



A-LEVEL

Mathematics

Statistics 4 – MS04

Mark scheme

6360

June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS04

- GN1** There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question
- GN2** In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks
- GN3** When applying AFWF, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks
- GN4** Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage is shown
- GN5** In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds
- GN6** Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$

Q	Solution	Marks	Total	Comments
1				
(a)	$P(X = 4) = 0.25 \times 0.75^3 = \underline{\mathbf{0.105 \text{ to } 0.106}}$	B1	1	AWFW (0.10547)
(b)	Require $pq^{n-1} < 0.001$ ie $(0.25)(0.75)^{n-1} < 0.001$ or $(0.75)^{n-1} < 0.004$ or $(0.75)^n < 0.003$ Thus $(n-1)\log(0.75) < \log(0.004)$ or $n-1 > \log(0.004)/\log(0.75)$ $n-1 = 19.2 \Rightarrow n_{\min} = \underline{\mathbf{21}}$ or Trial & improvement with 20 & 21 $\Rightarrow = 21$ (4) Trial & improvement with 21 $\Rightarrow = 21$ (3) Trial & improvement $\Rightarrow \neq 21$ (1) No method shown and $\Rightarrow = 21$ (2) No method shown and $\Rightarrow \neq 21$ (0)	M1 AF1 M1 A1	4	Used; allow pq^n Correct method for n CAO
		Total	5	

Q	Solution	Marks	Total	Comments
2 (a)	$H_0: \sigma_S = 45$ or $\sigma_S^2 = 2025$ $H_1: \sigma_S > 45$ or $\sigma_S^2 > 2025$ SL: $\alpha = 0.05$ (1-tailed) DF: $\nu = \underline{11}$ CV: $\chi^2 = \underline{19.6 \text{ to } 19.7}$ $\chi^2 = \frac{\sum(x-\bar{x})^2}{\sigma_S^2} = \frac{37100.25}{45^2}$ $= \underline{18.3}$	B1 B1 B1 M1 A1 AF1	Both CAO AFWW (19.675) (2-tailed \Rightarrow 21.9 & 3.8 (AWRT)) Used; OE AWRT (18.321) OE; F on CV and χ^2 (calc)	6
(b)	$H_0: \sigma_S = 2\sigma_N$ $H_1: \sigma_S \neq 2\sigma_N$ SL: $\alpha = 0.10$ (2-tailed) DF: $\nu_1 = \underline{11}$ & $\nu_2 = \underline{7}$ CV: $F_U = \underline{3.6(0)}$ $s_S^2 = \frac{37100.25}{11} = \underline{3372.75}$ $s_N^2 = \frac{2033.50}{7} = \underline{290.5}$ $F = \frac{3372.75}{4 \times 290.5}$ $= \underline{2.9 \text{ to } 2.91}$	B1 B1 B1 B1 M1 A1 Adep1	OE both CAO both AFWW (3.603) ($F_L = 3.012^{-1} = 0.332$) Can be from (a) ($s_S = 58.075$) CAO both OE (581 or 1162) ($s_N = 17.044$) Allow '2 or 1' instead of '4' AFWW (2.9025) ('2' \Rightarrow 5.81 & '1' \Rightarrow 11.61) Dependent on 2.9 to 2.91 and 3.6 OE	7
Notes	1 Use of '2 or 1' \Rightarrow B1 B1 B1 B1 M1 A0 Adep0 (max = 5) 2 Use of ' $s_S = 45$ ' \Rightarrow B1 B1 B1 B0 M1 A0 Adep0 (max = 4)			
		Total	13	

