AQA

Please write clearly in block capitals.

Centre number | | | | | Candidate number | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Unit Decision 2

Tuesday 26 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 Paul is choosing a team to participate in a local quiz league. Paul auditions five people: Kevin, Marilyn, Nathan, Sarah and Tom. Paul assesses each person's ability to answer questions on four different topics by giving them a score out of 20 for each topic. Paul then produces the following table of results.

	Art	**History**	**Language**	**Politics**
Kevin	13	8	12	17
Marilyn	9	13	18	19
Nathan	10	11	13	11
Sarah	14	12	17	9
Tom	11	9	14	14

Paul wants to choose a team where each member of the team is assigned to a different topic so that the total score of the team is **maximised**.

(a) Modify the table of results so that the Hungarian algorithm can be used.

[2 marks]

(b) Using the Hungarian algorithm by reducing the **columns first**, determine the person who will **not** be selected as a member of the team.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

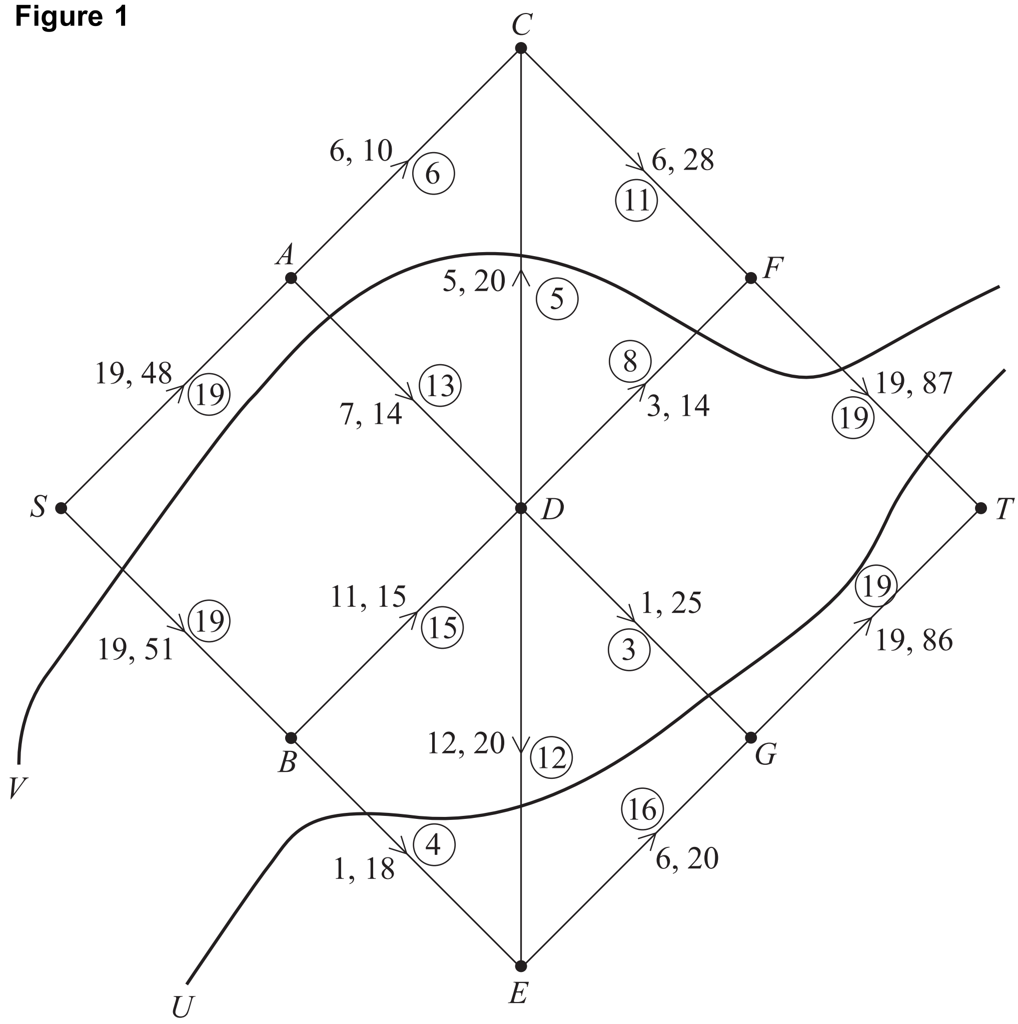
Do not write outside the box

9

Turn over ▶

2 **Figure 1** shows a system of pipes through which gas flows. The numbers on each edge represent the minimum and maximum capacities for each pipe in $\text{cm}^3\,\text{s}^{-1}$, and the numbers in the circles represent a feasible flow of $38\,\text{cm}^3\,\text{s}^{-1}$.

