AQA

A-LEVEL
Mathematics

Mechanics 5 – MM05
Mark scheme

6360
June 2015

Version/Stage: 1.0: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1.(a)	$0.8 = 0.2\omega$ $\omega = 4$ $\text{Period} = \frac{2\pi}{4} = \frac{\pi}{2}$	M1 A1 A1	3	M1: Use of $0.8 = 0.2\omega$ or $0.2 = 0.8\omega$ A1: Correct ω A1: Correct period (1.57)
1.(b)	$v^2 = 4^2(0.2^2 - 0.1^2)$ $v = \sqrt{0.48} = \frac{2\sqrt{3}}{5} = 0.693\,\text{m s}^{-1}$	M1A1 A1	3	M1: Seeing $(0.2^2 - 0.1^2)$ or $(0.4^2 - 0.3^2)$ in an equation. A1: Correct equation. A1: Correct speed.
1. (c)	$s = 0.2 - 0.2\cos(4t)$	B1B1B1	3	B1: Seeing 0.2-... B1: Seeing 0.2cos... B1: Seeing 4*t*.
1. (d)	$0.1 = 0.2 - 0.2\cos(4t)$ $\cos(4t) = 0.5$ $t = 0.262\,\text{s}$	M1 A1 A1	3	M1: Using 0.1 and answer to (c0 to form an equation. A1: Correct equation. A1: Correct time.
	Total		**12**	

Q	Solution	Mark	Total	Comment
2	$T_1 = \frac{1}{4} \times 2\pi\sqrt{\frac{1.2}{g}} = 0.54966$ $T_2 = \frac{1}{4} \times 2\pi\sqrt{\frac{0.7}{g}} = 0.41981$ $T_1 + T_2 = 0.54966 + 0.41981 = 0.969\,\text{s (to 3 sf)}$	M1A1 A1 A1	4	M1: Obtaining two times and adding. A1: One correct time. A1: Second correct time. A1: Correct total time.
	Total		**4**	

Q	Solution	Mark	Total	Comment
3. (a)	$\dot{\theta}=2$ $\theta=2t$ $\ddot{\theta}=0$ $\dot{r}=\frac{1}{2}ae^{a\theta}\dot{\theta}=ae^{2at}$ $\ddot{r}=\frac{1}{2}a^2e^{a\theta}\dot{\theta}^2+\frac{1}{2}ae^{a\theta}\ddot{\theta}=2a^2e^{2at}$ Radial: $\ddot{r}-r\dot{\theta}^2=2a^2e^{2at}-4\times\frac{1}{2}e^{2at}$ $=2e^{2at}(a^2-1)$ Transverse: $2\dot{r}\dot{\theta}+r\ddot{\theta}=2ae^{2at}\times 2=4ae^{2at}$	B1 B1 B1 M1 A1 M1A1	**7**	B1: Seeing or clear use of $\dot{\theta}=2$. B1: Correct expression for $\dot{r}$ in terms of t B1: Correct expression for $\ddot{r}$ in terms of t M1: Use or radial formula. A1: Correct radial acceleration. M1: Use or transverse formula. A1: Correct transverse acceleration.
3. (b)	$e^{2at}=1$ $(2(a^2-1))^2+(4a)^2=20^2$ $4a^4+8a^2+4=400$ $a^4+2a^2-99=0$ $(a^2+11)(a^2-9)=0$ $a=\pm 3$	B1 M1 A1 M1 A1	**5**	B1: Using initial value to get expression to eliminate t. M1: Finding magnitude. A1: Correct magnitude in terms of a. M1: Solving for a^2 A1: Correct values of a
	Total		**12**	

Q	Solution	Mark	Total	Comment
4. (a)	$V_{AB} = 2mga\sin\theta$ $V_{BC} = mg\frac{a}{2}\cos\theta$ $V_A = \frac{1}{2}\times\frac{2mg}{a}(5a - a - 2a\sin\theta)^2$ $= 4mga(2-\sin\theta)^2$ $= 4mga(4 - 4\sin\theta + \sin^2\theta)$ $V_B = \frac{1}{2}\times\frac{8mg}{a}(5a - a - a\cos\theta)^2$ $= 4mga(4-\cos\theta)^2$ $= 4mga(16 - 8\cos\theta + \cos^2\theta)$ $V = \frac{mga}{2}(4\sin\theta + \cos\theta + 32 - 32\sin\theta + 8\sin^2\theta + 128 - 64\cos\theta$ $= \frac{mga}{2}(168 - 63\cos\theta - 28\sin\theta)$	M1A1 M1A1 M1A1 A1	 7	M1: Two GPE terms found. A1: Both correct. M1: EPE for one string. A1: Correct EPE M1: EPE for other string. A1: Correct EPE A1: Correct total from correct working.
4. (b)	$\frac{dV}{d\theta} = \frac{mga}{2}(63\sin\theta - 28\cos\theta)$ $\frac{mga}{2}(63\sin\theta - 28\cos\theta) = 0$ $\tan\theta = \frac{28}{63} = \frac{4}{9}$ $\theta = 0.418$	M1A1 M1A1 A1	 5	M1: Differentiating energy expression. A1: Correct derivative. M1: Solving to obtain $\tan\theta$ A1: Correct value for $\tan\theta$ A1: Correct θ. (Accept 23.96°)
4. (c)	$\frac{d^2V}{d\theta^2} = \frac{mga}{2}(63\cos\theta + 28\sin\theta)$ $\theta = 0.418 \Rightarrow \frac{d^2V}{d\theta^2} = \frac{mga}{2}\times(68.9) > 0$ $\therefore$ Stable	M1 A1 A1	 3	M1: Finding second derivative. A1: Correct derivative. A1: Correct conclusion.
	Total		**15**	

Q	Solution	Mark	Total	Comment
5. (a)	$2g = 24e$ $e = \frac{2g}{24}$ $x = \frac{2g}{24} + 0.35 = \frac{49}{60} + \frac{21}{60} = \frac{7}{6}$	M1A1 A1	**3**	M1: Using Hookes Law. A1: Correct extension. A1: Correct distance. Accept 1.17
5. (b) (i)	$2\frac{d^2x}{dt^2} = 2g - 14\frac{dx}{dt} - 24(x - 0.35)$ $\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 12x = 9.8 + 12 \times 0.35$ $\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 12x = 14$	M1A1 A1 A1	**4**	M1: Suitable four term equation. A1: Three terms correct. A1: All terms correct.
5. (b) (ii)	**PI** $x = \frac{7}{6}$ **CF** $\lambda^2 + 7\lambda + 12 = 0$ $(\lambda + 3)(\lambda + 4) = 0$ $\lambda = -3$ or $\lambda = -4$ $x = Ae^{-3t} + Be^{-4t}$ $x = Ae^{-3t} + Be^{-4t} + \frac{7}{6}$ $x = 1.8, t = 0$ $1.8 = A + B + \frac{7}{6}$ $A + B = \frac{19}{30}$ $\frac{dx}{dt} = -3Ae^{-3t} - 4Be^{-4t}$ $\dot{x} = 0, t = 0$ $0 = -3A - 4B$ $A = -\frac{4B}{3}$ $A = \frac{38}{15}, B = -\frac{19}{10}$ $x = \frac{38}{15}e^{-3t} - \frac{19}{10}e^{-4t} + \frac{7}{6}$	B1 M1 A1 dM1A1 dM1A1 A1 A1 A1	**10**	B1: Correct PI. M1: Quadratic equation for λ. A1: correct values for λ. dM1: Using initial values to obtain an equation containing A and B. A1: Correct equation. dM1: Using derivative to obtain a second equation for A and B. A1: Correct equation. A1: Value of A correct. A1: Value of B correct. A1: Correct final expression.
5. (b) (iii)	Heavy damping	B1	**1**	B1: Correct statement.
	Total		**18**	

Q	Solution	Mark	Total	Comment
6. (a)	$T = M\frac{dv}{dt}$ $(mg - T)\delta t = (m + \delta m)(v + \delta v) - \delta m v - mv$ $= m\delta v$ $mg - M\frac{dv}{dt} = m\frac{dv}{dt}$ $mg = (m + M)\frac{dv}{dt}$ $m = M - \lambda t$ $\frac{dv}{dt} = \frac{mg}{m + M}$ $\frac{dv}{dt} = \frac{(M - \lambda t)g}{2M - \lambda t}$	B1 M1A1 A1 dM1 A1	6	B1: Correct equation for block. M1: Impulse-momentum equation with $mg - T$ or $T - mg$. A1: Correct equation. A1: Correct differential equation. M1: Use of expression for mass and it's derivative to complete expression A1: Correct final expression.
6. (b)	$\frac{dv}{dt} = \frac{(M - \lambda t)g}{2M - \lambda t}$ $= g - \frac{Mg}{2M - \lambda t}$ $v = gt + \frac{Mg}{\lambda}\ln(2M - \lambda t) + c$ $v = 0, t = 0 \Rightarrow c = -\frac{Mg}{\lambda}\ln(2M)$ $v = gt + \frac{Mg}{\lambda}\ln\left(\frac{2M - \lambda t}{2M}\right)$	M1 dM1 A1 dM1 A1	5	M1: Splitting RHS into two terms, with one a constant. dM1: Integrating to obtain linear and ln terms.. A1: Correct integral. dM1: Use of initial conditions. A1: Correct final expression.
6. (c)	$t = \frac{M}{\lambda}$ $v = \frac{Mg}{\lambda} + \frac{Mg}{\lambda}\ln\left(\frac{1}{2}\right)$ $= \frac{Mg}{\lambda}(1 - \ln 2)$	M1 A1 A1	3	M1: Use of correct value of *t*. A1: Correct substitution into correct result. A1: Simplified correct time.
	Total		**14**	
	TOTAL		**75**	