Q	Solution	Marks	Total	Comments
3 (a)	<p><i>Pots4U:</i> mean = <u>25.4</u> Sd = <u>0.333 to 0.334</u> or Var = <u>0.111 to 0.112</u></p> <p><i>WeRPots:</i> mean = <u>25.2</u> Sd = <u>0.262 to 0.263</u> or Var = <u>0.068 to 0.069</u></p> <p>$s_p^2 = \frac{14 \times 0.111429 + 9 \times 0.068889}{15 + 10 - 2}$ = <u>0.094 to 0.095</u></p> <p>or $s_p = \underline{0.307 \text{ to } 0.308}$</p> <p>$\nu = 15 + 10 - 2 = \underline{23}$</p> <p>$t_{23}(0.025) = \underline{2.07}$</p> <p>CI: $(25.4 - 25.2) \pm 2.069 \times 0.307868 \sqrt{\frac{1}{15} + \frac{1}{10}}$</p> <p>Thus $\underline{0.2 \pm 0.26}$</p> <p>or $\underline{(-0.06, 0.46)}$</p>	B1 B1 B1 M1 A1 B1 B1 M1 m1 A1	10	CAO AFWW (0.333809) AFWW (0.111429) Both means CAO AFWW (0.262467) AFWW (0.068889) OE AFWW (0.094783) AFWW (0.307868) CAO AWRT (2.069) $(\bar{x}_1 - \bar{x}_2) \pm ts\sqrt{a}$ $ts_p \sqrt{\frac{1}{15} + \frac{1}{10}}$ CAO/AWRT (0.2600) AWRT
(b)	<p>Since CI includes 0</p> <p>No evidence, at 5% level, of a difference in mean weight</p>	Bdep1 BF1	2	Dependent on 0 ∈ CI F on Bdep1; OE in context
		Total	12	

Q	Solution	Marks	Total	Comments
4(a) (i)(A)	$E(X) = \int_0^{\infty} x \frac{1}{\theta} e^{-\frac{x}{\theta}} dx$ $= \left[-xe^{-\frac{x}{\theta}} \right]_0^{\infty} - \int_0^{\infty} -e^{-\frac{x}{\theta}} dx$ $= 0 + \theta \times \int f(x) dx = \theta \times 1 = \underline{\theta}$	M1 m1 A1	 (3)	Used; ignore limits Integration by parts; ignore limits Fully correct convincing derivation
(B)	$P(X > x) = \int_x^{\infty} \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = \left[-e^{-\frac{x}{\theta}} \right]_x^{\infty}$ <p>or</p> $P(X < x) = \int_0^x \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = \left[-e^{-\frac{x}{\theta}} \right]_0^x$ $= \left(0 + e^{-\frac{x}{\theta}} \right) \text{ or } 1 - \left(-e^{-\frac{x}{\theta}} + 1 \right) = \underline{e^{-\frac{x}{\theta}}}$	M1 A1	 (2)	Correct integration with limits Fully correct convincing derivation
Note	1 Use of $1 - F(x) = 1 - (1 - e^{-x/\theta}) = e^{-x/\theta} \Rightarrow$ M0 A0			
			5	
(ii)	$P(m < X \leq \mu) = 0.5 - e^{-1}$ $= 0.5 - 0.36788 = \underline{\mathbf{0.132}}$	M1 A1	 2	AWRT (0.13212)
(b) (i)	$P(250 < L < 1250) = P(0.25 < X < 1.25)$ $= e^{-\frac{0.25}{2}} - e^{-\frac{1.25}{2}}$ $= 0.88250 - 0.53526 = \underline{\mathbf{0.347}}$	M1 A1 A1	 3	Use of correct values of X OE AWRT (0.34724)
(ii)	$P(F_0) = P(X > 250) = \mathbf{0.88250}$ $P(F_0 \text{ in } \geq 5 \text{ in } 6) = 6 \times 0.8825^5 \times 0.1175 + 0.8825^6$ $= 0.37737 + 0.47237 = \underline{\mathbf{0.849 \text{ to } 0.85}}$	BF1 M1 AF1 A1	 4	F on (b)(i) At least one binomial term F on (b)(i) plus both binomial terms AWFW (0.84974)
Notes	1 B(6, 0.34724) $\Rightarrow 1 - 0.97847 = 0.0215 \Rightarrow$ BF0 M1 A1F A0 2 B(6, 0.53526) $\Rightarrow 1 - 0.85397 = 0.1460 \Rightarrow$ BF0 M1 A1F A0 3 B(6, p) \Rightarrow BF0 M1 A1F A0 only if seen correct expressions			
		Total	14	

