Version 1.0

General Certificate of Education (A-level)
January 2011

Mathematics MPC4

(Specification 6360)

Pure Core 4

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Copyright © 2011 AQA and its licensors. All rights reserved.

Copyright
AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$R=\sqrt{29}$	B1		Accept 5.4 or 5.38, 5.39, 5.385….
	$R\sin\alpha=5$ or $R\cos\alpha=2$ or $\tan\alpha=\frac{5}{2}$	M1		
	$\alpha=68.2^{\circ}$	A1	3	Condone $\alpha=68.20^{\circ}$
(b)(i)	(maximum value =) $\sqrt{29}$	B1ft	1	ft on R
(ii)	$\sin(x+\alpha)=1$	M1		Or $x+\alpha=90$, $x+\alpha=\frac{\pi}{2}$
	$x=21.8^{\circ}$ only	A1	2	No ISW
	Total		**6**	

MPC4 (cont)

Q	**Solution**	**Marks**	**Total**	**Comments**
2 (a)(i)	$\text{f}\left(-\frac{1}{3}\right)=9\left(-\frac{1}{3}\right)^3+18\left(-\frac{1}{3}\right)^2-\left(-\frac{1}{3}\right)-2$ $=9\left(-\frac{1}{27}\right)+18\left(\frac{1}{9}\right)-\left(-\frac{1}{3}\right)-2$ $=-\frac{1}{3}+2+\frac{1}{3}-2=0$	M1		$\text{f}\left(-\frac{1}{3}\right)$ attempted **NOT** long division
	$\Rightarrow(3x+1)$ is a factor	A1	2	Shown = 0 plus statement
(ii)	$(\mathbf{f}(x)=)\ (3x+1)\left(3x^2+kx-2\right)$	M1		3 and – 2
	$k=5$	A1		
	$(\mathbf{f}(x)=)\ (3x+1)(3x-1)(x+2)$	A1	3	
(iii)	$9x^3+21x^2+6x=x\left(9x^2+21x+6\right)$	M1		x and attempt to factorise quadratic equation.
	$=3x(3x+1)(x+2)$	A1		Correct factors
	$\dfrac{9x^3+21x^2+6x}{\text{f}(x)}=\dfrac{3x}{3x-1}$	A1	3	cso no ISW
(b)	$9\left(\frac{2}{3}\right)^3+p\left(\frac{2}{3}\right)^2-\frac{2}{3}-2=-4$	M1		Condone missing brackets, but must have $=-4$
	$p=-9$	A1	2	
			10	
2(a)(ii)	**Alternative** Using long division			
	$3x^2+5x-2$ $3x+1\overline{)9x^3+18x^2-x-2}$ $9x^3+3x^2$ $\overline{15x^2-x}$ $15x^2+5x$	(M1)		$3x^2+ax+b$
	$\overline{-6x-2}$ $-6x-2$	(A1)		$3x^2+5x-2$
	$(\text{f}(x)=)\ (3x+1)(3x-1)(x+2)$	(A1)	(3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(iii)	**Alternative** $\frac{\mathrm{f}(x)+\mathrm{q}(x)}{\mathrm{f}(x)}$, where q is a quadratic expression $= 1+\frac{(3x+1)(x+2)}{(3x+1)(3x-1)(x+2)}$ $=1+\frac{1}{3x-1}$	(M1) (A1) (A1)	(3)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$3+9x=A(3+5x)+B(1+x)$	M1		PI by correct A and B
	$x=-1 \qquad x=-\frac{3}{5}$	m1		Substitute two values of *x* and solve for *A* and *B*.
	$A=3 \qquad B=-6$	A1	3	
	Alternative Equating coefficients			
	$3+9x=A(3+5x)+B(1+x)$	(M1)		
	$3=3A+B$ $9=5A+B$	(m1)		Set up simultaneous equations and solve. Condone 1 error.
	$A=3 \qquad B=-6$	(A1)	(3)	
	Alternative Cover up rule			
	$x=-1 \qquad A=\dfrac{3-9}{3-5}$	(M1)		
	$x=-\dfrac{3}{5} \qquad B=\dfrac{3-\frac{27}{5}}{1-\frac{3}{5}}$			$x=-1$ and $x=-\frac{3}{5}$ and attempt to find *A* and *B*.
	$A=3 \qquad B=-6$	(A1 A1)	(3)	SC NMS *A* and *B* both correct; 3/3 One of *A* and *B* correct 1/3
(b)	$(1+x)^{-1}=1-x+kx^2$	M1		
	$=1-x+x^2$	A1		
	$(3+5x)^{-1}=3^{-1}\left(1+\frac{5}{3}x\right)^{-1}$	B1		
	$\left(1+\frac{5}{3}x\right)^{-1}=1-\frac{5}{3}x+\left(\frac{5}{3}x\right)^2$	M1		Condone missing brackets; allow one sign error
	$=1-\frac{5}{3}x+\frac{25}{9}x^2$	A1		
	$\dfrac{3+9x}{(1+x)(3+5x)}$ $=3\left(1-x+x^2\right)-6\times3^{-1}\left(1-\dfrac{5}{3}x+\dfrac{25}{9}x^2\right)$	M1		Use PFs and simplify to $a+bx+cx^2$ **or** expand product of $(3+9x)$ and binomial expansions and simplify to $a+bx+cx^2$
	$=1+\dfrac{1}{3}x-\dfrac{23}{9}x^2$	A1	7	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
(c)	$\frac{5x}{3} < 1$ oe or $\frac{5x}{3} > -1$ oe	M1		Condone $\le$ instead of $<$
	$\lvert x \rvert < \frac{3}{5}$ or $-\frac{3}{5} < x < \frac{3}{5}$	A1	**2**	CAO
			12	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\frac{dx}{dt} = 3e^t$ $\quad \frac{dy}{dt} = 2e^{2t} + 2e^{-2t}$	M1 A1		Both derivatives attempted and one correct Both correct
	$t = 0 \quad$ gradient $= \frac{4}{3}$	A1	3	cso Condone $\frac{dy}{dx} = \frac{4}{3}$
(ii)	$y = \frac{4}{3}(x-3)$ oe	B1ft	1	ft on non-zero gradient
(b)	$e^{2t} = \frac{x^2}{9}$ or $9e^{2t} = x^2$ or $e^t = \frac{x}{3}$ or $e^{2t} = \left(\frac{x}{3}\right)^2$ or $t = \ln\left(\frac{x}{3}\right)$ or $2t = \ln\left(\frac{x^2}{9}\right)$	M1		
	$y = \frac{x^2}{9} - \frac{9}{x^2}$	A1	2	Equation required
			6	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$m = 10 \times 2^{-\frac{14}{8}}$	M1		
	≈ 3 (gm)	A1	2	Condone 2.97 or better NOT 2.9 as final answer
(b)	$2^{-\frac{d}{8}} = \frac{1}{16}$	M1		
	$\frac{d}{8} = 4 \Rightarrow d = 32$	A1	2	cso
(c)	$0.01m_0 = m_0 \times 2^{-\frac{t}{8}}$	M1		m_0 can be numerical
	$\ln(0.01) = -\frac{t}{8}\ln(2)$	M1		Take logs correctly from their equation leading to a linear equation in t.
	$t = 53.15$			
	$n = 54$	A1	3	cso
			7	

