



**General Certificate of Education (A-level)
June 2011**

Mathematics

MM04

(Specification 6360)

Mechanics 4

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Copyright © 2011 AQA and its licensors. All rights reserved.

Copyright

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

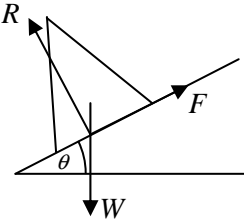
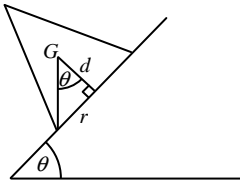
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

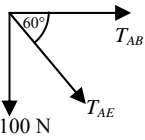
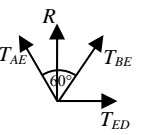
MM04

Q	Solution	Marks	Total	Comments
1(a)	$\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	B1	1	Clear use of $\sum \mathbf{F}_i = \mathbf{0}$
(b)	$\mathbf{r}_1 \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & 0 & -1 \\ \mathbf{j} & 2 & 1 \\ \mathbf{k} & 1 & 0 \end{vmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ $\mathbf{r}_2 \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & 3 & 4 \\ \mathbf{j} & -1 & 0 \\ \mathbf{k} & 0 & -2 \end{vmatrix} = \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$ $\mathbf{r}_3 \times \mathbf{F}_3 = \begin{vmatrix} \mathbf{i} & 4 & -3 \\ \mathbf{j} & 0 & -1 \\ \mathbf{k} & -5 & 2 \end{vmatrix} = \begin{pmatrix} -5 \\ 7 \\ -4 \end{pmatrix}$ $\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} -4 \\ 12 \\ 2 \end{pmatrix}$	M1 A3,2,1 m1 A1F	6	Any attempt at $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ -1 each 'type' of error Summing 3 vector product expressions Follow through their three answers for $\mathbf{r} \times \mathbf{F}$ $\mathbf{F} \times \mathbf{r}$ loses final A1
(c)	<p>Resultant force = $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and moment about</p> <p>O is non zero \Rightarrow couple</p>	E2,1	2	E1 $\sum \mathbf{F} = 0$ E1 Moment $\neq 0$
Total			9	

MM04 (cont)

Q	Solution	Marks	Total	Comments	
2(a)	Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{8h^3\pi}{3}$	B1	5	AG	
	Alternative: $\left[\pi \int_0^{8h} \left(\frac{1}{8}x\right)^2 dx = \frac{8h^3}{3}\pi \right]$	(B1)			
	$\pi \int xy^2 dx = \pi \int_0^{8h} x \left(\frac{x}{8}\right)^2 dx = \pi \int_0^{8h} \frac{x^3}{64} dx$	M1			Use of $\int xy^2 dx$
	$= \left[\frac{x^4}{256} \right]_0^{8h}$	A1			Correct integration of kx^3
	$= 16h^4$				
(b)(i)		B1	3 forces		
		B1	2	Through a common point at base of cone	
(ii)					
		<p>G above point of rotation</p> $\tan \theta = \frac{r}{d}$ $= \frac{h}{8h - 6h} = \frac{1}{2}$	M1	$\tan \theta$ seen anywhere	
	$\theta = \tan^{-1} \frac{1}{2} = 26.6^\circ$	A1F	3	Use of \tan^{-1} to get θ ; ft incorrect ratio	
(c)	Resolve perp to plane $N = W \cos \alpha$	B1	4	Both correct	
	parallel to plane $F = W \sin \alpha$				
	Limiting friction $F = \mu N \Rightarrow \tan \alpha = \frac{6}{13}$	M1			Eliminate W , obtain $\tan \alpha$
	$\alpha = 24.8^\circ < 26.6^\circ$ or $\frac{6}{13} < \frac{1}{2}$	A1			Comparison
	\Rightarrow slides first	A1F		Conclusion; ft (b)(ii)	
Total			14		

MM04 (cont)

