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General Certificate of Education (A-level)
January 2011

Mathematics MPC3

(Specification 6360)

Pure Core 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

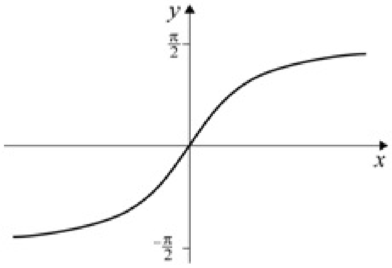
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{dy}{dx} = k\left(x^3-1\right)^5$	M1		Where k is an integer or function of x
	$= 6 \times 3x^2 \left(x^3-1\right)^5$ (ISW)	A1	2	But note $\frac{dy}{dx} = k\left(x^3-1\right)^5 + px^2$ M0 **Or** $\left(u = x^3 - 1\right)$ $\left(y = u^6\right)$ $\frac{dy}{du} = 6u^5$ and $\frac{du}{dx} = 3x^2$ M1 $= 6\left(x^3-1\right)^5 \times 3x^2$ A1 Note $\frac{dy}{dx} = 6 \times 3x^2\left(x^3-1\right)^5 + c$ scores M1 A0 (penalise + c in differential once only in paper)
(b)(i)	$\frac{dy}{dx} = \pm x \times \frac{1}{x} \pm \ln x$	M1		Product rule attempted **and** differential of ln x
	$= 1 + \ln x$ (ISW)	A1	2	
(ii)	$(x = e)$ $y = e$ PI	B1		Must have replaced ln e by 1 Condone $y = 2.72$ (AWRT)
	$\frac{dy}{dx} = 1 + \ln e \ (= 2)$	M1		Correct substitution into their $\frac{dy}{dx}$ But must have scored M1 in (b)(i)
	$y - e = 2(x - e)$ or $y = 2x - e$ OE, ISW	A1	3	Must have replaced ln e by 1
	Total		**7**	

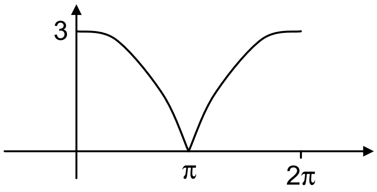
MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x)=(x^2-4)\ln(x+2)-15$ $\left.\begin{array}{l}\mathrm{f}(3.5)=-0.9\\ \mathrm{f}(3.6)=0.4\end{array}\right\}$ Attempt at evaluating both f (3.5) and f (3.6)	M1		**Or** reverse $\left.\begin{array}{l}\mathrm{f}(3.5)=0.9\\ \mathrm{f}(3.6)=-0.4\end{array}\right\}$ M1 But must see $\mathrm{f}(x)=15-(x^2-4)\ln(x+2)$ before A1 may be earned Condone $\left.\begin{array}{l}\mathrm{f}(3.5)<0\\ \mathrm{f}(3.6)>0\end{array}\right\}$ **Only** if $\mathrm{f}(x)$ defined M1 **Or** $\left.\begin{array}{l}x=3.5\ \ y=14.1\ (<15)\\ x=3.6\ \ y=15.4\ (>15)\end{array}\right\}$ M1
	Change of sign, $\therefore 3.5<\alpha<3.6$ OE	A1	2	Either side of 15, $\therefore 3.5<\alpha<3.6$ OE A1
(b)	$(x^2-4)\ln(x+2)=15$ $x^2-4=\dfrac{15}{\ln(x+2)}$ $x^2=4+\dfrac{15}{\ln(x+2)}$	M1		Either of these lines correct Condone poor use of brackets for M1 only
	$x=\pm\sqrt{4+\dfrac{15}{\ln(x+2)}}$ AG	A1	2	Must have **both** middle lines and no errors seen
(c)	$(x_1=3\cdot5)$ $x_2=3.578$ CAO	B1		
	$x_3=3.568$ CAO	B1	2	
				Sight of AWRT 3.58 or 3.57 scores B1 B0 Or ± 3.578 or ± 3.568 scores B1 B0 $x_1=3.578,\ x_2=3.568$ scores B1B0
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}y} = k\sec^2(3y+1)$	M1		Where k is an integer Condone omission of $\frac{\mathrm{d}x}{\mathrm{d}y}$ But $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sec^2(3y+1)$ scores M1 A0
	$= 3\sec^2(3y+1)$ ISW	A1	2	**Alternative methods** $y = \frac{1}{3}\left(\tan^{-1}x - 1\right)$ $\frac{\mathrm{d}x}{\mathrm{d}y} = k\left(1+x^2\right)$ M1 $= 3\left(1+\tan^2(3y+1)\right)$ A1 **Or** $x = \frac{\sin(3y+1)}{\cos(3y+1)}$ $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\pm k\cos^2(3y+1) \pm k\sin^2(3y+1)}{\cos^2(3y+1)}$ M1 $= \frac{3}{\cos^2(3y+1)}$ A1
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(3\times -\frac{1}{3}+1\right)$	M1		Substitution of $y = -\frac{1}{3}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x}$ BUT must have scored M1 in (a)(i)
	$= 3\sec^2 0$			
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}$ CSO	A1	2	Condone 0.333 or better **Or** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3\sec^2(3y+1)}$ $= \frac{1}{3\sec^2 0}$ $= \frac{1}{3}$ As above
3(b)		M1		Approx correct shape with no turning points, through (0,0) and only 1 curve Asymptotic at both $\pm\frac{\pi}{2}$ and both values shown
		A1	2	Condone ± 90 (degrees) Condone $y = \tan x$ also drawn but clearly identified, otherwise M0
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$-3 \leqslant \mathrm{f}(x) \leqslant 3$	M1		$-3 \leqslant x \leqslant 3,\ -3 < \mathrm{f}(x) < 3$ $-3 < \mathrm{f} < 3,\ -3 < y < 3$ $-3 \leqslant \mathrm{f} < 3,\ -3 < \mathrm{f} \leqslant 3$
		A1	2	Allow $-3 \leqslant y \leqslant 3,\ -3 \leqslant \mathrm{f} \leqslant 3$
(b)(i)	$y = 3\cos\frac{1}{2}x$ $\frac{y}{3} = \cos\frac{1}{2}x$			
	$\cos^{-1}\frac{y}{3} = \left(\frac{1}{2}x\right)$ $x = 2\cos^{-1}\frac{y}{3}$	M1		Or $\cos^{-1}\frac{x}{3} =$ } Either order
	$y = 2\cos^{-1}\frac{x}{3}$	M1		Swap x and y
	$\mathrm{f}^{-1}(x) = 2\cos^{-1}\frac{x}{3}$	A1	3	
(ii)	$\frac{x}{3} = \cos\frac{1}{2}$	M1		If incorrect in (b)(i) **BUT** answer in form $p\cos^{-1}(qx)$ (condone p, q =1)
	$x = 3\cos\frac{1}{2}$ ISW	A1	2	Then $qx = \cos\left(\frac{1}{p}\right)$ M1 **or** $x = \mathrm{f}(1)$ M1 $x = 3\cos\frac{1}{2}$ A1
(c)(i)	$\mathrm{gf}(x) = \left\lvert 3\cos\frac{1}{2}x \right\rvert$	B1	1	
(ii)		M1		Modulus graph in 1st quadrant, starting from a +ve y-intercept, at least 2 continuous parts, first descending, then second increasing IGNORE CURVE OUTSIDE RANGE
		A1		Correct curvature, curves reaching x-axis, condone multiple curves (no turning points at axis)
		A1	3	Approximately symmetrical graph with 3, π, 2π indicated (must have scored previous 2 marks) Condone $y = 3\cos\frac{1}{2}x$ also drawn but clearly identified, otherwise M0
(d)	STRETCH + direction	M1		Either in x-direction or y-direction
	s.f. 3, parallel to y-axis	A1		Either order
	s.f. 2, parallel to x-axis	A1	3	
	Total		**14**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\int \frac{1}{3+2x}\,\mathrm{d}x$			
	$= k\ln(3+2x)$	M1		Where k is a rational number
	$= \frac{1}{2}\ln(3+2x)+c$	A1	2	
				Or if substitution $u=3+2x,\ \mathrm{d}u=2\mathrm{d}x$
				$\int = \int \frac{1}{u}\frac{\mathrm{d}u}{2} = k\ln u$ M1
				$= \frac{1}{2}\ln(3+2x)+c$ A1
(b)	$u=x \qquad \mathrm{d}v=\sin\frac{x}{2}$	M1		$\int \sin\frac{x}{2}(\mathrm{d}x) = k\cos\frac{x}{2},\ \frac{\mathrm{d}}{\mathrm{d}x}(x)=1$ where k is a constant
	$\mathrm{d}u=1 \qquad v=-2\cos\frac{x}{2}$	A1		All correct
	$\int = -2x\cos\frac{x}{2} - \int -2\cos\frac{x}{2}(\mathrm{d}x)$	m1		**Correct** substitution of their terms into parts formula (watch signs carefully)
	$= -2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + c$	A1	4	CAO
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	x $\quad$ y	B1		Using 4 correct x-values, PI
	$0.05 \quad \cos\sqrt{1.15} = 0.4780$ $0.15 \quad \cos\sqrt{1.45} = 0.3585$ $0.25 \quad \cos\sqrt{1.75} = 0.2454$ $0.35 \quad \cos\sqrt{2.05} = 0.1386$	M1		At least 3 correct y-values, (condone unsimplified correct expressions), Or correct values rounded to 2 s.f. or truncated to 2 s.f.
	$0.1\times\Sigma y$	m1		Used and must be working in radians
	$= 0.122$ CAO	A1	4	Must be 3 s.f.
(b)	$\frac{\mathrm{d}u}{\mathrm{d}x} = 3$	M1		$\mathrm{d}u = 3\mathrm{d}x$ OE
	$\int = \int\left(\frac{u\pm1}{3}\right)\sqrt{u}\times k\,\mathrm{d}u$	m1		All in terms of u, with $k = 3$ or $\frac{1}{3}$ Condone omission of du
	$=\left(\frac{1}{9}\right)\int u^{\frac{3}{2}} \pm u^{\frac{1}{2}}(\mathrm{d}u)$	m1		$p\int u^{\frac{3}{2}} \pm u^{\frac{1}{2}}(\mathrm{d}u)$ (must have scored first 2 marks)
	$=\frac{1}{9}\left[\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]$	A1		OE
	$=\left(\frac{1}{9}\right)\left[\left(\frac{2}{5}\times4^{\frac{5}{2}} - \frac{2}{3}\times4^{\frac{3}{2}}\right) - \left(\frac{2}{5}-\frac{2}{3}\right)\right]$	m1		Must have earned all previous method marks and then correct substitution, into their integral, of 1, 4 for u or 0, 1 for x **and** subtracting
	$=\frac{116}{135}$ ISW	A1	6	Or equivalent fraction
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\cos x = -0.2$	M1		Or $\tan x = (\pm)\sqrt{24}$
	$x = 1.77,\ 4.51$ AWRT	A1		One correct value
		A1	3	Second correct value and no extra values in interval 0 to 6.28… Ignore answers outside interval SC $x = 1.8,\ 4.5$ with or without working M1 A1 A0 SC (using degrees) 101.54, 281.54 M1 A1 A0 101.5, 281.5 M1 A0 A0 SC No working shown 2 correct answers 3/3 1 correct answer 2/3
(b)	LHS $= \dfrac{\operatorname{cosec} x(1-\operatorname{cosec} x) - \operatorname{cosec} x(1+\operatorname{cosec} x)}{(1+\operatorname{cosec} x)(1-\operatorname{cosec} x)}$	M1		Correctly combining fractions but condone poor use, or omission, of brackets
	$= \dfrac{\operatorname{cosec} x - \operatorname{cosec}^2 x - \operatorname{cosec} x - \operatorname{cosec}^2 x}{1-\operatorname{cosec}^2 x}$	A1		Allow recovery from incorrect brackets
	$= \dfrac{-2\operatorname{cosec}^2 x}{-\cot^2 x}$ or $\dfrac{-2(1+\cot^2 x)}{-\cot^2 x}$ $2\sec^2 x = 50$	m1		Correct use of relevant trig identity eg $\operatorname{cosec}^2 x = 1 + \cot^2 x$
	$\sec^2 x = 25$ AG	A1	4	All correct with no errors seen INCLUDING correct brackets on 1st line
	Or			
	$\dfrac{\operatorname{cosec} x}{1+\operatorname{cosec} x} - \dfrac{\operatorname{cosec} x}{1-\operatorname{cosec} x} = 50$			
	$\operatorname{cosec} x(1-\operatorname{cosec} x) - \operatorname{cosec} x(1+\operatorname{cosec} x)$ $= 50(1+\operatorname{cosec} x)(1-\operatorname{cosec} x)$	(M1)		Correctly eliminating fractions but condone poor use, or omission, of brackets
	$\operatorname{cosec} x - \operatorname{cosec}^2 x - \operatorname{cosec} x - \operatorname{cosec}^2 x$ $= 50(1-\operatorname{cosec}^2 x)$	(A1)		Allow recovery from incorrect brackets
	$48\operatorname{cosec}^2 x = 50$ $\sin^2 x = \dfrac{24}{25} \Rightarrow \cos^2 x = \dfrac{1}{25}$	(m1)		Correct use of relevant trig identity eg $\sin^2 x = 1 - \cos^2 x$
	$\sec^2 x = 25$ AG	(A1)		All correct with no errors seen INCLUDING correct brackets on 1st line

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(c)	$\sec x = \pm 5$	M1		Or $\cos x = \pm\ 0.2$ Or $\tan x = \pm\sqrt{24}$
	$x = 1.77,\ 4.51,\ 1.37,\ 4.91$ (AWRT)	A1		3 correct
		A1	3	4 correct and no other answers in interval Ignore answers outside interval SC 1.8, 4.5, 1.4, 4.9 With or without working M1 A1 SC their 2 answers from (a) +1.37, 4.91 (AWRT) 2/3 SC For this part, if in degrees max mark is M1 A0 SC No working shown 4 correct answers 3/3 3 correct answers 2/3 0, 1, 2 correct answers 0/3
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$e^{-2x} = 4$			
	$-2x = \ln 4$	M1		
	$x = -\frac{1}{2}\ln 4$ ISW	A1	2	OE, eg $\ln \frac{1}{2}, -\ln 2, \frac{\ln 4}{-2}$
(b)(i)	$(y=)3$	B1	1	Condone $(0,3)$ but not $(3,0)$
(ii)	$y = 0$			
	$4e^{-2x} - e^{-4x} = 0$			
	$4e^{2x} - 1 = 0$	M1		$ae^{\pm 2x} \pm b = 0$
	$e^{2x} = \frac{1}{4}$ or $e^{-2x} = 4$	A1		
	$x = \ln\frac{1}{2}$ ISW	A1	3	OE, eg $-\frac{1}{2}\ln 4, -\ln 2, \frac{1}{2}\ln\frac{1}{4}$ and no extra solutions
	Or			
	$4e^{-2x} = e^{-4x}$			
	$\ln 4 - 2x = -4x$	(M1)		
	$2x = -\ln 4$	(A1)		OE
	$x = -\frac{1}{2}\ln 4$	(A1)		OE
(iii)	$(y' =) -8e^{-2x} + 4e^{-4x}$	B1		
	$4e^{-4x} = 8e^{-2x}$			
	$2e^{2x} - 1 = 0$ or $e^{-2x} - 2 = 0$ or $e^{2x} = \frac{1}{2}$ or $e^{-2x} = 2$ or $\ln 4 - 4x = \ln 8 - 2x$	M1		Equating $\frac{dy}{dx} = 0$ and getting $ae^{\pm 2x} \pm b = 0$ from $\frac{dy}{dx} = pe^{-2x} + qe^{-4x}$
	$x = \frac{1}{2}\ln\frac{1}{2}$ ISW	A1	3	OE, eg $\frac{1}{2}(\ln 4 - \ln 8)$ and no extra solutions

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8(b)(iv)	$V = \pi \int_0^{\ln 2} \left(4e^{-2x} - e^{-4x}\right)^2 dx$	B1		Must be completely correct including dx seen on this line or next line Limits, brackets and π PI from later working
	$= (\pi) \int 16e^{-4x} + e^{-8x} - 8e^{-6x} \left(dx\right)$	B1		Correct expansion, PI from later working
	$= (\pi) \left[-4e^{-4x} - \frac{1}{8}e^{-8x} + \frac{4e^{-6x}}{3} \right]_{(0)}^{(\ln 2)}$	B1		$\frac{16}{-4}e^{-4x}$ OE
		B1		$-\frac{1}{8}e^{-8x}$ OE
		B1		$\frac{-8}{-6}e^{-6x}$ OE may be two separate terms
	$= (\pi) \left[\left(-4e^{-4\ln 2} - \frac{1}{8}e^{-8\ln 2} + \frac{4}{3}e^{-6\ln 2} \right) - \left(-4e^0 - \frac{1}{8}e^0 + \frac{4}{3}e^0 \right) \right]$	M1		Correct substitution of $x = \ln 2$ and 0 into their integrated expression $\left(\text{must be of form } ae^{-4x} + be^{-6x} + ce^{-8x}\right)$ and subtracting. PI
	$= \frac{5247}{2048}\pi$	A1	7	OE exact fraction eg $\frac{251856}{98304}\pi$
	Total		**16**	
	TOTAL		**75**	