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General Certificate of Education (A-level)
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Mathematics MM05

(Specification 6360)

Mechanics 5

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	Period $= 2\pi\sqrt{\dfrac{2}{9.8}} = 2.84$ s	M1A1	2	M1: Use of formula for period. A1: Correct period
(b)	Max Speed $= l\theta\omega = 2\times\dfrac{\pi}{20}\times\sqrt{\dfrac{9.8}{2}}$ $= 0.695$ m s^{-1}	M1A1 A1	3	M1: Use of $l\theta\omega$ or $\theta\omega$. A1: Correct expression. A1: Correct speed.
	Total		**5**	
2(a)	$V = mga\sin\theta + \dfrac{2mg}{2a}(2a\cos\theta - a)^2 + \dfrac{2mg}{2a}(2a\sin\theta - a)^2$ $= mga\left(2\sin\theta + 4\cos^2\theta - 4\cos\theta + 1 + 4\sin^2\theta - 4\sin\theta + 1\right)$ $= mga\left(4(\cos^2\theta + \sin^2\theta) - 4\cos\theta - 3\sin\theta + 6\right)$ $= mga\left(4 + 1 + 1 - 4\cos\theta - 3\sin\theta\right)$ $= mga(6 - 4\cos\theta - 3\sin\theta)$ **AG**	M1A1 A1 dM1 A1	5	M1: Sum of GPE and 2 EPEs A1: One correct EPE A1: All correct. dM1: Expressions expanded. A1: Simplified to the correct answer using $\sin^2\theta + \cos^2\theta = 1$
(b)	$\dfrac{dV}{d\theta} = mga(4\sin\theta - 3\cos\theta)$ $\dfrac{dV}{d\theta} = 0$ $\tan\theta = \dfrac{3}{4}$ $\theta = 0.644$	M1A1 dM1 A1	4	M1;Differentiating V. A1: Correct derivative. dM1: Seeing $\tan\theta = \dfrac{3}{4}$ A1: Correct angle in radians.
(c)	$\dfrac{d^2V}{d\theta^2} = mga(4\cos\theta + 3\sin\theta)$ $= mga\left(4\times\dfrac{4}{5} + 3\times\dfrac{3}{5}\right)$ $\dfrac{d^2V}{d\theta^2}(= 5mga) > 0$ $\therefore$ Stable Equilibrium	M1 dM1 A1	3	M1: finding second derivative. dM1: Substituted and simplified. A1: Correct conclusion. (Allow 5)
	Total		**12**	

Q	Solution	Marks	Total	Comments
3(a)	$T_{AC} = \frac{45}{0.4}(0.6 + x - 0.4) = 22.5 + 112.5x$ $T_{BC} = \frac{45}{0.4}(0.6 - x - 0.4) = 22.5 - 112.5x$ Resultant $= T_{BC} - T_{AC}$ $= 22.5 - 112.5x - (22.5 + 112.5x)$ $= -225x$ AG	M1A1 A1 A1	4	M1: Two tensions in terms of x with ± a constant term. A1: First correct tension. A1: second correct tension. A1: Tensions summed correctly.
(b)	$9\frac{d^2x}{dt^2} = -225x$ $\frac{d^2x}{dt^2} = -\frac{225}{9}x = -25x$ $\therefore$ SHM	M1 A1	2	M1: Showing second derivative = –25x. A1: Correct deduction about SHM.
(c)	$\omega = 5$ Period $= \frac{2\pi}{5}$	M1 A1	2	M1: Using ω from part (b). A1: Correct period. Accept AWRT 1.26.
(d)	$a = 0.1$ $v^2 = 5^2(0.1^2 - 0.05^2)$ $v = 0.433 \text{ m s}^{-1}$	B1 M1A1 A1	4	B1: Use of a = 0.1 M1: Equation with -0.05^2 or $0.05^2 -$ A1: Correct equation. A1: Correct speed.
(e)	$x = 0.1\cos(5t)$	B1F B1F	2	B1F: Seeing a = 0.1 of their earlier value. B1F: Correct expression using their values.
	Total		**14**	

Q	Solution	Marks	Total	Comments
4(a)	$\theta = 4t$ $\dot{\theta} = 4$ $r = 1 + 2\cos\theta$ $\dot{r} = -2\sin\theta\dot{\theta} = -8\sin\theta$ $v^2 = \left(-8\sin\theta\right)^2 + \left(4(1+2\cos\theta)\right)^2$ $= 64\sin^2\theta + 16 + 64\cos\theta + 64\cos^2\theta$ $= 80 + 64\cos\theta$ $v = 4\sqrt{5+4\cos\theta}$ $k = 4$	B1 B1 M1A1 A1	 5	B1: Correct statement about θ or $\dot{\theta}$ B1: Correct expression for $\dot{r}$. M1: Attempts at v^2 with two components. A1: Correct v^2 A1: Correct *v* with 4 as a factor.
(b)	$\ddot{\theta} = 0$ $\ddot{r} = -32\cos\theta$ $\ddot{r} - r\dot{\theta}^2 = -32\cos\theta - (1+2\cos\theta)\times 4^2$ $= -64\cos\theta - 16$ $0 = -64\cos\theta - 16$ $\cos\theta = -\frac{1}{4}$ $\sin\theta = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}$ $2\dot{r}\dot{\theta} + r\ddot{\theta} = 2(-8\sin\theta)\times 4 + (-8\cos\theta)\times 0$ $= \pm 2\times 8\times\frac{\sqrt{15}}{4}\times 4$ $= \pm 16\sqrt{15}$ Magnitude $= 16\sqrt{15} = 62.0$	M1 M1 A1 A1 M1 A1	 6	M1: $\ddot{\theta}$ and $\ddot{r}$. M1: Radial component set equal to zero. A1: Correct value for $\cos\theta$. A1: Correct value for $\sin\theta$. M1: Finding transverse component. A1: Correct magnitude.
	Total		**11**	

