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Centre number	
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Surname	
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Candidate signature	

A-level
MATHEMATICS
Unit Mechanics 4

Tuesday 26 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
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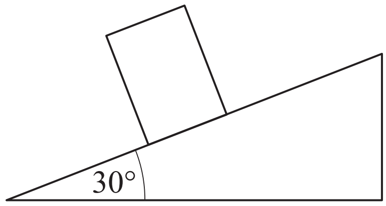
Answer **all** questions.

Answer each question in the space provided for that question.

1 A uniform solid cylinder has vertical height h and base radius r. It is placed with its axis in a vertical plane and its circular base in contact with a rough inclined plane. The plane is slowly tilted until the cylinder is on the point of toppling when the angle of inclination of the plane is $30°$, as shown in the diagram.



Assume that the plane is rough enough so that no slipping occurs.

Express h in terms of r.

[3 marks]

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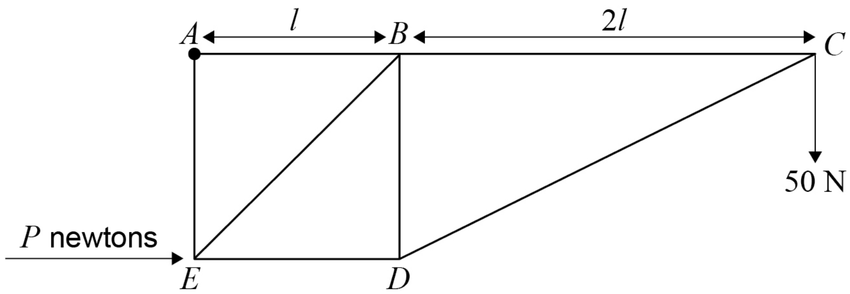
2 A framework is composed of seven light smoothly-jointed rods AB, AE, BC, BD, BE, CD and DE.

$ABDE$ is a square and BCD is a right angled triangle. The rod AB has length l and rod BC has length $2l$. The framework is freely hinged at A to a fixed support, so that it can move in a vertical plane about a horizontal axis through A. An object of weight $50\,\text{N}$ is attached to the framework at C. The framework is kept in equilibrium with the rod AE vertical by a horizontal force of magnitude P newtons applied at E, as shown in the diagram.



(a) Find P.

[2 marks]

(b) (i) Show that the magnitude of the reaction force on the framework at A is $k\sqrt{10}$ newtons where k is an integer to be found.

[2 marks]

(ii) Draw a diagram showing the direction of the reaction force at A.

[1 mark]

(c) State which rods are in compression.

[1 mark]

(d) Find the magnitudes of the forces in each of the rods CD, BC and BE.

[5 marks]

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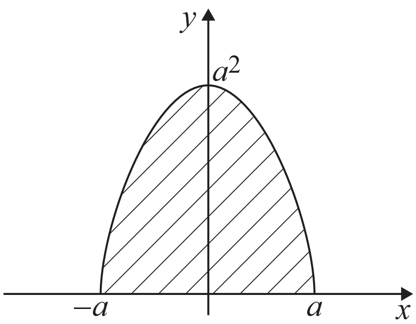
3 A uniform lamina, S, is bounded by the curve $y = a^2 - x^2$ and the x axis, as shown in **Figure 1**.

Figure 1

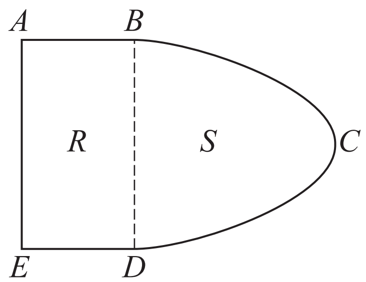


(a) Find the coordinates of the centre of mass of the lamina, S.

[7 marks]

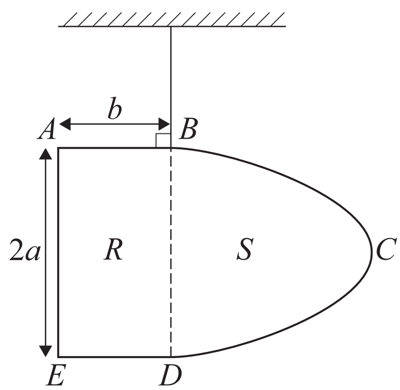
(b) A large sign, $ABCDE$, is formed by joining the straight edge of the lamina, S, to an edge of a rectangular lamina, R, so that R and S are in the same plane, as shown in **Figure 2**.

Figure 2



The rectangular lamina, R, has length $2a$ and width b. The density of the lamina R is twice the density of the lamina S. When the sign is freely suspended from the point B, it hangs with its axis of symmetry horizontal, as shown in **Figure 3**.

Figure 3



Express b in terms of a.

[5 marks]

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4 A system consists of three forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$. The forces $\mathbf{F}_1 = -2\mathbf{i} + \mathbf{k}$ and $\mathbf{F}_2 = 4\mathbf{j} - 2\mathbf{k}$, act at the points whose coordinates are $(2,\ 1,\ -1)$ and $(3,\ -2,\ 0)$ respectively.

(a) In the case where $\mathbf{F}_3$ acts at the origin, the system is equivalent to a couple.

Find the moment of the couple.

[4 marks]

(b) In the case where $\mathbf{F}_3$ does not act at the origin, the system is in equilibrium.

Find a vector equation for the line of action of $\mathbf{F}_3$.

[9 marks]

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5 A uniform rectangular lamina $ABCD$ has mass m, with $AB = 4a$ and $BC = 3a$. The centre of mass of the lamina is G.

(a) Find the moment of inertia of the lamina about a fixed axis which is perpendicular to its plane and passes through A.

[3 marks]

(b) The lamina is smoothly hinged at A. It is free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passes through A. Initially the lamina is held with AG horizontal and then released. At time t after release, AG is at an angle θ below the horizontal.

(i) Show that $\dot{\theta}^2 = \dfrac{3g\sin\theta}{5a}$.

[4 marks]

(ii) Hence find $\ddot{\theta}$ in terms of a, g and θ.

[2 marks]

(iii) Show that the magnitude of the reaction force which the hinge exerts on the lamina is $\dfrac{mg}{4}\sqrt{p + q\sin^2\theta}$ where p and q are integers to be found.

[8 marks]

(iv) Hence state the maximum value of the reaction force, in terms of m and g.

[1 mark]

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6 (a) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass M and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{1}{2}Mr^2$.

[5 marks]

(b) A uniform circular solid pulley with radius r is free to rotate about a fixed smooth horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. A light inextensible string passes over the rough rim of the pulley and has a particle of mass $3m$ attached to one end and a particle of mass $6m$ attached to the other end to form a system.

The system is released from rest at time $t = 0$. At time t, the pulley has turned through an angle θ. During the motion, the string does not slip on the pulley.
The string between the pulley and the $3m$ mass is vertical and the magnitude of the tension is T_1, and the string between the pulley and the $6m$ mass is vertical and the magnitude of the tension is T_2, where $\mathrm{T}_2 : \mathrm{T}_1 = 4 : 3$.

(i) Find the angular acceleration of the pulley in terms of g and r.

[4 marks]

(ii) Find the mass of the pulley in terms of m.

[5 marks]

(iii) When the angular speed of the pulley is ω, the string breaks and a constant braking couple of magnitude mgr is applied to the pulley to bring it to rest. After breaking, the string does not exert any further force on the pulley.

Find an expression for the angle turned by the pulley from the instant the string breaks until the pulley first comes to rest, in terms of r, ω and g.

[4 marks]

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