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AQA, OCR, Edexcel

GCSE

GCSE Maths

Level 9 Paper (A** Paper)

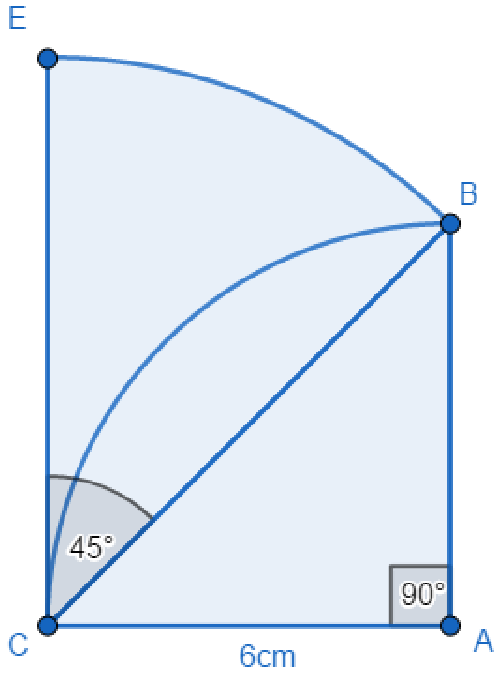
Name:

Guidance

1. Read each question carefully.
2. Don't spend too long on each question.
3. Attempt every question.
4. Always show your workings.

Revise GCSE Maths:
www.MathsMadeEasy.co.uk/gcse-maths-revision/

1. Calculate the area between the arc CB, the arc EB and the line CE. ECA = 90°



$$E\hat{C}A = 90°$$

$$B\hat{C}A = E\hat{C}A - E\hat{C}B = 90° - 45° = 45°$$

$$C\hat{B}A = 180° - B\hat{A}C - B\hat{C}A$$

$$= 180° - 90° - 45°$$

$$= 45°$$

ACB is an isosceles triangle, $BA = BC = 6cm$

$$Area = 12 \times 6 \times 6 = 18cm^2$$

$$Area\ of\ Sector\ ABC\ = \frac{1}{4} \times \pi \times 6^2$$

$$= 9\,\pi\ cm^2$$

$$Area\ of\ Segment = (9\,\pi - 18)cm^2$$

$$CB = \sqrt{6^2 + 6^2}$$

$$= 6\sqrt{2}$$

$$Area\ of\ Sector\ CBE = \frac{1}{8} \times \pi \times 6\sqrt{2} = 9\pi$$

Area Required

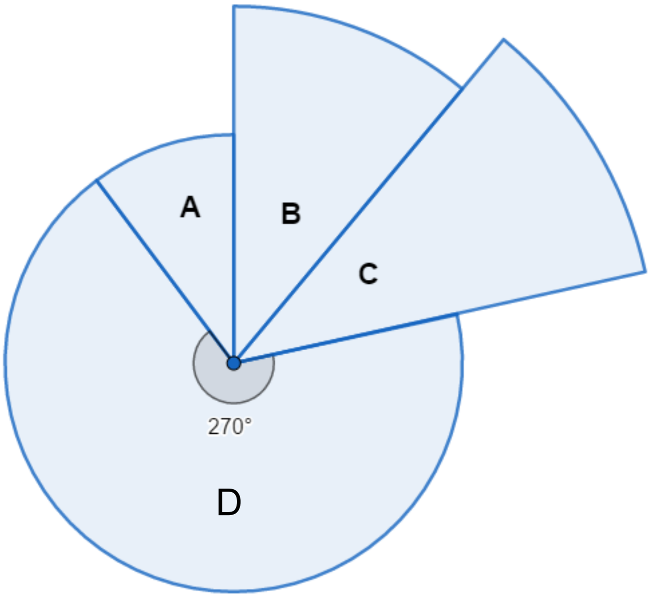
$$9\pi - (9\pi - 18) = 9\pi - 9\pi + 18 = 18cm^2$$

Area =………………..cm^2

(3 marks)

2. In the diagram below the sectors A, B, and C all subtend the same angle at the centre of the circle.
   The radii of the sectors A, B, and C are in the ratio 1: 2: 3.