Q	Solution	Marks	Total	Comments
5(a)	$75\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=75g-\frac{450}{12}x-15\frac{\mathrm{d}x}{\mathrm{d}t}$	M1A1 A1		M1: equation with four terms of the correct format. A1: Correct terms. A1: Correct signs. A1: Correct result from correct working.
	$75\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=750-37.5x-15\frac{\mathrm{d}x}{\mathrm{d}t}$			
	$10\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\frac{\mathrm{d}x}{\mathrm{d}t}+5x=100$ **AG**	A1	4	
(b)	CF			
	$10\lambda^2+2\lambda+5=0$	M1		M1: Correct auxiliary equation. M1: Correct complex solutions. A1: Correct CF
	$\lambda=\frac{-2\pm\sqrt{2^2-4\times5\times10}}{2\times10}=-0.1\pm0.7\mathrm{i}$	M1		
	$x=\mathrm{e}^{-0.1t}(A\sin(0.7t)+B\cos(0.7t))$	A1		
	PI			
	$x=20$	B1		B1: Correct PI.
	$x=\mathrm{e}^{-0.1t}(A\sin(0.7t)+B\cos(0.7t))+20$			
	$x=0, t=0\Rightarrow B=-20$	dM1 A1		dM1: Using initial conditions to find *B*. A1: Correct value for *B*.
	$x=\mathrm{e}^{-0.1t}(A\sin(0.7t)-20\cos(0.7t))+20$			
	$\dot{x}=-0.1\mathrm{e}^{-0.1t}(A\sin(0.7t)-20\cos(0.7t))+$ $\mathrm{e}^{-0.1t}(0.7A\cos(0.7t)-14\sin(0.7t))$	dM1		dM1: Correct $\dot{x}$ dM1: Using initial conditions to find *A*. A1: Correct value for *A* and correct final expression.
	$\dot{x}=12.5, t=0\Rightarrow A=15$	dM1		
	$x=\mathrm{e}^{-0.1t}(15\sin(0.7t)-20\cos(0.7t))+20$	A1	10	
(c)	$v=\mathrm{e}^{-0.1t}(12.5\cos(0.7t)+12.5\sin(0.7t))$	M1A1		M1: Setting derivative equal to zero. A1: Correct equation including correct derivative.
	$v=0$			
	$\tan(0.7t)=-1$	dM1 A1		dM1: Value for $\tan(0.7t)$. A1: Correct value for $\tan(0.7t)$.
	$t=\frac{15\pi}{14}=3.37$ s	A1	5	A1: Correct time.
	Total		**19**	

Q	Solution	Marks	Total	Comments
6(a)	$(v+\delta v)(m+\delta m)+(-\delta m)(v-U)-mv=0$ $mv+v\delta m+m\delta v-v\delta m+U\delta m-mv=0$ $m\delta v+U\delta m=0$ $m\dfrac{\delta v}{\delta t}+U\dfrac{\delta m}{\delta t}=0$ $m\dfrac{\mathrm{d}v}{\mathrm{d}t}+U\dfrac{\mathrm{d}m}{\mathrm{d}t}=0$ $m\dfrac{\mathrm{d}v}{\mathrm{d}t}=-U\dfrac{\mathrm{d}m}{\mathrm{d}t}$ **AG**	M1A1 A1 A1	 4	M1: Impulse momentum equation equal to zero with correct format. A1: Correct equation. A1: Correct simplified equation. A1: Correct differential equation from correct working.
(b)(i)	$\dfrac{dm}{dt}=-\lambda$ $m=2M-\lambda t$ $(2M-\lambda t)\dfrac{\mathrm{d}v}{\mathrm{d}t}=U\lambda$ $\displaystyle\int\frac{U\lambda}{2M-\lambda t}\,\mathrm{d}t=\int 1\,\mathrm{d}v$ $-U\ln(2M-\lambda t)=v+c$ $v=0, t=0\Rightarrow c=-U\ln(2M)$ $v=U\ln(2M)-U\ln(2M-\lambda t)$ $\left(=U\ln\left(\dfrac{2M}{2M-\lambda t}\right)\right)$	M1A1 M1 A1 dM1 A1 A1	 7	M1: Obtaining expression for m at time t. A1: Correct expression for m. M1: Variables separated and integral formed. A1: Correct v with or without c. dM1: Finding c. A1: Correct c. A1: Correct final result.
(b)(ii)	$2M-\lambda t=M$ $t=\dfrac{M}{\lambda}$ $v=U\ln\left(\dfrac{2M}{2M-M}\right)=U\ln 2$	M1 M1 A1	 3	M1: Equation to find t when $m=M$. M1: Finding t when $m=M$. A1: Correct v.
	Total		**14**	
	TOTAL		**75**	