AQA

Please write clearly in block capitals.

Centre number [ ][ ][ ][ ][ ]   Candidate number [ ][ ][ ][ ]

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level MATHEMATICS

Unit Mechanics 4

Wednesday 28 June 2017   Morning   Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- The blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
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6	
7	
8	
TOTAL	

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Answer **all** questions.

Answer each question in the space provided for that question.

1 A light lamina is placed in the $x - y$ plane. Three masses are attached to the lamina. A mass of 1 kg is attached at the point $(d, 0)$, a mass of 1.5 kg is attached at the point $(0, -2d)$ and a mass of 2 kg is attached at the point $(-3d, 4d)$, where d is a positive constant. The lamina is free to rotate about an axis through O perpendicular to the plane of the lamina. The moment of inertia of the system about this axis of rotation is 513 kg m^2.

Find the value of d.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	**Answer space for question 1**

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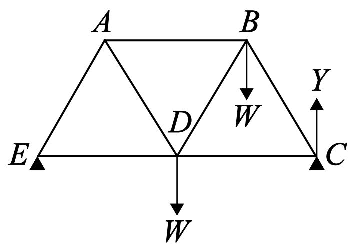
2 A framework consists of seven light inextensible smoothly jointed rods AB, BC, CD, AD, AE, BD and DE. They form equilateral triangles ADE, ABD and BCD. The framework is in equilibrium in a vertical plane resting on supports at C and E. The rods AB, CD and DE are horizontal. Loads of weight W are attached at B and D. The reaction on the support at C is Y, as shown in the diagram.



(a) Show that $Y = \dfrac{5W}{4}$.

[3 marks]

(b) Find the magnitudes of the forces in the rods BC and BD in terms of W.

[5 marks]

(c) State whether the rod AB is in tension or compression, giving a reason for your answer.

[2 marks]

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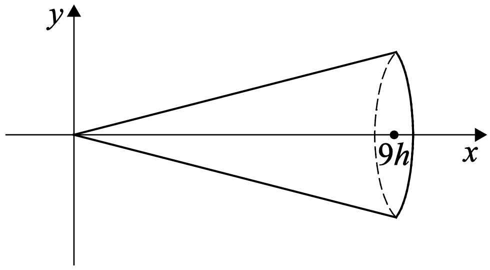
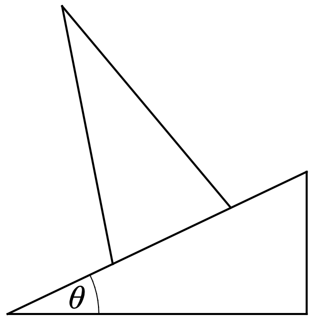
3 A uniform solid cone is formed by rotating the finite region bounded by the lines with equations $y = \frac{1}{3}x$, $y = 0$ and $x = 9h$ through 2π radians about the x-axis. The cone is shown in the diagram below.



(a) Use integration to find the distance of the centre of mass of the cone from the origin.

[5 marks]

(b) The cone rests in equilibrium with its plane face on a rough plane inclined at an angle of θ to the horizontal as shown in the diagram.



Given that the plane is sufficiently rough to prevent slipping, find the maximum value of θ for the cone to remain in equilibrium without toppling. Give your answer to the nearest degree.

[3 marks]

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4 A uniform rod AB has mass $2m$ and length l. The rod is free to rotate about a fixed smooth axis which passes through A and is perpendicular to the rod. The rod has angular speed ω when it collides with a stationary particle, P, of mass m. Immediately before the collision P is at a distance d from A. The particle sticks to the rod and immediately after the collision the angular speed of the rod is $\frac{2}{3}\omega$.

Find d in terms of l.

[5 marks]

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5 A system of forces $2p\mathbf{i}+\mathbf{j}+2\mathbf{k}$, $p\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $2\mathbf{i}-p\mathbf{j}+2p\mathbf{k}$ act at the points with coordinates $(2p, 1, 0)$, $(1, 2, 0)$ and $(-2, -1, 2)$ respectively, where p is a constant.

Show that the total moment of this system of forces about the origin is independent of p.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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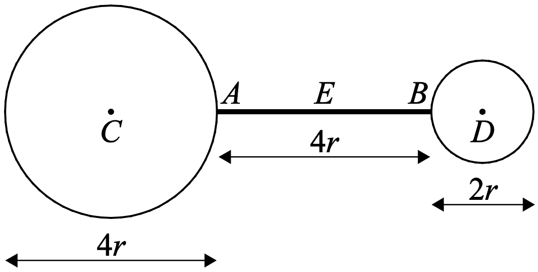
6 (a) Prove, by integration, that the moment of inertia of a uniform solid sphere, of mass m and radius r, about a diameter is $\dfrac{2mr^2}{5}$.

[6 marks]

(b) A body consists of two uniform solid spheres and a uniform rod. The uniform rod, AB, has mass $2m$ and length $4r$. The larger sphere has mass $4m$ and diameter $4r$ and is rigidly attached at A. The smaller sphere has mass m and diameter $2r$ and is rigidly attached at B. The centres of the spheres are C and D, and $CABD$ is a straight line, as shown in the diagram.



The rod is smoothly pivoted at E, the midpoint of AB. The body is free to rotate about a horizontal axis through E and perpendicular to AB. Initially the body is at rest with AB horizontal.

(i) Show that the moment of inertia of the smaller sphere about the axis through E is $\dfrac{47mr^2}{5}$.

[2 marks]

(ii) Find the moment of inertia of the body about the axis through E.

[5 marks]

(iii) Find the maximum angular velocity of the body after it is released from rest.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

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7 The equations of the lines of actions of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are given by

$$\mathbf{r} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -1 \end{bmatrix} \text{ and } \mathbf{r} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 2 \end{bmatrix} \text{ respectively.}$$

The unit of length is the metre. The system formed by these two forces is equivalent to a single force $\mathbf{F} = \begin{bmatrix} k \\ 0 \end{bmatrix}$ N acting at the origin along with a clockwise couple of magnitude 39 N m.

(a) Explain why $\mathbf{F}_1$ can be written as $\mathbf{F}_1 = \begin{pmatrix} 3a \\ -a \end{pmatrix}$.

[1 mark]

(b) Find $\mathbf{F}_1$ and $\mathbf{F}_2$ and the value of k.

[11 marks]

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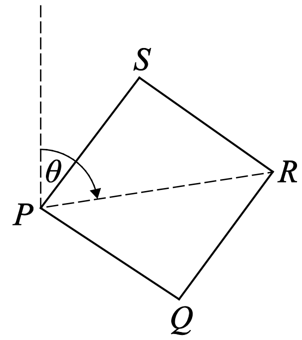
8 A uniform square lamina $PQRS$ has mass m and side $6a$. The lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis, which passes through P and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about this axis is $24ma^2$.

The lamina is held at rest with S vertically above P and then released. At time t seconds after release PR makes an angle θ with the upward vertical as shown in the diagram.



(a) Show that $\dot{\theta}^2 = \dfrac{g}{4a}(1 - \sqrt{2}\cos\theta)$.

[4 marks]

(b) Find, at time t, the magnitude of the component of the force acting on the lamina at P, in the direction perpendicular to PR, in terms of m, g and θ .

[6 marks]

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END OF QUESTIONS

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