   Calculate the total area of the shape in terms of r, the radius of the circle.



$$360° - 270° = 90°$$

Sector fraction (A,B,C) = $\frac{angle}{360} = \frac{30}{360} = \frac{1}{12}$

Sector fraction (D) = $\frac{angle}{360} = \frac{270}{360} = \frac{3}{4}$

Ratios

$$1: 2: 3$$

$$r: 2r: 3r$$

$$Area = \pi r^2$$

$$area\ A = \frac{1}{12} \times \pi \times r^2$$

$$= \frac{\pi r^2}{12}$$

$$area\ B = \frac{1}{12} \times \pi \times (2r)^2$$

$$= \frac{1}{12} \times \pi \times 4r^2$$

$$= \frac{4\pi r^2}{12}$$

$$area\ C = \frac{1}{12} \times \pi \times (3r)^2$$

$$= \frac{1}{12} \times \pi \times 9r^2$$

$$= \frac{9\pi r^2}{12}$$

$$area\ A = \frac{3}{4} \times \pi \times r^2$$

$$= \frac{3\pi r^2}{4}$$

$$= \frac{9\pi r^2}{12}$$

$$Total\ Area = \frac{\pi r^2}{12} + \frac{4\pi r^2}{12} + \frac{9\pi r^2}{12} + \frac{9\pi r^2}{12}$$

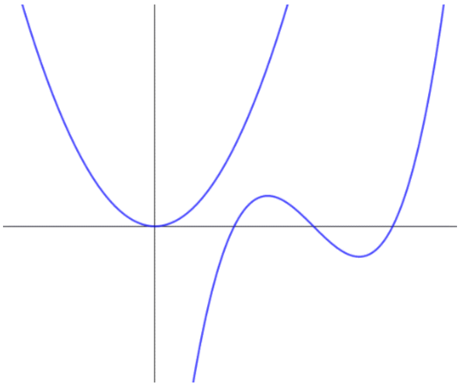
$$= \frac{23\pi r^2}{12}$$

$$\frac{23\pi r^2}{12}$$

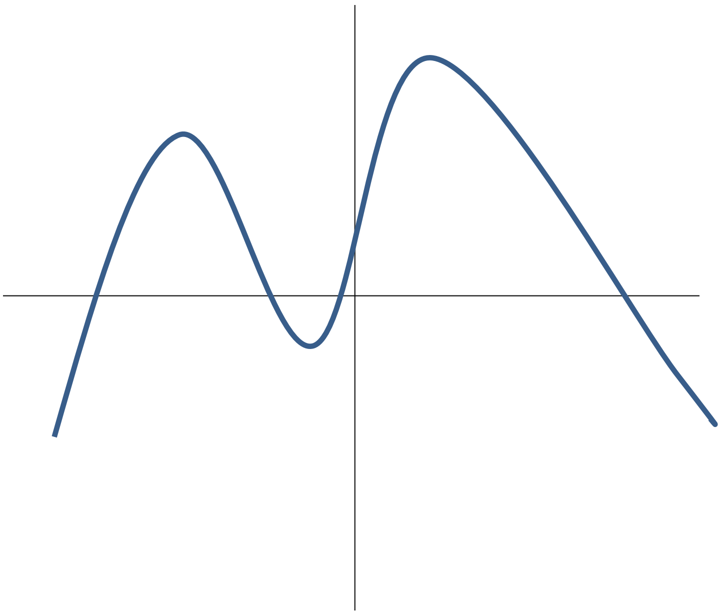
....

(4 marks)

3. Quadratic and cubic graphs have one and two turning points respectively. This can be seen on the diagram below.



Sketch the graph of a quartic graph. A quartic graph has x^4 as the leading power of x, and three turning points.



Describe the general rule for the number of turning points of a graph with a leading power of x^n.