Q	Solution	Marks	Total	Comments																																								
5																																												
(a)	<p>Table 1</p> <table border="1"> <thead> <tr> <th><i>D</i></th> <th>0–5</th> <th>5–6</th> <th>6–7</th> <th>7–8</th> <th>8–9</th> <th>9–10</th> <th>>10</th> </tr> </thead> <tbody> <tr> <td>Probability</td> <td>0</td> <td>$\frac{1}{54}$</td> <td>$\frac{7}{54}$</td> <td>$\frac{19}{54}$</td> <td>$\frac{3}{8}$</td> <td>$\frac{1}{8}$</td> <td>0</td> </tr> </tbody> </table>	<i>D</i>	0–5	5–6	6–7	7–8	8–9	9–10	>10	Probability	0	$\frac{1}{54}$	$\frac{7}{54}$	$\frac{19}{54}$	$\frac{3}{8}$	$\frac{1}{8}$	0																											
<i>D</i>	0–5	5–6	6–7	7–8	8–9	9–10	>10																																					
Probability	0	$\frac{1}{54}$	$\frac{7}{54}$	$\frac{19}{54}$	$\frac{3}{8}$	$\frac{1}{8}$	0																																					
	$P(5 < X < 6) = \int_0^1 \frac{1}{18}x^2 dx = \left[\frac{x^3}{54} \right]_0^1 = \frac{1}{54}$ $P(6 < X < 7) = \left[\frac{x^3}{54} \right]_1^2 = \frac{7}{54}$ <p>Thus</p> $P(8 < X < 9) = 1 - \frac{27}{54} - \frac{1}{8} = \frac{3}{8}$	<p>M1 A1</p> <p>A1</p> <p>B1</p>	4	<p>Ignore limits CAO</p> <p>CAO</p> <p>CAO; accept $\left(\frac{81}{216}, \frac{27}{72}, \frac{9}{24} \right)$</p>																																								
Note	$1 \ P(8 < X < 9) = \left[\frac{5x}{4} - \frac{x^2}{8} \right]_3^4 = 3 - \left[\frac{15}{4} - \frac{9}{8} \right] = 3 - \frac{21}{8} = \frac{3}{8}$																																											
(b)	<p>H_0: suggested model is appropriate H_1: suggested model is not appropriate</p> <p>SL: $\alpha = 0.05$ (1-tailed)</p> <p>DF: $\nu = 5 - 1 = \underline{4}$</p> <p>CV: $\chi^2 = \underline{9.49}$</p>	<p>B1</p> <p>B1</p> <p>BF1</p>		<p>At least H_0</p> <p>CAO</p> <p>AWRT; F on ν (9.488)</p>																																								
Note	<p>1 Only F on $\nu = 6 \Rightarrow \chi^2 = 12.592$ or $\nu = 5 \Rightarrow \chi^2 = 11.070$ or $\nu = 3 \Rightarrow \chi^2 = 7.815$</p>																																											
	<table border="1"> <thead> <tr> <th><i>d</i></th> <th>0–5</th> <th>5–6</th> <th>6–7</th> <th>7–8</th> <th>8–9</th> <th>9–10</th> <th>>10</th> </tr> </thead> <tbody> <tr> <td><i>O</i></td> <td>3</td> <td>11</td> <td>63</td> <td>182</td> <td>222</td> <td>53</td> <td>6</td> </tr> <tr> <td><i>E</i></td> <td>0</td> <td>10</td> <td>70</td> <td>190</td> <td>202.5</td> <td>67.5</td> <td>0</td> </tr> <tr> <td><i>O–E</i></td> <td></td> <td>4</td> <td>–7</td> <td>–8</td> <td>19.5</td> <td>–8.5</td> <td></td> </tr> <tr> <td>$(O-E)^2/E$</td> <td></td> <td>1.6</td> <td>0.7</td> <td>0.337</td> <td>1.878</td> <td>1.070</td> <td></td> </tr> </tbody> </table>	<i>d</i>	0–5	5–6	6–7	7–8	8–9	9–10	>10	<i>O</i>	3	11	63	182	222	53	6	<i>E</i>	0	10	70	190	202.5	67.5	0	<i>O–E</i>		4	–7	–8	19.5	–8.5		$(O-E)^2/E$		1.6	0.7	0.337	1.878	1.070		<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>AF1</p>		<p>$E = 540 \times p$</p> <p>Combining 2 classes at least once</p> <p>Attempt at $\sum (O - E)^2 / E$</p> <p>AWFW (5.58499)</p> <p>F on CV(upper) and $\chi^2(\text{calc})$ OE in context</p>
<i>d</i>	0–5	5–6	6–7	7–8	8–9	9–10	>10																																					
<i>O</i>	3	11	63	182	222	53	6																																					
<i>E</i>	0	10	70	190	202.5	67.5	0																																					
<i>O–E</i>		4	–7	–8	19.5	–8.5																																						
$(O-E)^2/E$		1.6	0.7	0.337	1.878	1.070																																						
Note	$1 \ \sum (O - E)^2 / E = \sum O^2 / E - \sum E = 545.58499 - 540 = 5.585$																																											
		Total	12																																									