Q	Solution	Marks	Total	Comments
3(a)	System is symmetrical about vertical line through B , hence reactions are equal	E1	1	
(b)	Resolve vertically $2R = 250$ $R = 125 \text{ N}$	M1 A1	2	Attempt to resolve
(c)	 <p>At A resolve vertically $T_{AE} \sin 60^\circ + 100 = 0$ $T_{AE} = \frac{100}{\sin 60^\circ} = 115.4(\dots) = 115 \text{ N}$</p> <p>At A resolve horizontally $T_{AB} + T_{AE} \cos 60^\circ = 0$ $T_{AB} = \frac{100}{\sin 60^\circ} \times \cos 60^\circ = 57.73(\dots) = 57.7 \text{ N}$</p>	M1 A1	4	Attempt at equation involving AE AG $\left[\text{or } \frac{200\sqrt{3}}{3} \text{ seen} \right]$ Attempt at equation involving AB AG $\left[\text{or } \frac{100\sqrt{3}}{3} \text{ seen} \right]$
(d)	By symmetry, $T_{BE} = T_{BD} = T$ Resolve vertically at B : $2T \cos 30^\circ + 50 = 0$ $ T_{BE} = \frac{25}{\cos 30^\circ} = 28.86 = 28.9 \text{ N}$	M1 A1 A1	3	Use of symmetry Accept $\frac{50\sqrt{3}}{3}$
	<p>Alternative: At E resolve vertically</p>  <p>$R + T_{AE} \cos 30^\circ + T_{BE} \cos 30^\circ = 0$ (M1) $125 - \frac{100}{\sin 60^\circ} \cos 30^\circ + T_{BE} \cos 30^\circ = 0$ (A1) $T_{BE} = 28.86 = 28.9 \text{ N}$ (A1)</p>	(M1) (A1) (A1)	(3)	3 term equation involving R, T_{AE}, T_{BE} Correct equation, +/- signs and values correct Accept $\frac{50\sqrt{3}}{3}$
(e)	At E resolve horizontally $T_{ED} + T_{BE} \cos 60^\circ = T_{AE} \cos 60^\circ$ $ T_{ED} = \frac{25}{\cos 30^\circ} \cos 60^\circ - \frac{100}{\sin 60^\circ} \cos 60^\circ = 43.3(\dots) = 43.3 \text{ N}$ $= 43.3 \text{ N}$	M1 A1 A1F	3	3 term equation including T_{ED}, T_{BE}, T_{AE} Correct equation, +/- signs and values correct Follow through T_{AE} ; accept $\frac{75\sqrt{3}}{3}$
	Total		13	

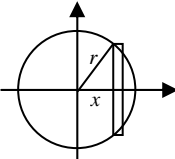
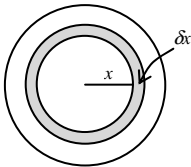
MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$MI_{\text{DISC}} = \frac{1}{2}mr^2 = \frac{1}{2}(1.5)(0.2)^2$ $= 0.03\text{kg m}^2$	M1	2	Use of $\frac{1}{2}mr^2$
		A1		AG
(b)(i)	$V = r\omega$ hence $2 = 0.2\omega$ $\omega = 10\text{ rad s}^{-1}$	M1	2	Attempt to use $r\omega$
		A1		
(ii)	$\text{KE gained by pulley} = \frac{1}{2}I\omega^2$ $= \frac{1}{2}(0.03)(10)^2$ $= 1.5\text{J}$	M1	4	Attempt at KE for pulley
		A1		Correct KE for pulley
(iii)	Conservation of energy: Gain $\text{KE}_{\text{PULLEY}} + \text{KE}_{\text{BUCKET}}$ $= \text{loss in PE for bucket}$			
	$1.5 + \frac{1}{2}(1)(2)^2 = 1gd$ $d = \frac{5}{14}$	M1		3 term equation
(iii)	$g - T = 0.2\ddot{\theta}$	M1A1	5	Equation of motion for bucket $mg - T = mr\ddot{\theta}$
	$0.2T = 0.03\ddot{\theta}$	M1A1		Equation of motion for pulley $rT = I\ddot{\theta}$ A1 for correct substitutions in each case
	$0.2T = \frac{0.03(9.8 - T)}{0.2}$ $T = 4.2\text{N}$	A1		CAO
	Alternative: $a = r\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{a}{0.2}$	(M1)		Connecting $a, \ddot{\theta}$
	For bucket $u = 0, v = 2, s = \frac{5}{14}$ using $v^2 = u^2 + 2as$ gives $a = \frac{28}{5} = 5.6$ $\Rightarrow \ddot{\theta} = \frac{5.6}{0.2} = 28\text{ rad s}^{-1}$	(A1)		
	Either $g - T = 0.2\ddot{\theta}$ or $0.2T = 0.03\ddot{\theta}$ $T = 4.2\text{N}$	(M1A1)		One other equation
		(A1)	(5)	CAO
	Total		13	

MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	<p>Moments about O:</p> $-3(2) - a(6) + 2a(3) - 4(5) + 8(2)$ $= -6 - 6a + 6a - 20 + 16$ $= -10 \Rightarrow \text{no 'a'} \Rightarrow \text{independent}$ <p>Magnitude = 10</p> <p>Alternative:</p> $\mathbf{r}_1 \times \mathbf{F}_1 = \begin{vmatrix} \mathbf{i} & -1 & -8 \\ \mathbf{j} & 2 & 0 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 16 \end{pmatrix}$ $\mathbf{r}_2 \times \mathbf{F}_2 = \begin{vmatrix} \mathbf{i} & 2 & a \\ \mathbf{j} & 6 & -3 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -6 - 6a \end{pmatrix}$ $\mathbf{r}_3 \times \mathbf{F}_3 = \begin{vmatrix} \mathbf{i} & 3 & -4 \\ \mathbf{j} & -5 & 2a \\ \mathbf{k} & 0 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6a - 20 \end{pmatrix}$ <p>Total moment = $16 - 6 - 6a + 6a - 20$ $= -10$ 'a' cancels / no 'a' Magnitude = 10</p> <p>(b)(i) $\mathbf{F} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} -4 \\ 8 \end{pmatrix} + \begin{pmatrix} -8 \\ 0 \end{pmatrix} = \begin{pmatrix} -8 \\ 5 \end{pmatrix}$</p> <p>(ii)</p> <p>Moments about O: $-10 = 5d$ $d = -2$</p>	<p>M1 A2,1</p> <p>A1</p> <p>A1F</p> <p>(M1)</p> <p>(A1)</p> <p>(A1) (A1) (A1F)</p> <p>M1 A1</p> <p>M1</p> <p>A1F</p>	<p>5</p> <p>(5)</p> <p>2</p> <p>2</p>	<p>One correct $F \times d$ pairing seen -1 each type of error</p> <p>'a' cancels / comment about no 'a' Must be positive value stated; ft single slip</p> <p>One correct $\mathbf{r} \times \mathbf{F}$ seen</p> <p>Second $\mathbf{r} \times \mathbf{F}$ correct</p> <p>\sum (k components) Comment about 'a' ft single slip</p> <p>Substitute $a = 4$ and add forces</p> <p>y-component $\times d =$ answer (a)</p> <p>ft (a)</p>
	Total		9	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	 <p>Let density = ρ</p> $m = \frac{4}{3}\pi r^3 \rho \Rightarrow \rho = \frac{3m}{4\pi r^3}$ <p>Mass of elemental disc / cylinder $= \pi(r^2 - x^2) \delta x \rho$</p> <p>MI of elemental disc =</p> $\frac{1}{2} [\pi(r^2 - x^2) \delta x \rho] [r^2 - x^2]$ <p>MI of sphere = $\int_{-r}^r \frac{1}{2} \pi \rho (r^2 - x^2)^2 dx$</p> $= \frac{3m}{8r^3} \int_{-r}^r (r^4 + x^4 - 2r^2 x^2) dx$ $= \frac{3m}{8r^3} \left[r^4 x + \frac{x^5}{5} - 2r^2 \frac{x^3}{3} \right]_{-r}^r$ $= \frac{3m}{8r^3} \left[2 \left(r^5 + \frac{r^5}{5} - \frac{2r^5}{3} \right) \right]$ $= \frac{3m}{4r^3} \times \frac{8r^5}{15} = \frac{2}{5} mr^2$ <p>Alternative:</p>  <p>Let density = ρ</p> $m = 4\pi r^3 \rho$ $\Rightarrow \rho = \frac{3m}{4\pi r^3}$ <p>Mass of elemental shell = $4\pi x^2 \delta x \rho$</p> <p>MI of elemental shell about diameter</p> $= \frac{2}{3} (4\pi x^2 \delta x \rho) x^2$ <p>MI of sphere = $\int_0^r \frac{8}{3} \pi x^4 \rho dx$</p> $= \frac{2m}{r^3} \int_0^r x^4 dx$ $= \frac{2m}{r^3} \left[\frac{x^5}{5} \right]_0^r$ $= \frac{2mr^5}{5r^3}$ $= \frac{2mr^2}{5}$	B1 M1 A1 m1 A1F A1		ρ, m linked anywhere Attempt to use $\pi r^2 h \rho$ Use of $\frac{1}{2} mr^2$ Attempt at integral, dep on first M1 Their expression integrated correctly – correct number of terms only Correct limit used, ρ replaced to obtain answer given AG
		(B1)		ρ, m linked anywhere
		(M1)		Attempt to use surface area of sphere $\times \delta x \times \rho$
		(A1)		Use of $\frac{2}{3} mr^2$
		(m1)		Attempt at integral; dep on first M1
		(A1F)		Their expression integrated correctly
				Correct limits used, ρ replaced
		(A1)	(6)	AG

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(i)	$MI_{\text{ROD}} = \frac{4}{3}(3m)(2l)^2 = 16ml^2$	B1	1	Use of $\frac{4}{3}ml^2$ with ' m ' = $3m$ and ' l ' = $2l$
(ii)	$MI_{\text{SPHERE (about } G)} = \frac{2}{5}(5m)(l)^2 = 2ml^2$	B1	4	Use of $\frac{2}{5}ml^2$ with ' m ' = $5m$ and ' l ' = l
	$MI_{\text{SPHERE (about } P)} = 2ml^2 + 5m(5l)^2$ $= 127ml^2$	M1 A1		Use of parallel axis theorem
(iii)	$I_{\text{PENDULUM}} = 127ml^2 + 16ml^2 = 143ml^2$	A1		AG
	Angular momentum of clay before collision = $mv(3l) = 3mvl$	B1		
	Angular momentum for pendulum after collision = $I_1\omega = 143ml^2\omega$	B1	Correct for pendulum	
	Angular momentum for clay after collision = $I_2\omega = m(3l)^2\omega$	B1	Correct for clay	
	Conservation of angular momentum: $3mvl = 143ml^2\omega + 9ml^2\omega$ $3mvl = 152ml^2\omega$ $\omega = \frac{3v}{152l}$	M1A1 A1F	6	Attempt at conservation of momentum ft one slip
	Total		17	
	TOTAL		75	