A graph with a leading power of x^n *has* $n-1$ *turning points.*

(3 marks)

4. The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is a re-arrangement of the general quadratic equation

$$ax^2 + bx + c = 0$$

By completing the square on the general quadratic equation, prove this result.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac}{4a^2} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac - b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

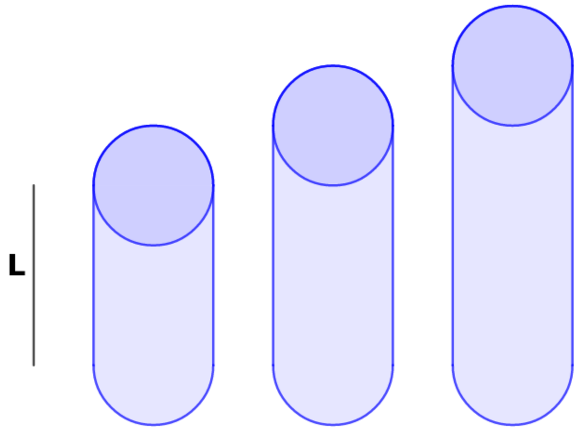
$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{\pm\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(5 marks)

5. Three cylinders with the same radii have lengths in the ratio 3:4:5. Calculate the surface areas of the three cylinders.



Ratios

$$3 : 4 : 5$$

$$1 : \frac{4}{3} : \frac{5}{3}$$

Heights of Cylinders

$$Small\ cylinder\ height = L$$

$$Middle\ cylinder\ height = \frac{4}{3}L$$

$$Largest\ cylinder\ height = \frac{5}{3}L$$

Surface area of small cylinder.

$$Two\ circles = \pi r^2 + \pi r^2$$

$$= 2\pi r^2$$

$$Curved\ surface\ area = 2\pi r \times L$$

$$= 2\pi rL$$

$$Total\ Area = 2\pi rL + 2\pi r^2$$

Surface area of middle cylinder.

$$Two\ circles = \pi r^2 + \pi r^2$$

$$= 2\pi r^2$$

$$Curved\ surface\ area = 2\pi r \times \frac{4}{3}L$$

$$= \frac{8\pi r}{3}L$$

$$Total\ Area = \frac{8\pi r}{3}L + 2\pi r^2$$

Surface area of middle cylinder.

$$Two\ circles = \pi r^2 + \pi r^2$$

$$= 2\pi r^2$$

$$Curved\ surface\ area = 2\pi r \times \frac{5}{3}L$$

$$= \frac{10\pi r}{3}L$$

$$Total\ Area = \frac{10\pi r}{3}L + 2\pi r^2$$

Surface area = ……………………

(4 marks)

6. The following information is known about a parallelogram ABCD.

$A = (0,6)$
AB has gradient 3
AD has gradient $-\frac{1}{2}$
The x-coordinate of B is 2
The y-coordinate of D is 2.

Find the co-ordinates of the point C.

Equation of Line AB	Equation of Line AD
$y = 3x + 6$ $x = 2$ $y = 3 \times 2 + 6$ $= 12$ $B(2,12)$	$y = -\frac{1}{2}x + 6$ $2 = -\frac{1}{2}x + 6$ $-4 = -\frac{1}{2}x$ $x = 8$ $D(8,2)$

ABCD is a parallelogram, A to C will be the same as B to D

B to D is 6 right and 10 down

From A to C
$(0,6) + (6,-10) = (6,-4)$

Similarly

B to A is 2 left and 6 down

From D to C
$(8,2) + (-2,-6) = (6,-4)$

C = ……6…… , ……-4……

(4 marks)

7. a. The points $A(8,10)$ and $B(d,e)$ form a line AB. Find the equation of the line AB in terms of d and e.

$$Gradient \ = \ m = \frac{e-10}{d-8}$$

$$y = mx + c$$

$$y = \frac{e-10}{d-8}x + c$$

$$A(8,10) \rightarrow x = 8\,, y = 10$$

$$10 = \frac{e-10}{d-8} \times 8 + c$$

$$10 = \frac{8e-80}{d-8} + c$$

$$c = 10 - \frac{8e-80}{d-8}$$

$$c = \frac{10(d-8)}{d-8} - \frac{8e-80}{d-8}$$

$$c = \frac{10d-80}{d-8} - \frac{8e-80}{d-8}$$

$$c = \frac{10d-80-8e+80}{d-8}$$

$$c = \frac{10d-8e}{d-8}$$

$$y = \frac{e-10}{d-8}x + \frac{10d-8e}{d-8}$$

$$E: y = -3x + 6$$

b. What is a possible set of values of d and e if the line segment AB is parallel to E?