Figure 1



(a) Using **Figure 1**, calculate the value of the cut:

(i) U

(ii) V.

[2 marks]

(b) Interpret your answers to part **(a)** in context.

[2 marks]

(c) (i) Using the feasible flow, shown in **Figure 1**, as the initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 2** on page 7.

[3 marks]

(ii) Use flow augmentation on **Figure 2** to find the maximum flow from S to T. You should indicate any flow-augmenting paths in the table and modify the potential increases and decreases of the flow on **Figure 2**.

[4 marks]

(iii) State the value of the maximum flow.

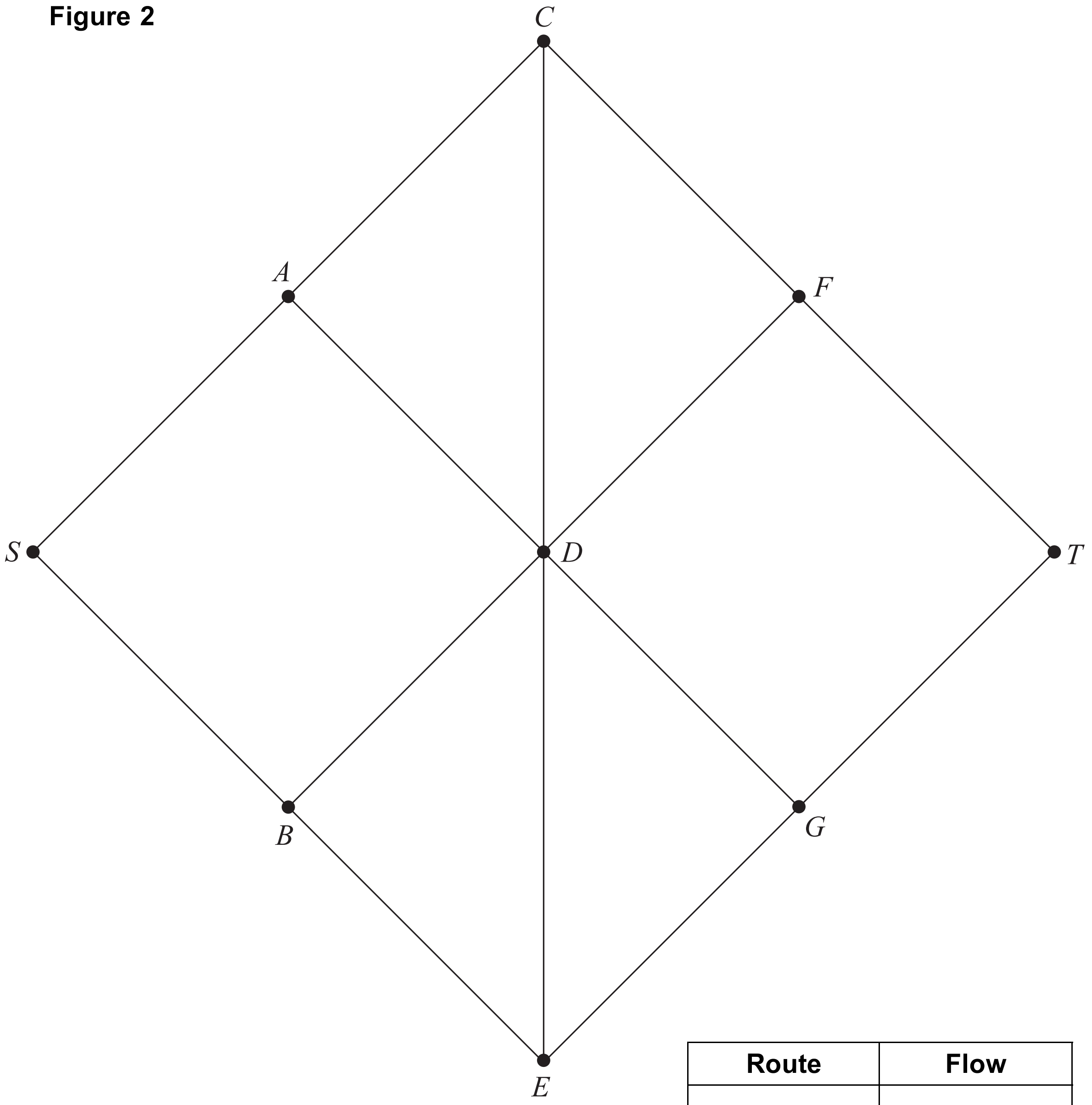
[1 mark]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

Figure 2



Route	Flow

Maximum flow is ______________________

12

Turn over ▶

Do not write outside the box

3 John and Winnie play a zero-sum game. The game is represented by the following pay-off matrix for John.

Winnie

John

Strategy	W_1	W_2	W_3
J_1	0	−2	4
J_2	−2	7	−6

(a) Find the value of the game for John.

