1

2

3

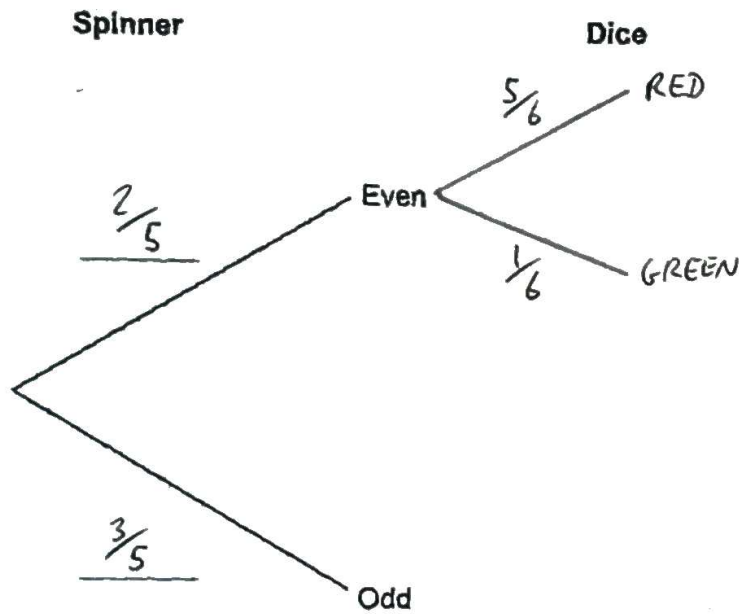
4



- 16 A fair spinner has five equal sections numbered 1, 2, 3, 4 and 5
A fair six-sided dice has five red faces and one green face.
The spinner is spun.
If the spinner shows an even number, the dice is thrown.

- 16 (a) Complete the tree diagram for the spinner and the dice.

[2 marks]



- 16 (b) Work out the probability of getting an even number and the colour green.

[2 marks]

$$\frac{2}{5} \times \frac{1}{6} = \frac{2}{30} = \frac{1}{15}$$

Answer $\frac{1}{15}$



- 17 A is the point (2, -5)
B is the point (4, -9)

17 (a) Show that the gradient of the straight line passing through A and B is -2

[2 marks]

$$\text{GRADIENT} = \frac{\text{CHANGE IN } y}{\text{CHANGE IN } x} = \frac{(-9) - (-5)}{4 - 2} = \frac{-9 + 5}{2} = \frac{-4}{2}$$

$$= -2$$

- 17 (b) C is the point (-301, 601)

Does C lie on the straight line passing through A and B?

You must show your working.

[2 marks]

GRADIENT OF LINE
THROUGH A AND C:

$$\frac{\text{CHANGE IN } y}{\text{CHANGE IN } x} = \frac{601 - (-5)}{(-301) - 2} = \frac{601 + 5}{-303} = \frac{606}{-303} = -2$$

THE GRADIENT OF THE LINE THROUGH A AND C
IS THE SAME AS IN THE PREVIOUS QUESTION, SO
ALL 3 POINTS ARE ON THE SAME LINE

Answer YES



18

Bottles of drink are for sale at three shops.

The normal price of a bottle is the same at each shop.

Shop A
Buy 1 bottle
Get 2 more bottles at half price

Shop B
Buy 2 bottles
Get 3 more bottles at half price

Shop C
30% off a bottle

$$70\% \text{ of } x = 0.7x$$

What is the cheapest way to buy exactly 8 bottles?

You can buy from more than one shop.

You must show your working.

$$\text{COST OF BOTTLE} = x$$

[3 marks]

IN SHOP A, BUYING 2 BOTTLES GETS YOU 4 BOTTLES
AT HALF PRICE, WHICH IS ALWAYS BETTER THAN SHOP B.