Line are parallel, so they have the same gradient. For every +1 in x there is -3 in y.

$$A(8,10)$$
$$B(8+n, 10-3n)$$

$$n = 1 \rightarrow B(9,7)$$
$$n = 2 \rightarrow B(10,4)$$
$$n = 3 \rightarrow B(11,1)$$
$$n = -1 \rightarrow B(7,13)$$
$$n = -2 \rightarrow B(6,16)$$
$$n = -3 \rightarrow B(5,19)$$

c. What is a possible set of values of c and d if the line segment AB is perpendicular to E?

Perpendicular lines have gradients such that the product of their gradients is equal to -1.

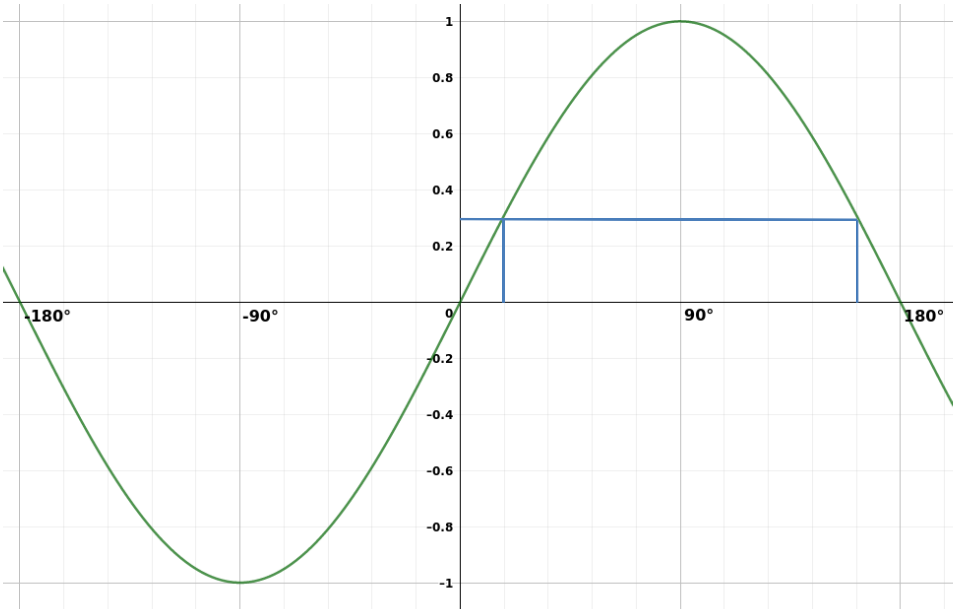
$$-3 \times \frac{1}{3} = 1$$
$$Gradient = \frac{1}{3}$$

$$A(8,10)$$
$$B\left(8+n, 10-\frac{n}{3}\right)$$
$$B\left(9, \frac{29}{3}\right), or\ B\left(10, \frac{28}{3}\right), or\ (11,9), etc$$

$$B\left(7, \frac{31}{3}\right), or\ B\left(6, \frac{32}{3}\right), or\ (5,11), etc$$

(2 marks, 2 marks, 3 marks)

8. The graph of $y = sin(x)$ for $-180 \leq x \leq 180$ has been drawn on the axes below. A solution to $sin(x) = a$ is $18°$.



Find another solution, and the approximate value of a.

To find another solution, $\sin(x)$ *is symmetric about the* $x = 90°$*.*

$$90° - 18° = 72°$$
$$90° + 72° = 162°$$
$$x = 162°$$

Alternatively:

$$180° - 18° = 162°$$
$$x = 162°$$

Start by locating $18°$*on the* $x - axis$*, then looking up to your graph and then reading across to the* $y - axis$*, this gives the value of* a*.*

$$a = 0.3 \pm 0.02$$

The domain of $y = sin(x)$ is no longer limited to $-180 \leq x \leq 180$.
Write down two more solutions to $sin(x) = a$.

The graph of $\sin(x)$ *repeats itself every* $360°$.