[7 marks]

(b) Find the optimal mixed strategy for Winnie.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 3

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

11

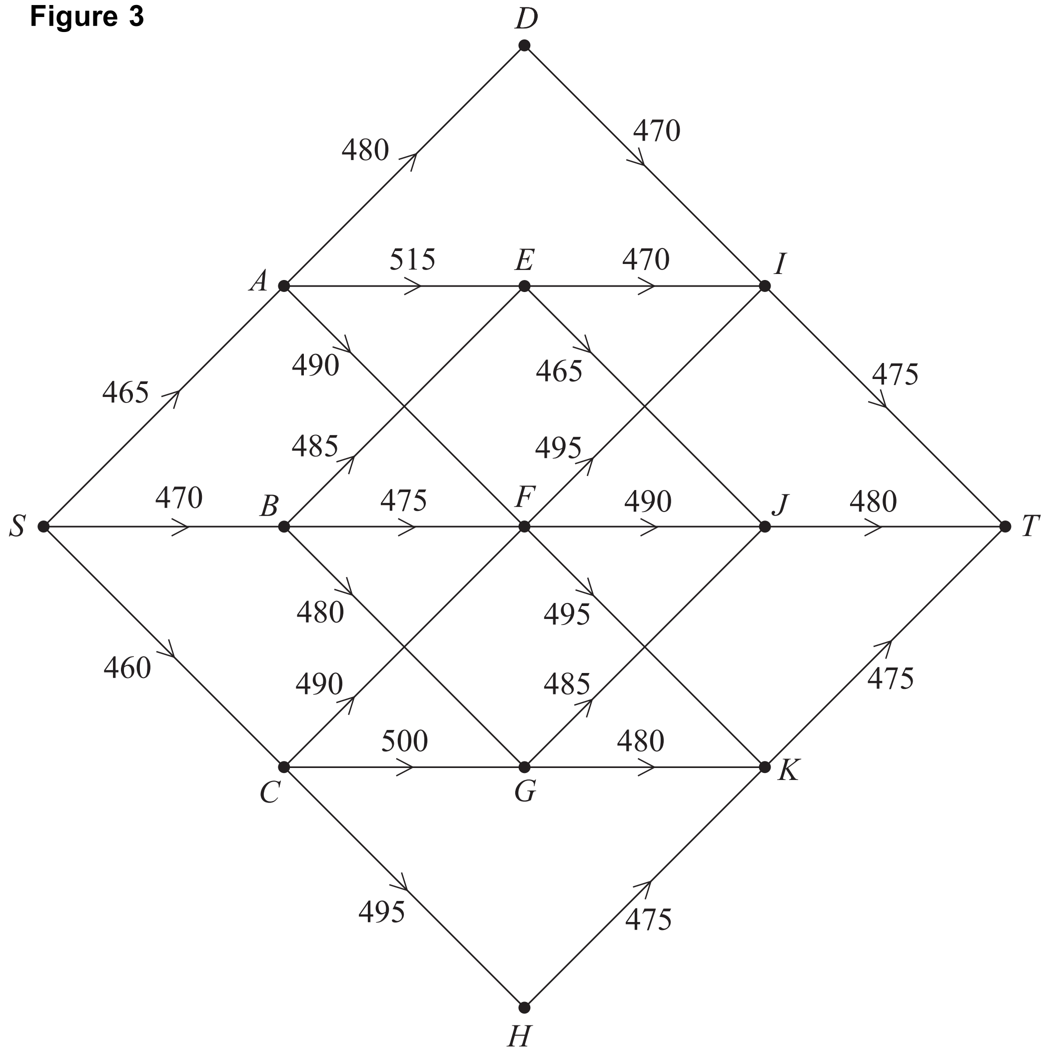
Turn over ▶

Do not write outside the box

4 The owner of a haulage company is planning a four-day continental journey from S to T for one of its drivers. The network in **Figure 3** shows different roads and possible stop-off points A, B, … K.

The number on each edge shows the driving time, in minutes, along each road.

Figure 3



The driver requests that the longest amount of time driving on any one day of the journey be kept to a minimum.

Working backwards from T, use dynamic programming to find all optimal routes for the driver and state the longest amount of time driving on any one day of the journey.