BUYING 6 FROM SHOP A, 4 WILL BE HALF PRICE, SO
 $\text{COST} = 2x + 4\left(\frac{x}{2}\right) = 2x + 2x = 4x.$

THEN, BUYING 2 MORE FROM SHOP A, 1 WOULD BE
HALF PRICE, SO $\text{COST} = x + \frac{x}{2} = 1.5x$

BUYING 2 MORE FROM SHOP C INSTEAD, $\text{COST} = 0.7x \times 2$
 $= 1.4x$

ONLY OTHER OPTION IS ALL FROM SHOP C, SO TOTAL = 5.4x

$\text{COST} = 8 \times 0.7x = 5.6x$, NOT AS GOOD.

Answer 6 FROM SHOP A, 2 FROM SHOP C

7

Turn over ►

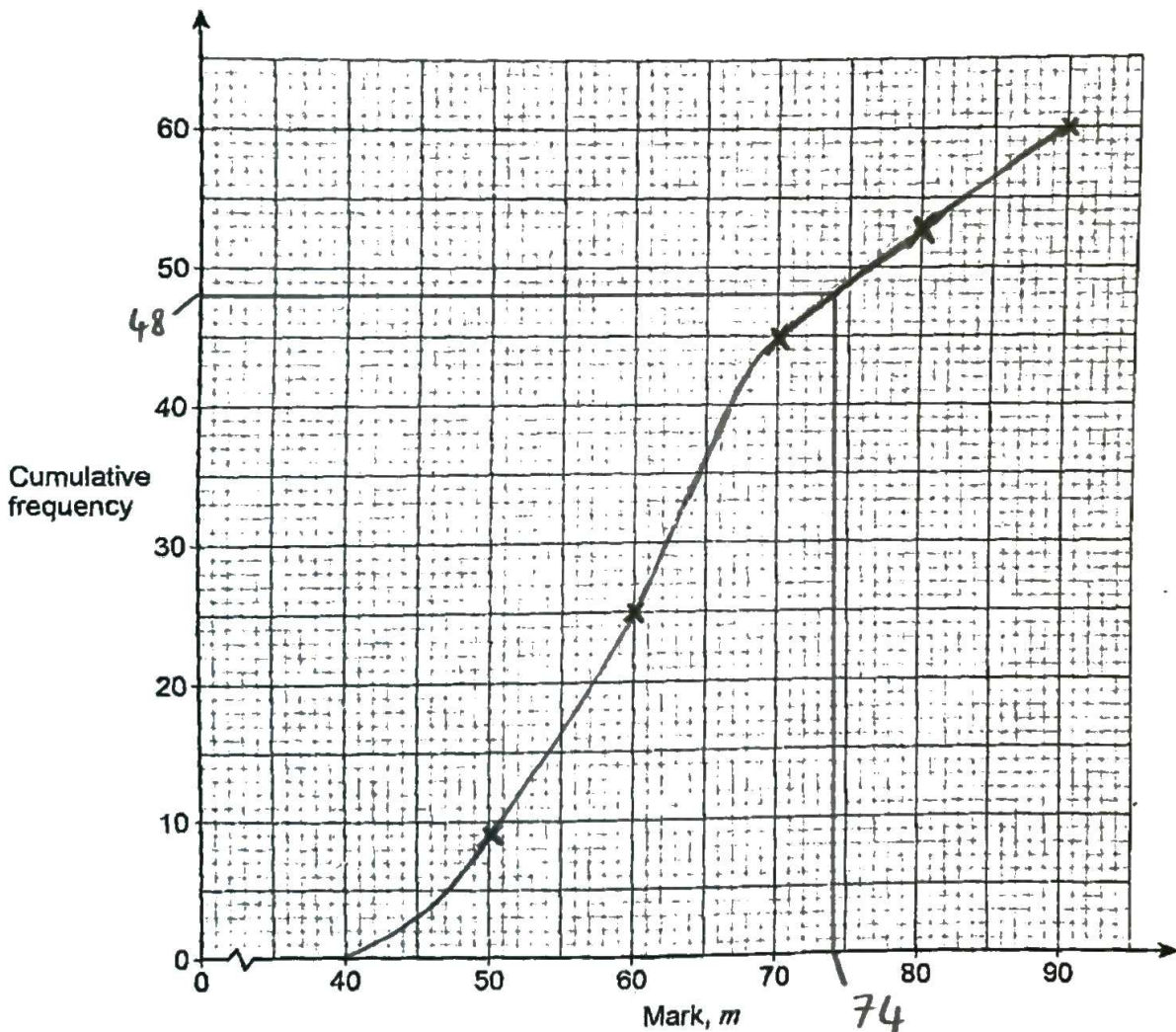


19 Here is some information about the marks of 60 students in a test.

Mark, m	Frequency	Mark, m	Frequency
$40 < m < 50$	9	≤ 50	9
$50 < m < 60$	16	≤ 60	25
$60 < m < 70$	20	≤ 70	45
$70 < m < 80$	8	≤ 80	53
$80 < m < 90$	7	≤ 90	60

19 (a) On the grid, draw a cumulative frequency graph.