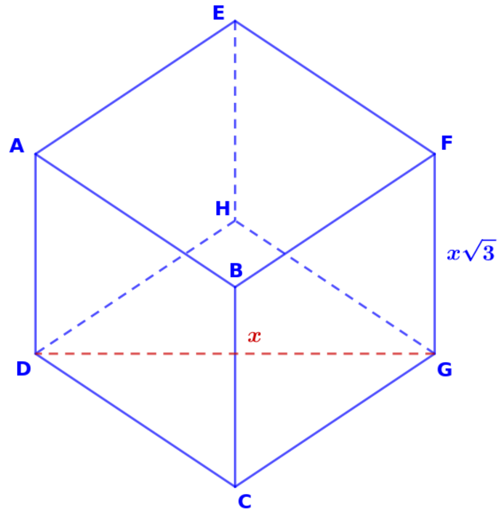
$$18° + 360° = 378°$$

$$162° + 360° = 522°$$

Any multiple of $360°$*added to or subtracted from* $18°$ *or* $162°$ *works.*

(2 marks, 2 marks)

9. A cuboid is pictured below.



$$DC = CG$$

Using an algebraic argument, show that the shape is not a cube.

We can give and DC and CG an algebraic value.

$$DC = CG = y$$

The shape is a cuboid, so DCG forms a right angle triangle with x as the hypotenuse, so we can use Pythagoras to find y:

$$x^2 = y^2 + y^2$$
$$x^2 = 2y^2$$
$$\frac{x^2}{2} = y^2$$
$$y = \sqrt{\frac{x^2}{2}}$$
$$y = \frac{\sqrt{2}}{2}x$$

If this shape were a cuboid then all sides should be the same length, but

$$\frac{\sqrt{2}}{2}x \neq x\sqrt{3}$$

Find the length CE in terms of x.

HDGC is a rectangle

$$HC = DG = x$$

The shape is a cuboid

$$EH = FG = x\sqrt{3}$$

Create a right-angle triangle EHC, where CE is the hypotenuse, and HC and EH are the shorter sides.

$$CE^2 = x^2 + \left(x\sqrt{3}\right)^2$$
$$CE^2 = x^2 + 3x^2$$
$$CE^2 = 4x^2$$
$$CE = \sqrt{4x^2}$$
$$CE = 2x$$

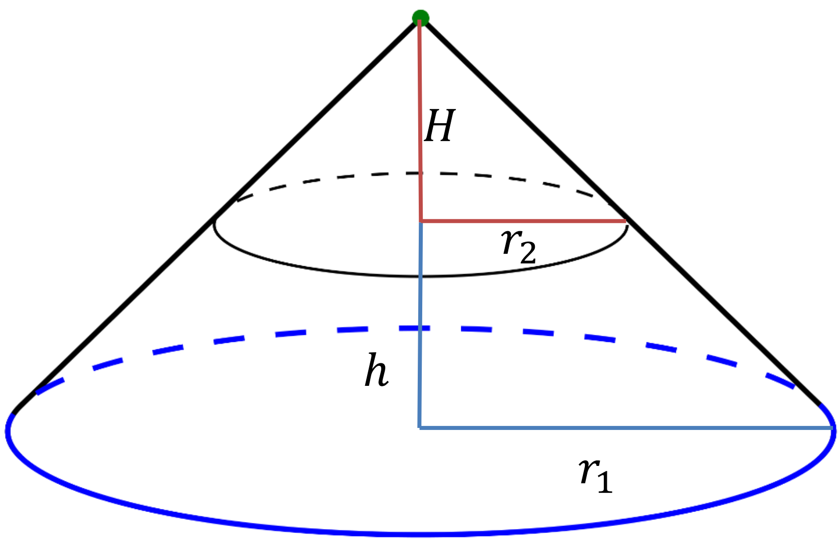
(2 marks, 2 marks)

10. Given a frustum of height h, base radius r_1 and top radius r_2, show that the volume of the frustum can be written as:

$$\frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$$

$$Volume\ of\ cone = V = \frac{\pi r^2 h}{3}$$

If we imagine a full cone, we can split it into our frustrum with a smaller cone on top.