You must complete the table on page 13 as your solution.

[11 marks]

QUESTION PART REFERENCE

Do not write outside the box

Answer space for question 4

Stage	State	From	Value
1	I	T	
	J	T	
	K	T	
2			

11

Do not write outside the box

5 A linear programming problem is being solved using the simplex method. After one iteration of the simplex method the following tableau is obtained.

P	x	y	z	s	t	u	value
1	0	a	35	0	10	0	100
0	0	$\frac{1}{2}$	$\frac{7}{2}$	1	$-\frac{1}{2}$	0	b
0	1	$\frac{3}{2}$	1	0	$\frac{1}{2}$	0	c
0	0	$-\frac{3}{2}$	d	0	$-\frac{1}{2}$	1	15

The constants a, b, c and d are rational numbers, and $b > 0$, $c > 0$.

It is known that at least one further iteration is required.

(a) State the inequality that a must satisfy.

[1 mark]

(b) It is known that the second row of the tableau should be used as the pivot row for the second iteration of the simplex method.

Find and simplify a relationship between b and c.

[2 marks]

(c) A second iteration of the simplex method is carried out using the second row of the tableau as the pivot row.

(i) Determine an expression for the entry in the shaded cell after the second iteration of the simplex method is performed.

[2 marks]

(ii) Determine the entries which will appear in the objective (top) row after the second iteration of the simplex method has been carried out.

[4 marks]

(iii) No further iterations of the simplex method are required.

Determine the interval satisfied by a.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

11

Turn over ▶

Do not write outside the box

6 **Figure 4** on page 17 shows an activity network for a building project being carried out by a construction company. The duration of each activity is given in days.

(a) Find the earliest start time and the latest finish time for each activity and insert these values on **Figure 4**.

[2 marks]

(b) Find the critical path.

[1 mark]

(c) Draw a Gantt chart for the building project on **Figure 5**, assuming that each activity is to start as late as possible.

[3 marks]

(d) The duration of activities F, G and H can be reduced at an additional cost. The following table shows the additional cost for reducing the duration of each of these activities by one day, as well as the minimum possible duration for each activity.

Activity	Additional cost per day (£)	Minimum duration (days)
F	250	3
G	500	3
H	650	3

(i) Determine which activities should have their durations reduced in order to complete the project as early as possible.

For **each** such activity state the smallest reduction in its duration needed to achieve this.

(ii) The construction company will receive a £5000 bonus if the building project is completed 5 days ahead of the initial schedule.

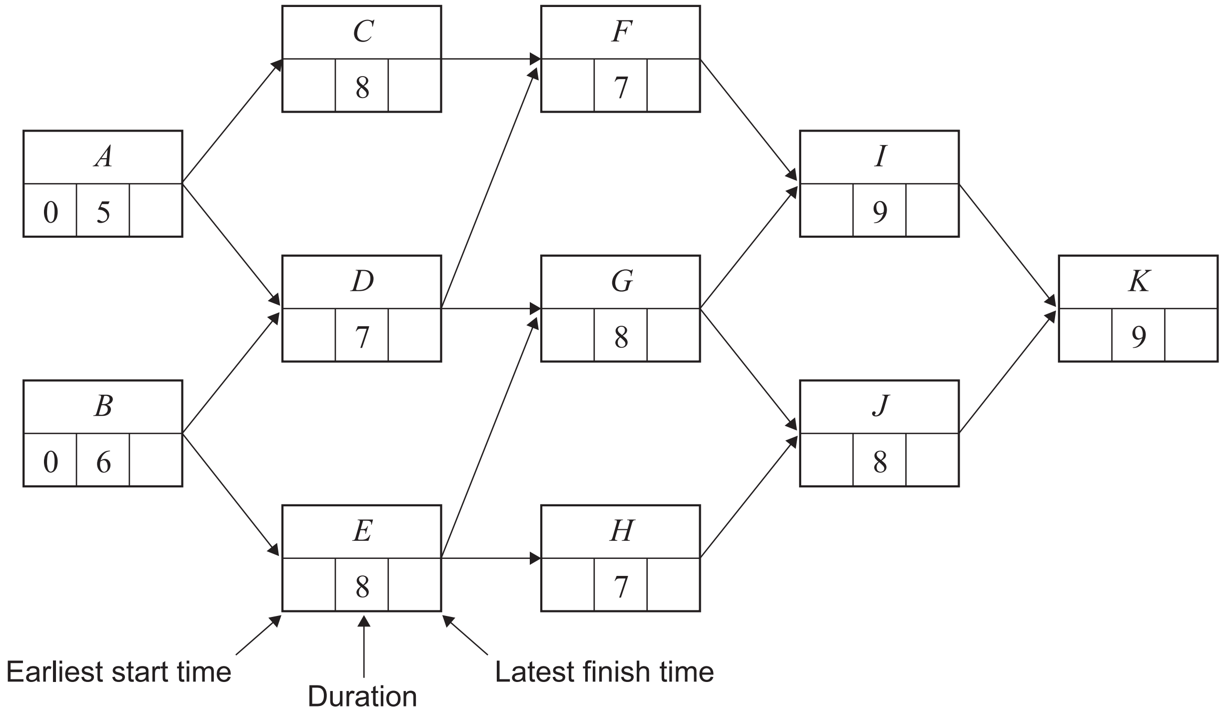
State, with justification, whether or not the construction company should reduce the durations of any of the activities F, G and H.