[3 marks]



19 (b) Use your graph to estimate the lowest mark of the top 20% of students.

[2 marks]

$$10\% \text{ OF } 60 = \frac{60}{10} = 6, \Rightarrow 20\% \text{ OF } 60 = 12,$$

$$60 - 12 = 48$$

Answer 74

20 Work out the diameter of the circle $x^2 + y^2 = 64$

Circle your answer.

[1 mark]

8

16

32

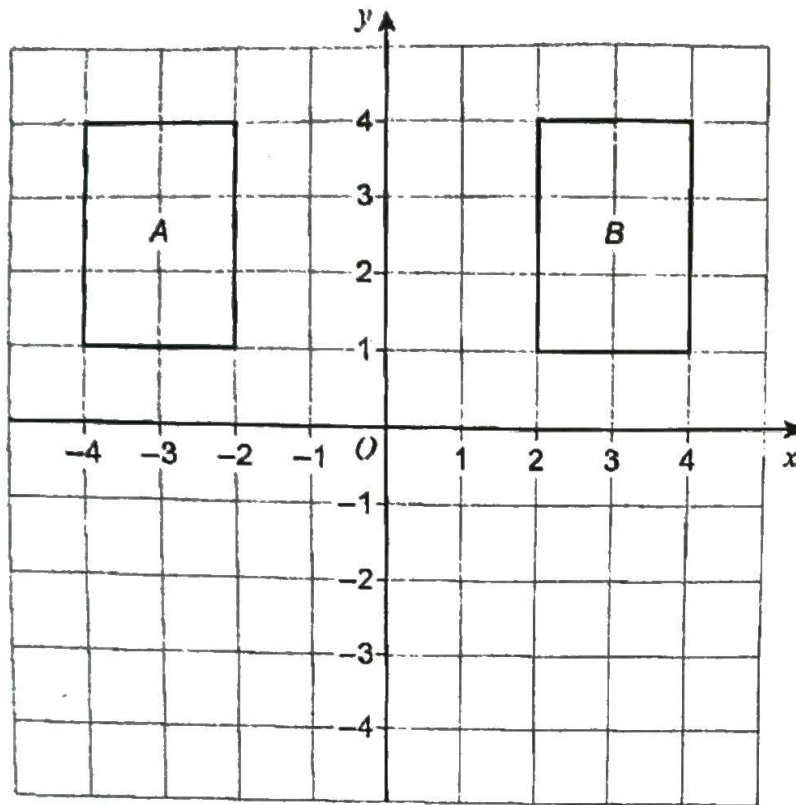
128

Turn over for the next question

$$r = \sqrt{64} = 8, \quad d = 2 \times 8 = 16$$



21 (a) The diagram shows rectangles A and B.



Rectangle A can be mapped to rectangle B by a **single** transformation.

Javed says,

"The **only** single transformation is a reflection in the y -axis because the rectangles are on opposite sides of the y -axis."

Is he correct?

Tick a box.