$$Volume\ of\ full\ cone = \frac{\pi {r_1}^2 (h + H)}{3}$$

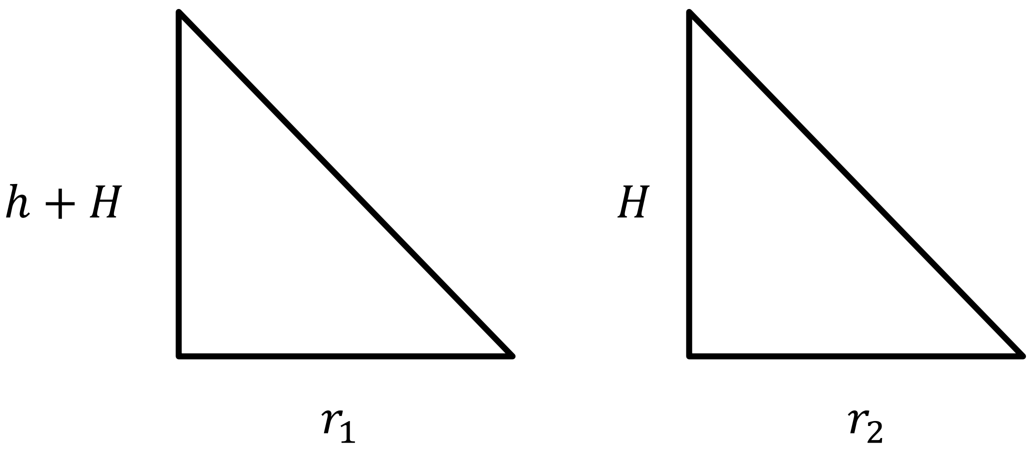
$$Volume\ of\ smaller\ cone = \frac{\pi {r_2}^2 H}{3}$$

$$Volume\ of\ frustrum = full\ cone - smaller\ cone$$

$$= \frac{\pi {r_1}^2 (h + H)}{3} - \frac{\pi {r_2}^2 H}{3}$$

$$= \frac{\pi {r_1}^2 (h + H) - \pi {r_2}^2 H}{3}$$

$$= \frac{\pi}{3}[{r_1}^2 (h + H) - {r_2}^2 H]$$



Looking at the slanted sides and radii we can created similar triangles, which gives the following equation:

$$h + H = \frac{r_1 H}{r_2}$$

Substituting this into the equation:

$$V = \frac{\pi}{3}[r_1{}^2(h+H) - r_2{}^2 H]$$

$$= \frac{\pi}{3}\left[r_1{}^2\frac{r_1 H}{r_2} - r_2{}^2 H\right]$$

$$= \frac{\pi}{3}H[r_1{}^2\frac{r_1}{r_2} - r_2{}^2]$$

$$= \frac{\pi}{3}H[\frac{r_1{}^3}{r_2} - \frac{r_2{}^3}{r_2}]$$

$$= \frac{\pi}{3}H\left[\frac{r_1{}^3 - r_2{}^3}{r_2}\right]$$

From similar triangles again:

$$H = \frac{hr_2}{r_1 - r_2}$$

Substituting into the above:

$$V = \frac{\pi}{3}H\left[\frac{r_1{}^3 - r_2{}^3}{r_2}\right]$$

$$= \frac{\pi}{3}\frac{hr_2}{r_1 - r_2}\left[\frac{r_1{}^3 - r_2{}^3}{r_2}\right]$$

$$= \frac{\pi}{3}\left[\frac{h({r_1}^3 - {r_2}^3)}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3}\left[\frac{h(r_1 - r_2)(r_1^2 + r_1 r_2 + r_2^2)}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3} h(r_1^2 + r_2^2 + r_1 r_2)$$

(4 marks)

11. In the diagram below a hemisphere has been fixed to a cone. They have the same radius.

Given that the curved surface area of the cone is 108π, what is the surface area of the whole object in terms of s, the slanted height of the cone.

$$Curved\ Surface\ Area\ of\ Cone\ = \pi r L$$

$$Curved\ Surface\ Area\ of\ Cone = 108\pi$$

$$L = S$$

$$108\pi = \pi r S$$

$$108 = rS$$

$$r = \frac{108}{S}$$

$$Surface\ area\ of\ sphere = 4\pi r^2$$

$$= 4\pi\left(\frac{108}{S}\right)^2$$

$$= \frac{46656\pi}{S^2}$$

$$Curved\ surface\ area\ of\ hemisphere = \frac{sphere}{2}$$

$$= \frac{23328\pi}{S^2}$$

$$Total\ surface\ area = \frac{23328\pi}{S^2} + 108\pi$$

Surface area = ……………….