[5 marks]

Do not write outside the box

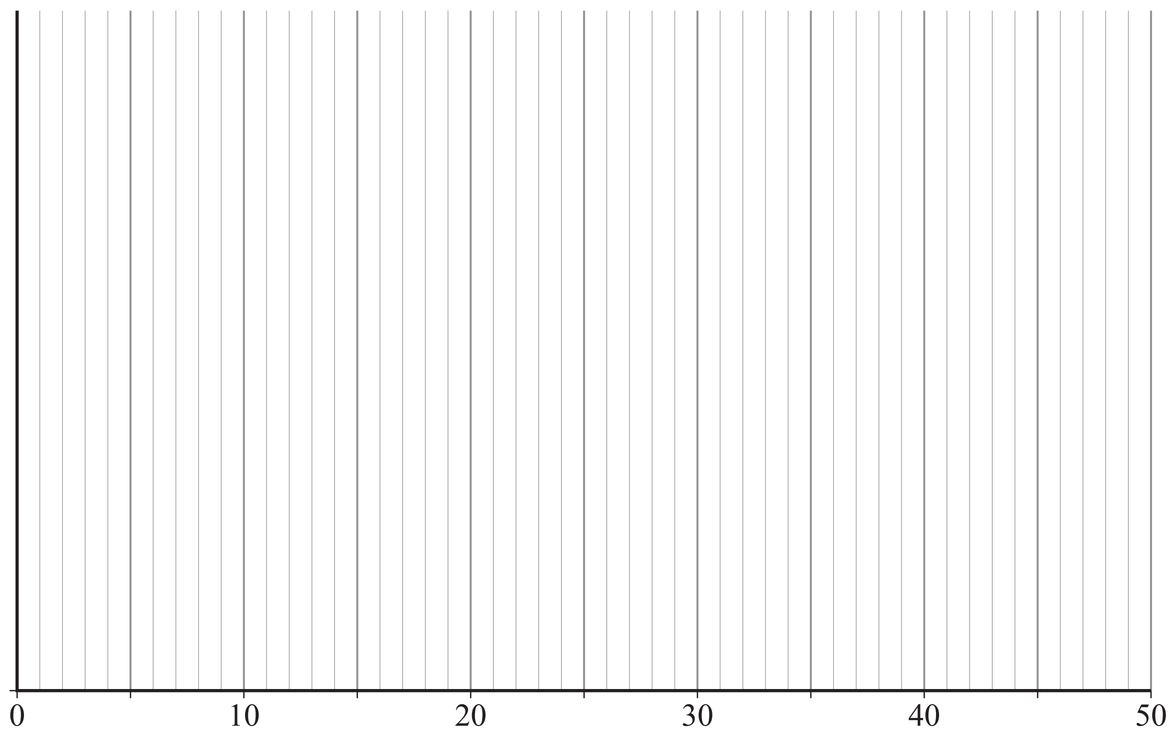
Answer space for question 6

Figure 4



Critical path = ____________________

Figure 5



Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

11

Turn over ▶

Do not write outside the box

7 Gary and Pete play a zero-sum game. The game is represented by the following pay-off matrix for Gary.

Pete

Gary: Strategy	W	X	Y	Z
A	-2	5	$2x$	3
B	-3	6	$x-3$	-5
C	$x-3$	$3x-3$	-2	7
D	$x-1$	1	-2	6

x is an integer.

(a) In the case where $x = -1$, a stable solution exists.

Find any saddle points that exist.

[4 marks]

(b) If a strategy is dominated, it can be removed from the game.

In the case where $x < -3$, no stable solution exists.

Find any strategies which can be removed from the game, giving reasons for your answers.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 7**

Turn over ▶

QUESTION PART REFERENCE	**Answer space for question 7**

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7

10

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED