A-LEVEL
Mathematics

MM05 - Mechanics 5
Mark scheme

6360

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Q	Solution	Mark	Total	Comment
1	$T = 2\pi\sqrt{\frac{l}{g}}$	B1		B1: Quoting formula for period.
	$1.05 \times 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{L}{g}}$	M1		M1: Increasing length by 5%.
	$L = 1.05^2 l = 1.1025l$	A1		A1: Correct length or ratio.
	10.25% increase needed	A1	4	A1: Correct percentage increase in length.
	Total		4	

Q	Solution	Mark	Total	Comment
2(a)	$a\omega = 0.16 \Rightarrow a = \dfrac{4}{25\omega}$	B1		B1: Using $a\omega = 0.16$ M1: Substituting into SHM formula. A1: Correct equation. bM1: Use of their ω to find period. A1: Correct period.
	$\dfrac{3}{125} = \omega^2(a^2 - 0.02^2)$	M1 A1		
	$\dfrac{3}{125} = \omega^2\left(\dfrac{16}{625\omega^2} - \dfrac{1}{2500}\right)$ $60 = 64 - \omega^2$	dM1		
	$\omega = 2$			
	$\text{Period} = \dfrac{2\pi}{2} = \pi \text{ seconds}$	A1	5	
(b)	$m\dfrac{d^2x}{dt^2} = -k(x+e) + mg$			M1: Forming an equation to find *k.* A1: Correct equation. M1: Solving for *k*. A1: Correct *k*.
	$\text{But } e = \dfrac{mg}{k}$	M1A1		
	$m\dfrac{d^2x}{dt^2} = -kx - mg + mg$			
	$m\dfrac{d^2x}{dt^2} = -kx$	M1		
	$\omega^2 = \dfrac{k}{m}$			
	$k = 2^2 \times 1.5 = 6 \text{ N m}^{-1}$	A1	4	
(c)	$a = \dfrac{0.16}{2} = 0.08$	B1		B1: Correct amplitude. M1: Equation to find extension at equilibrium. A1: Correct extension. M1: Equation to give max tension. A1: Correct maximum tension.
	At equilibrium position: $1.5 \times 9.8 = 6e$	M1		
	$e = 2.45$	A1F		
	At maximum extension: $T = 6(2.45 + 0.08)$	M1		
	$= 15.2 \text{ N}$	A1	5	
	Total		**14**	

Q	Solution	Mark	Total	Comment
3(a)	$V_A = 2mg \times 4\cos\theta$ $V_B = -3mg \times 3\sin\theta$ $V_C = 5mg \times 4\sin\left(\frac{\pi}{3}-\theta\right)$ $= 20mg\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)$ $V = 8mg\cos\theta - 9mg\sin\theta + 10\sqrt{3}mg\cos\theta - 10mg\sin\theta$ $= mg\left(\left(8+10\sqrt{3}\right)\cos\theta - 19\sin\theta\right)$	**B1** **B1** **M1** **A1** **A1**	**5**	B1: Correct GPE for *A*. B1: Correct GPE for *B*. M1: Attempts GPE for *C*. A1: Correct GPE for *C*. A1: Results combined and simplified to required result.
(b)	$\frac{dV}{d\theta} = mg\left(-\left(8+10\sqrt{3}\right)\sin\theta - 19\cos\theta\right)$ $= mg\left(-\left(8+10\sqrt{3}\right)\sin\theta - 19\cos\theta\right)$ $\frac{dV}{d\theta} = 0$ $\tan\theta = \frac{-19}{8+10\sqrt{3}}$ $\theta = 2.50 \quad \text{or} \quad 5.64$	**M1** **A1** **M1** **A1**	**4**	M1: Differentiates *V*. A1: Correct derivative. M1: Finds value for tan θ. A1: Correct angles. Condone 143° and 323°
(c)	$\frac{d^2V}{d\theta^2} = mg\left(-\left(8+10\sqrt{3}\right)\cos\theta + 19\sin\theta\right)$ $\theta = 2.50$ $\frac{d^2V}{d\theta^2} = 31.7mg > 0$ $\therefore$ Stable $\theta = 5.64$ $\frac{d^2V}{d\theta^2} = -31.7mg < 0$ $\therefore$ Unstable	**M1** **M1** **A1** **A1**	**4**	M1: Obtains second derivative. M1: Substitutes and considers both values. A1: One correct conclusion. A1: Second correct conclusion. Accept rigorously reasoned alternatives.
	Total		**13**	

Q	Solution	Mark	Total	Comment
4(a)	$\frac{20}{0.5}e = 1.6\times 9.8$ $e = 0.392$ Length = 0.892 m	M1 A1 A1	3	M1: Equation to find extension. A1: Correct extension. A1: Includes 0.5.
(b)(i)	$1.6\frac{d^2x}{dt^2} = 1.6\times 9.8 - T$ $= 15.68 - \frac{20}{0.5}(x - 0.1\sin(10t) - 0.5)$ $\frac{d^2x}{dt^2} = 9.8 - 25x + 2.5\sin(10t) + 12.5$ $\frac{d^2x}{dt^2} + 25x = 22.3 + 2.5\sin(10t)$	M1 M1 A1 A1	4	M1: Equation of motion involving *mg* and *T*. M1: Attempts expression for tension. A1: Correct tension. A1: Required result from correct working.
(b)(ii)	CF $\lambda^2 + 25 = 0$ $\lambda = \pm 5i$ $x = A\cos(5t) + B\sin(5t)$ PI $x = C\cos(10t) + D\sin(10t) + E$ $\dot{x} = -10C\sin(10t) + 10D\cos(10t)$ $\ddot{x} = -100C\cos(10t) - 100D\sin(10t)$ $-100C\cos(10t) - 100D\sin(10t) +$ $25(C\cos(10t) + D\sin(10t) + E) = 22.3 + 2.5\sin(10t)$ $E = \frac{22.3}{25} = 0.892, C = 0$ $-100D + 25D = 2.5$ $D = -\frac{2.5}{75} = -\frac{1}{30}$ $x = 0.892 - \frac{1}{30}\sin(10t)$	M1 A1 M1 A1 M1 A1 A1 A1		M1: Roots of aux equation. A1: Correct form of CF. M1: Correct form of PI. A1: Correct derivatives. M1: Substitution to find constants. A1: Correct values of *E* and *C*. A1: Correct *D*. A1: Correct PI.

	$x = A\cos(5t) + B\sin(5t) + 0.892 - \frac{1}{30}\sin(10t)$ $x = 0.892, t = 0$ $0.892 = A + 0.892$ $A = 0$ $\dot{x} = 5B\cos(5t) - \frac{1}{3}\cos(10t)$ $\dot{x} = 0, t = 0$ $0 = 5B - \frac{1}{3}$ $B = \frac{1}{15}$ $x = \frac{1}{15}\sin(5t) + 0.892 - \frac{1}{30}\sin(10t)$	**M1** **M1** **A1** **A1**	**12**	M1: Equation to find *A*. M1: Equation to find *B*. A1: Correct *A* and *B*. A1: Correct expression for *x*.
	Total		**19**	

Q	Solution	Mark	Total	Comment
5(a)	$\dot{\theta} = \frac{2}{5}$	B1		B1Correct $\dot{\theta}$
	$\dot{r} = \cos\theta\dot{\theta} = \frac{2}{5}\cos\theta$	M1		M1: Expression for $\dot{r}$.
	$v^2 = (\dot{r})^2 + (r\dot{\theta})^2$	M1		M1: Finds v^2
	$= \frac{4}{25}\cos^2\theta + \frac{4}{25}\left(1 + 2\sin\theta + \sin^2\theta\right)$			A1: Correct v^2
	$= \frac{8}{25}\left(1 + \sin\theta\right)$	A1		A1: Correct constant and conclusion.
	$v = \frac{2\sqrt{2}}{5}\sqrt{r}$	A1	5	
	$\therefore$ Speed proportional to $\sqrt{r}$			
(b)	$\ddot{r} = -\frac{2}{5}\sin\theta\dot{\theta} = -\frac{4}{25}\sin\theta$			M1: Attempts both components.
	$\ddot{r} - r\dot{\theta}^2 = -\frac{4}{25}\sin\theta - (1+\sin\theta)\times\frac{4}{25}$	M1		A1: One correct component.
	$= -\frac{4}{25}(1 + 2\sin\theta)$	A1		A1: Second correct component.
	$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{8}{25}\cos\theta$	A1		
	$a^2 = \frac{16}{625}(1 + 4\sin\theta + 4\sin^2\theta) + \frac{64}{625}\cos^2\theta$	M1		M1: Expression for a^2
	$= \frac{16}{625}(5 + 4\sin\theta)$			A1: Correct magnitude of acceleration.
	$a = \frac{4}{25}\sqrt{5 + 4\sin\theta}$	A1		A1: Correct min. A1: Correct max.
	$a_{\max} = \frac{12}{25} = 0.48$	A1		
	$a_{\min} = \frac{4}{25} = 0.16$	A1	7	
	Total		12	

Q	Solution	Mark	Total	Comment
6(a)	$\frac{dr}{dt} = kr$			B1: Correct constant of proportionality.
	$0.002 = 0.001k$ $k = 2$	B1		
	$g(m+\delta m)\delta t = (m+\delta m)(v+\delta v) - mv$	M1		M1: Attempts impulse-momentum equation. A1: Correct equation.
	$gm\delta t + g\delta m\delta t = m\delta v + v\delta m + \delta m\delta v$			
	$mg = m\frac{dv}{dt} + v\frac{dm}{dt}$	A1		
	$m = \frac{4}{3}\pi r^3\rho$	M1		M1: Expression for mass.
	$\frac{dm}{dt} = 4\pi r^2\rho\frac{dr}{dt}$	M1		M1: Derivative of m with respect to t.
	$= 8\pi r^3\rho$ $= 6m$	A1		A1: Obtaining $6m$.
	$mg = m\frac{dv}{dt} + 6mv$			A1: Required result from correct working.
	$\frac{dv}{dt} = g - 6v$	A1	7	
(b)	$\int \frac{1}{g-6v}dv = \int 1\,dt$	M1		M1: Separation of variables.
	$-\frac{1}{6}\ln(g-6v) = t + c$	A1		A1: Correct integrals
	$g - 6v = Ae^{-6t}$	M1		M1: Finding constant of integration.
	$t = 0, v = U$	A1		A1: Correct constant.
	$A = g - 6U$			
	$v = \frac{g-(g-6U)e^{-6t}}{6}$	A1	5	A1: Correct expression for v.
(c)	$V \to \frac{g}{6}$	B1	1	B1: Correct limit.
	Total		**13**	