Q	Solution	Marks	Total	Comments
6 (a)(i)	$E(X_1) = \frac{1}{3}(\mu + 2\mu) = \underline{\mu}$ $E(X_2) = \frac{1}{2}\left(\mu + \frac{2\mu}{2}\right) = \underline{\mu}$	M1 m1 A1	3	Use of $E(X)$ At least one correct application Both derivations fully correct
(ii)	$\text{Var}(X_1) = \frac{1}{3^2}\left(\frac{\sigma^2}{n} + \frac{\sigma^2}{2n}\right) = \underline{\sigma^2/6n}$ $\text{Var}(X_2) = \frac{1}{2^2}\left(\frac{\sigma^2}{n} + \frac{\sigma^2}{2^2(2n)}\right) = \underline{9\sigma^2/32n}$	M1 m1 A1	3	Use of $\text{Var}(aX) = a^2\text{Var}(X)$ or σ^2/n Correct use of $\text{Var}(aX) = a^2\text{Var}(X)$ & σ^2/n in at least one expression Both expressions correct
(iii)	$\text{Efficiency} = \frac{1/\text{Var}(X_2)}{1/\text{Var}(X_1)} = \frac{32}{6 \times 9}$ $= \underline{32/54 \text{ or } 16/27 \text{ or } 0.59 \text{ to } 0.595}$	M1 AF1	2	Used CAO/AWFW (0.5926) F on (ii)
(b) (i)	$\text{Var}(Y) = c^2 \frac{\sigma^2}{n} + (1-c)^2 \frac{\sigma^2}{2^2 2n}$ $\frac{d}{dc} = \frac{\sigma^2}{n} \left(2c - \frac{2(1-c)}{8}\right) = 0$ $\Rightarrow c = \underline{1/9}$ <p>Thus $\text{Var}(Y) = \frac{\sigma^2}{n} \left(\frac{1}{81} + \frac{64}{81} \times \frac{1}{8}\right) = \underline{\sigma^2/9n}$</p>	M1 M1 A1 B1	4	Use of $\text{Var}(aX) = a^2\text{Var}(X)$ & σ^2/n Differentiation wrt c CAO CAO
(ii)	$H_0: \mu = 5$ $H_1: \mu \neq 5$ <p>SL: $\alpha = 0.01$ (2-tailed)</p> <p>CV: $z = \underline{2.57 \text{ to } 2.58}$</p> $y = \frac{1}{9} \times 4.8 + \frac{8}{9} \times \frac{12.3}{2} = \underline{6.0}$ <p>$\text{Var}(Y) = \underline{16/90 \text{ or } 8/45 \text{ or } 0.177 \text{ to } 0.178}$</p> $z = \frac{y-5}{\sqrt{\text{Var}(Y)}} = \frac{6.0-5}{\sqrt{16/90}} = \underline{2.37}$ <p>No evidence, at 1% level, to reject $\mu = 5$</p>	B1 B1 B1 M1 A1 Adepl	7	OE AWFW (2.5758) CAO CAO/AWFW Divisor of 16 must be 10, 20, 30, 90 or 180 AWRT (2.3717) Dependent on 2.37 and 2.57 to 2.58 OE in context
		Total	19	