Q	Solution	Marks	Total	Comments
6(a)(i)	$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$	B1		Condone numerator as tan x + tan x
	$2\tan x + \tan x\left(1-\tan^2 x\right) = 0$	M1		Multiplying throughout by their denominator
	$\tan x = 0$ $\text{or}\left(2+1-\tan^2 x\right) = 0 \Rightarrow \tan^2 x = 3$	A1	3	**AG** Must show tan x = 0 **and** tan^2x = 3
	Alternative			
	$\tan 2x = \dfrac{\sin 2x}{\cos 2x} = \dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x}$			
	$\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x} + \dfrac{\sin x}{\cos x} = 0$	(B1)		
	$2\sin x\cos^2 x + \sin x\left(\cos^2 x - \sin^2 x\right) = 0$			
	$\sin x(2\cos^2 x + \cos^2 x - \sin^2 x) = 0$	(M1)		
	$\left.\begin{array}{l}\Rightarrow \sin x = 0\\ \Rightarrow \tan x = 0\end{array}\right\}\begin{array}{l}\text{and}\\ \text{and}\end{array}\quad\left.\begin{array}{r}3\cos^2 x = \sin^2 x\\ \tan^2 x = 3\end{array}\right\}$	(A1)	(3)	
(ii)	$x = 60$ **AND** $x = 120$	B1	1	Condone extra answers outside interval eg 0 and 180
(b)(i)	$2\sin x\cos x = \cos x.\text{f}(x)$	M1		Where f(x) = cos^2x – sin^2x or 2cos^2x – 1 or 1 – 2sin^2x
	$2\sin x\cos x = \cos x\left(1-2\sin^2 x\right)$	A1		
	$\left(\cos x \neq 0\right)\quad 2\sin x = 1-2\sin^2 x$ $2\sin^2 x + 2\sin x - 1 = 0$	A1	3	**AG**