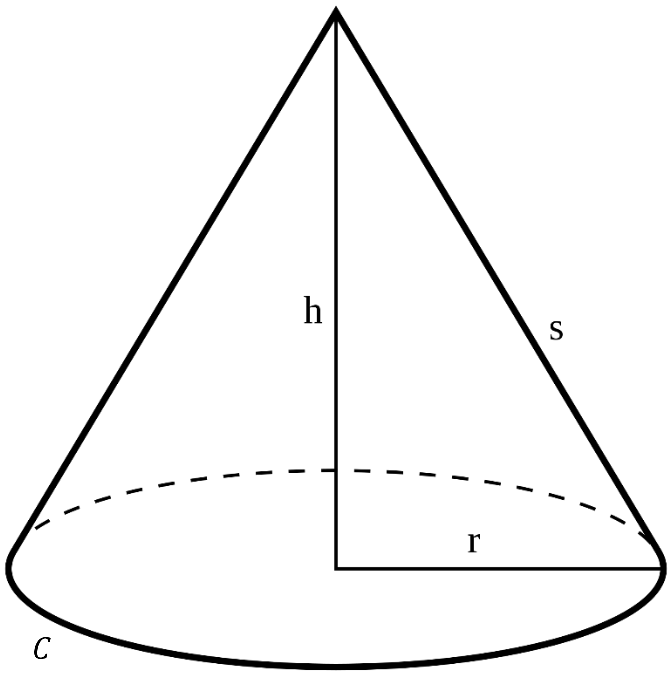
(4 marks)

12. By drawing a net of a cone, prove that the surface area of a cone is given by the formula

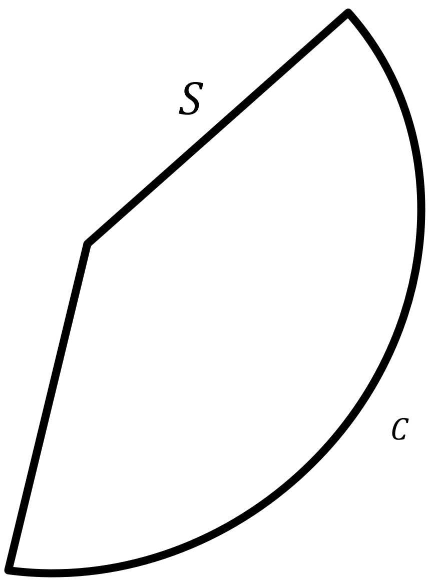
$$\pi rs + \pi r^2$$

where r is the radius of the cone and s is the slanted height.

Cone with height h, radius r, slanted side length of s, and circumference C.



With the curved surface rolling out into a sector as follows:



$$radius = s$$
$$arc\ legnth = C$$
$$= 2\pi r$$

If the angle in the sector is x, then the fractional amount of a full circle is $\frac{x}{360}$.

$$Area\ of\ sector = \pi s^2 \times \frac{x}{360}$$
$$= x \times \frac{\pi s^2}{360}$$

$$Arc\ legnth = 2\pi s \times \frac{x}{360}$$
$$= \frac{2\pi s x}{360}$$
$$= 2\pi r$$

$$\frac{2\pi s x}{360} = 2\pi r$$

$$\frac{sx}{360} = r$$

$$x = \frac{360r}{s}$$

Substituting into the formula for the area of the sector.

$$Area\ of\ sector = x \times \frac{\pi s^2}{360}$$
$$= \frac{360r}{s} \times \frac{\pi s^2}{360}$$
$$= \frac{r}{s} \times \pi s^2$$
$$= \pi r s$$
$$Area\ of\ curved\ surface\ area = \pi r s$$

$$Area\ of\ base = \pi r^2$$

$$Total\ area = \pi r s + \pi r^2$$

(3 marks)