Yes

No

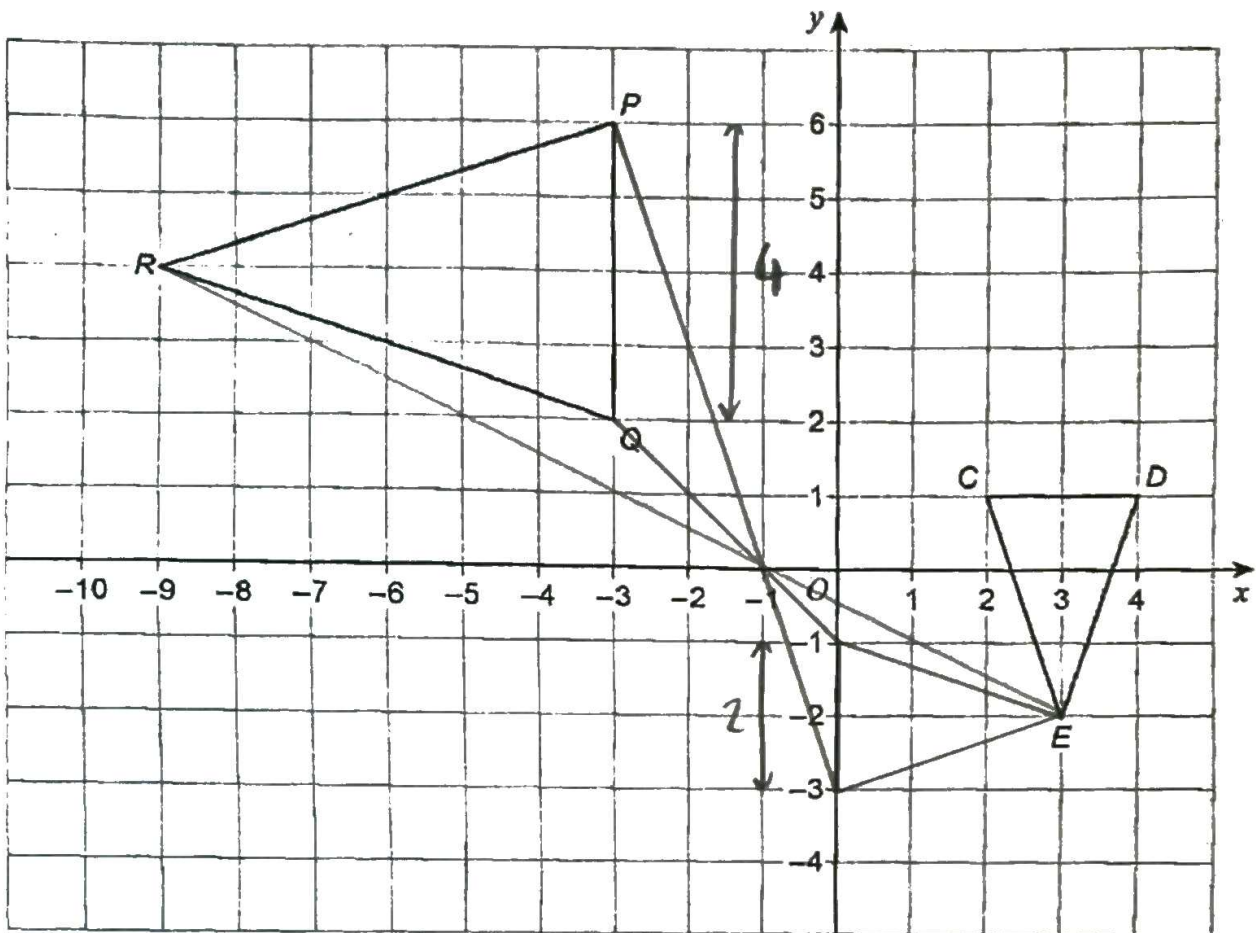
Give a reason for your answer.

[1 mark]

A CAN BE MAPPED TO B WITH A TRANSLATION
BY 6 IN THE POSITIVE X-DIRECTION



21 (b) This diagram shows triangles CDE and PQR .



CDE is mapped to PQR by combining two single transformations.
The first is a rotation of 90° anticlockwise about E .

Describe fully the second transformation.

[3 marks]

ENLARGEMENT SCALE FACTOR -2 ABOUT (-1, 0)

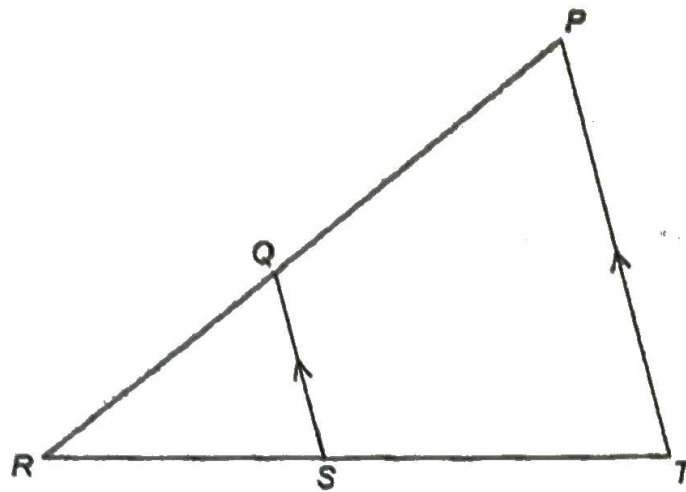
Turn over for the next question

Turn over ►



22

PRT and QRS are similar triangles.

Not drawn
accuratelyWhich of these is equivalent to $\frac{QR}{PR}$?SMALL
BIG

Circle your answer.

[1 mark]

$$\frac{RS}{ST}$$

$$\frac{QS}{PT}$$

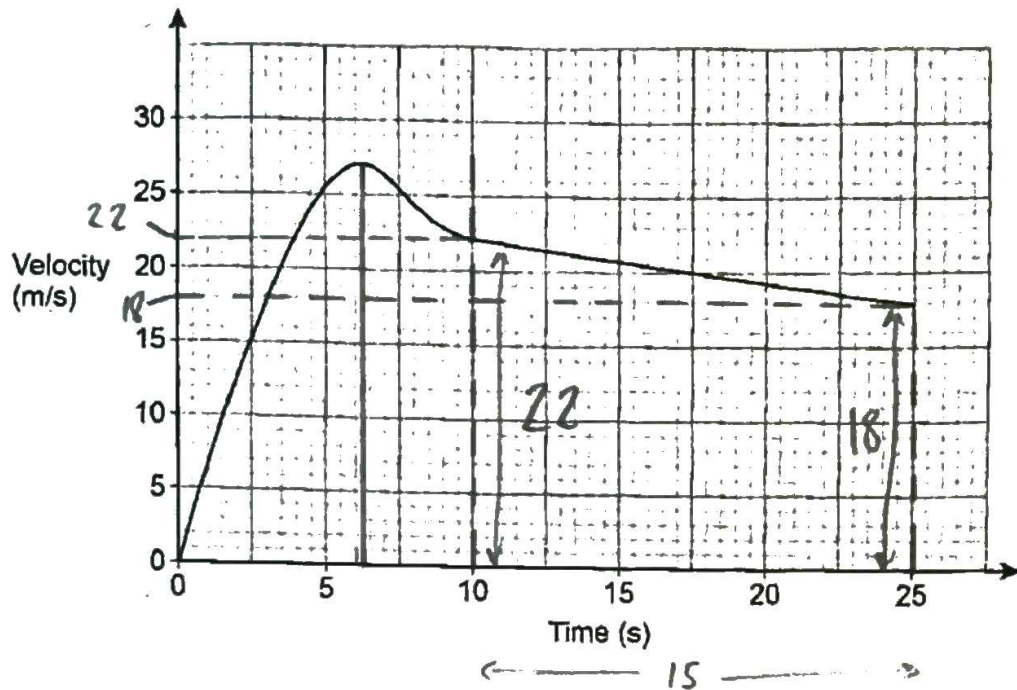
$$\frac{PT}{QS}$$

$$\frac{RT}{RS}$$



23

Here is a velocity-time graph of a motorbike for 25 seconds.



23 (a) After how many seconds was the acceleration zero?

[1 mark]

Answer 6.2 seconds

23 (b) Work out the distance travelled in the last 15 seconds.

[2 marks]

↙ AREA UNDER GRAPH

$$\text{AREA OF TRAPEZIUM} = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(18+22) \times 15$$

$$= \frac{1}{2}(40) \times 15 = 20 \times 15$$

$$= 2 \times 10 \times 15 = 2 \times 150$$

$$= 300$$

Answer 300 metres

Turn over ►



24 (a) Work out $\sqrt{12\frac{1}{4}}$ as an improper fraction.

[1 mark]

$$\sqrt{12\frac{1}{4}} = \sqrt{\frac{48}{4} + \frac{1}{4}} = \sqrt{\frac{49}{4}} = \frac{\sqrt{49}}{\sqrt{4}} = \frac{7}{2}$$

Answer $\frac{7}{2}$

24 (b) Work out $\sqrt[3]{16}$ as a power of 2

[2 marks]

$$\sqrt[3]{16} = 16^{\frac{1}{3}} = (2^4)^{\frac{1}{3}} = 2^{4 \times \frac{1}{3}} = 2^{\frac{4}{3}}$$

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

Answer $2^{\frac{4}{3}}$



25

In an office there are twice as many females as males.

$\frac{1}{4}$ of the females wear glasses.

$\frac{3}{8}$ of the males wear glasses.

84 people in the office wear glasses.

Work out the number of people in the office.

[4 marks]

LET F BE THE TOTAL NUMBER OF FEMALES IN THE
OFFICE, AND M BE THE TOTAL NUMBER OF MALES.

THEN, $F = 2M$.

$$\text{PEOPLE WEARING GLASSES} = 84 = \frac{1}{4}F + \frac{3}{8}M,$$

$$\text{BUT } F = 2M, \text{ SO: } 84 = \frac{1}{4}(2M) + \frac{3}{8}M$$

$$\begin{array}{r} 84 \times \\ 8 \\ \hline 672 \end{array}$$

$$= \frac{1}{2}M + \frac{3}{8}M$$

$$= \frac{4}{8}M + \frac{3}{8}M = \frac{7}{8}M$$

$$84 = \frac{7}{8}M \Rightarrow 84 \times 8 = 672 = 7M$$

$$\begin{array}{r} 096 \\ 7 \overline{)672} \end{array}$$

$$\Rightarrow M = \frac{672}{7} = 96$$

$$F = 2M = 2 \times 96 = 192, \therefore \text{TOTAL} = 96 + 192 = 288$$

Answer 288

Turn over for the next question

$$\begin{array}{r} 192 \\ 96 \\ \hline 288 \end{array}$$

Turn over ►



26

Expand and simplify $(x-4)(2x+3y)^2$

[4 marks]

$$(2x+3y)^2 = (2x+3y)(2x+3y)$$

$$= 4x^2 + 6xy + 6xy + 9y^2$$

$$= 4x^2 + 12xy + 9y^2$$

$$\Rightarrow (x-4)(2x+3y)^2 = (x-4)(4x^2 + 12xy + 9y^2)$$

$$= 4x^3 + 12x^2y + 9y^2x$$

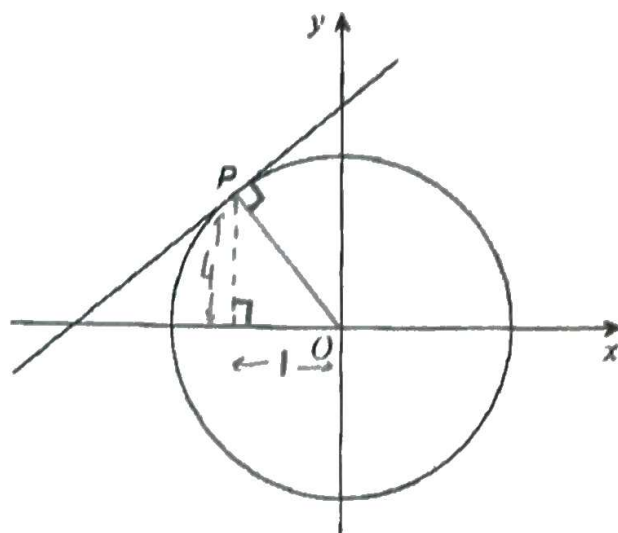
$$- 16x^2 - 48xy - 36y^2$$

$$= 4x^3 + 12x^2y + 9y^2x - 16x^2 - 48xy - 36y^2$$

Answer $4x^3 + 12x^2y + 9y^2x - 16x^2 - 48xy - 36y^2$



27

 $P(-1, 4)$ is a point on a circle, centre O Not drawn
accuratelyWork out the equation of the tangent to the circle at P .Give your answer in the form $y = mx + c$

[4 marks]

$$\text{GRADIENT OF OP} = -\left(\frac{4}{1}\right) = -4$$

TANGENT IS PERPENDICULAR TO RADIUS, SO

$$m = \frac{-1}{-4} = \frac{1}{4}$$

SUB $x = -1$, $y = 4$ INTO $y = mx + c$:

$$4 = \frac{1}{4}(-1) + c \Rightarrow 4 = -\frac{1}{4} + c \Rightarrow c = 4 + \frac{1}{4} = \frac{16+1}{4}$$

$$\therefore y = \frac{1}{4}x + \frac{17}{4}$$

Answer $y = \frac{1}{4}x + \frac{17}{4}$

8

Turn over ►



28

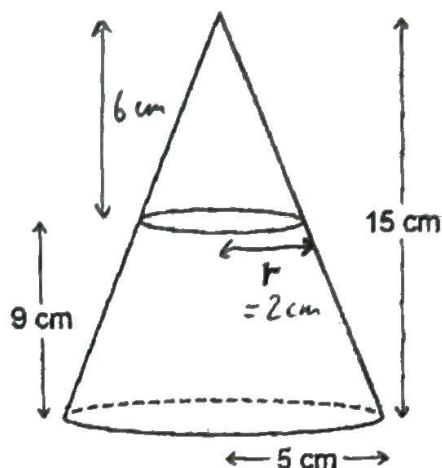
Volume of cone = $\frac{1}{3}\pi r^2 h$ where r is the radius and h is the perpendicular height.

A cone has a

horizontal base of radius 5 cm

height of 15 cm

The cone contains water to a depth of 9 cm



Work out the volume of the water, in cm^3

Give your answer in terms of π .

[4 marks]

$$\text{SIMILARITY OF CONES: } \frac{r}{6} = \frac{5}{15} = \frac{1}{3}$$

$$\Rightarrow r = \frac{1}{3} \times 6 = 2$$

$$\text{VOLUME OF CONE} = \frac{1}{3} \pi \times 5^2 \times 15$$

$$= \pi \times 25 \times \frac{15}{3} = \pi \times 25 \times 5$$

$$= 125\pi$$

$$\text{VOLUME OF EMPTY SECTION} = \frac{1}{3} \pi \times 2^2 \times 6 = \frac{1}{3} \pi \times 24 = 8\pi$$

$$\text{VOLUME OF WATER} = 125\pi - 8\pi = 117\pi \text{ cm}^3$$

Answer 117π cm^3



29

Simplify $\frac{2 \sin 45^\circ - \tan 45^\circ}{4 \tan 60^\circ}$ Give your answer in the form $\frac{\sqrt{a} - \sqrt{b}}{c}$ where a , b and c are integers.

$$= \frac{\sqrt{2}}{2}$$

[4 marks]

$$\tan 45 = 1, \quad \sin 45 = \frac{1}{\sqrt{2}}, \quad \tan 60 = \sqrt{3}$$

$$\Rightarrow \frac{2 \sin 45 - \tan 45}{4 \tan 60} = \frac{2 \left(\frac{\sqrt{2}}{2} \right) - 1}{4\sqrt{3}}$$

$$= \frac{\sqrt{2} - 1}{4\sqrt{3}} \left(\times \frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{6} - \sqrt{3}}{4\sqrt{3}^2} = \frac{\sqrt{6} - \sqrt{3}}{12}$$

Answer $\frac{\sqrt{6} - \sqrt{3}}{12}$

END OF QUESTIONS