(ii)	$\sin x = \dfrac{-2 \pm \sqrt{4 - 4 \times 2 \times (-1)}}{2 \times 2}$	M1		Correct use of quadratic formula or completing the square or correct factors
	$\sin x = \dfrac{-2 \pm 2\sqrt{3}}{4}$	A1		$\sqrt{12}$ must be simplified and must have ±
	$\left.\begin{array}{l} \sin x = \dfrac{-1-\sqrt{3}}{2} \text{ has no solution} \\ \sin x = \dfrac{\sqrt{3}-1}{2} \end{array}\right\}$	E1	3	Reject one solution and state correct solution.
			10	

MPC4

Q	Solution	Marks	Total	Comments
7 **(a)(i)**	$\int \frac{\mathrm{d}x}{\sqrt{x}} = \int \sin\left(\frac{t}{2}\right) \mathrm{d}t$	B1		Correct separation; condone missing integral signs.
	$2\sqrt{x} = -2\cos\left(\frac{t}{2}\right) \ (+k)$	M1		$p\sqrt{x} = q\cos\left(\frac{t}{2}\right)$ Condone missing + k
	$x = \left(-\cos\left(\frac{t}{2}\right) + C\right)^2$	A1	3	Must have previous line correct
(ii)	$(1,0) \quad 2 = -2 + k$ or $1 = (-1 + C)^2$	M1		Use $(1,0)$ to find a constant
	$k = 4$ or $C = 2$	A1ft		ft on $C = p - q$ from (a)(i)
	$x = \left(2 - \cos\left(\frac{t}{2}\right)\right)^2$	A1	3	cso applies to (a)(ii)
(b)(i)	Greatest height when $\cos(bt) = -1$	M1		
	Greatest height = 9 (m)	A1ft	2	ft is (their a + 1)^2
(ii)	$\cos\left(\frac{t}{2}\right) = 2 - \sqrt{5}$	M1		$\cos bt = a - \sqrt{5}$
	$t = 2\cos^{-1}\left(2 - \sqrt{5}\right) = 3.6$ (seconds 1dp)	A1	2	condone 3.6 or better (3.618…..)
			10	

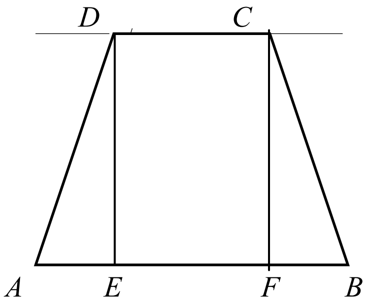
MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}6\\0\\3\end{bmatrix}-\begin{bmatrix}3\\-2\\4\end{bmatrix}=\begin{bmatrix}3\\2\\-1\end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OB}-\overrightarrow{OA}\right)$ implied by 2 correct components
(ii)	$\begin{bmatrix}3\\2\\-1\end{bmatrix}\bullet\begin{bmatrix}2\\-1\\3\end{bmatrix}=6-2-3=1$	M1 A1ft		Scalar product with correct vectors; allow one component error. ft on $\overrightarrow{AB}$
	$\cos\theta=\dfrac{sp}{\sqrt{14}\sqrt{14}}$	m1		Correct form for $\cos\theta$ with one correct modulus
	$\cos\theta=\dfrac{1}{14}\qquad\theta=85.9^{\circ}$	A1	4	cso 85.9 or better
(b)(i)	$\overrightarrow{OD}=\begin{bmatrix}3\\-2\\4\end{bmatrix}+2\begin{bmatrix}2\\-1\\3\end{bmatrix}=\begin{bmatrix}7\\-4\\10\end{bmatrix}$	M1		Implied by 2 correct components
	line $l_2\quad\mathbf{r}=\begin{bmatrix}7\\-4\\10\end{bmatrix}+\mu\begin{bmatrix}3\\2\\-1\end{bmatrix}$	A1ft	2	$\mathbf{r}=$ or $\begin{bmatrix}x\\y\\z\end{bmatrix}$ required ft on $\overrightarrow{AB}$
(ii)	$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{bmatrix}1+3p\\-4+2p\\7-p\end{bmatrix}$	M1		$\mu=p$ at C Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}\qquad\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$	B1ft		PI B1 is for $\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$
	$(1+3p)^2+(-4+2p)^2+(7-p)^2=56$	m1		
	$14p^2-24p+66=56$ $7p^2-12p+5=0$ $(7p-5)(p-1)=0$	m1		ft on $\overrightarrow{BC}$ Simplification to quadratic equation with all terms on one side
	$p=\dfrac{5}{7}$ and $p=1$	A1		Exact fraction required
	C is at $\left(9\dfrac{1}{7},-2\dfrac{4}{7},9\dfrac{2}{7}\right)$	A1	6	cso Accept as column vector
			14	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : Using equal angles $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\begin{bmatrix}1+3p\\-4+2p\\7-p\end{bmatrix}$	(M1)		$\mu=p$ at C Find $\overrightarrow{BC}$ in terms of p
	$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}$ $\quad \left\|\overrightarrow{BC}\right\|=\sqrt{56}$	(B1ft)		
	$(\cos\theta)=\dfrac{\overrightarrow{BA}\bullet\overrightarrow{BC}}{\sqrt{14}\sqrt{56}}=\dfrac{\begin{bmatrix}-3\\-2\\1\end{bmatrix}\bullet\begin{bmatrix}1+3p\\-4+2p\\7-p\end{bmatrix}}{\sqrt{14}\sqrt{56}}=\dfrac{1}{14}$	(m1)		Condone $\overrightarrow{AB}$ used. Allow $\left\|\overrightarrow{BC}\right\|$ in terms of p, in which case previous B1 is implied
	$-3-9p+8-4p+7-p=2$	(m1)		Reduce to linear or quadratic equation in p.
	$p=\dfrac{5}{7}$	(A1)		
	C is at $\left(9\dfrac{1}{7},-2\dfrac{4}{7},9\dfrac{2}{7}\right)$	(A1)	(6)	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(b)(ii)	**Alternative** : using symmetry (i)			
	$\left\lvert\overrightarrow{AD}\right\rvert=\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{56}$	(B1ft)		$\overrightarrow{AD}=\begin{bmatrix}4\\-2\\6\end{bmatrix}$
	$\left\lvert\overrightarrow{DC}\right\rvert=\left\lvert\overrightarrow{AB}\right\rvert-\left\lvert\overrightarrow{AD}\right\rvert\cos\theta-\left\lvert\overrightarrow{BC}\right\rvert\cos\theta$	(M1)		Substitute values and evaluate $\left\lvert\overrightarrow{AB}\right\rvert-\left\lvert\overrightarrow{AD}\right\rvert\cos\theta-\left\lvert\overrightarrow{BC}\right\rvert\cos\theta$
	$\left\lvert\overrightarrow{DC}\right\rvert=\dfrac{10}{\sqrt{14}}$	(A1ft)		F on $\overrightarrow{AB}$ and $\cos\theta$
	$\left\lvert\overrightarrow{DC}\right\rvert=p\left\lvert\overrightarrow{AB}\right\rvert\Rightarrow\dfrac{10}{\sqrt{14}}=p\sqrt{14}$	(m1)		Set up equation in p
	$p=\dfrac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	(A1)	(6)	
	Alternative using symmetry (ii) $\left\lvert\overrightarrow{AD}\right\rvert=\sqrt{56}$	(B1ft)		
	$\left\lvert\overrightarrow{AE}\right\rvert=\left\lvert\overrightarrow{AD}\right\rvert\cos\theta=\sqrt{56}\times\dfrac{1}{14}=\dfrac{2}{\sqrt{14}}$	(M1) (A1ft)		Substitute values and evaluate for $\left\lvert\overrightarrow{AD}\right\rvert\cos\theta$. F on $\cos\theta$
	$\left\lvert\overrightarrow{AE}\right\rvert=q\left\lvert\overrightarrow{AB}\right\rvert\Rightarrow\dfrac{2}{\sqrt{14}}=q\sqrt{14}$ and $\left\lvert\overrightarrow{AE}\right\rvert=\left\lvert\overrightarrow{FB}\right\rvert\Rightarrow p=1-2q$	(m1)		Set up equation to find p
	$q=\dfrac{2}{14}\quad p=\dfrac{5}{7}$	(A1)		
	C is at $\left(9\frac{1}{7},-2\frac{4}{7},9\frac{2}{7}\right)$	(A1)	(6)	
	TOTAL		**75**	