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General Certificate of Education (A-level)
June 2012

Mathematics MS2B

(Specification 6360)

Statistics 2B

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1(a)	$\bar{x} = \frac{\sum x}{n} = \frac{546}{15} = \frac{182}{5} = 36.4$	B1		oe
	$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{1407.6}{14} = 100.54$ **(or** $s = 10.03$ **)**	B1		$\sigma^2 = 93.84$ or $\sigma = 9.687$ iff $\sigma/\sqrt{14}$ used below
	$t_{crit} = \pm 2.624$	B1		ignore signs for t_{crit} (allow t = 2.62) (if z used then **max**(B1B1B0 M0A0A0)
	98% CI for μ: $36.4 \pm 2.624 \times \frac{s}{\sqrt{15}}$	M1		their $\bar{x} \pm t_{14} \times \frac{\text{their } s}{\sqrt{15}}$ **or** their $\bar{x} \pm t_{14} \times \frac{\text{their } \sigma}{\sqrt{14}}$
	$(29.6, 43.2)$ 36.4 ± 6.8	A1ft		(allow any of the following for t_{14}: 1.345; 1.761; 2.145; 2.624; 2.977)
	$= 29.6, 43.2$	A1	6	cao
(b)	$40.0 \in$ C.I.	E1ft		Must refer to 40 (dep M1)
	$\Rightarrow$ no change	E1ft	2	Dep on previous mark
			8	
2(a)	$H_0: \mu = 4.0$ $H_1: \mu > 4.0$	B1		(both)
	$z_{calc} = \frac{4.2-4}{1.1/\sqrt{40}}$	M1		**Alternative:** $P(\bar{X} > 4.2) = P(Z > 1.15)$ **M1A1**
	$= 1.15$	A1		awrt
	$z_{crit} = 1.6449$	B1		$= 1 - 0.87493$ $= 0.125$ **B1**
				$0.125 > 0.05 \Rightarrow$ accept H_0 **Adep1**
	Accept H_0 [or Reject H_1]	A1		Dep on B1M1B1
	Insufficient evidence at 5% level to support Julian's claim	E1	6	Dep on previous mark
(b)	Type II error.	B1ft		Follow through on conclusion in (a)
	Accepted H_0 when H_0 was false (oe)	E1	2	Dep on previous mark
				If Reject H_0 in (a) then: No error (B1ft) Rejected H_0 when H_0 was false (oe) (E1)
	Total		**8**	

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Q	Solution	Marks	Total	Comments
3(a)	for $-5 \le x \le 15$ $f(x) = \frac{d}{dx}F(x) = \frac{d}{dx}\left(\frac{x+5}{20}\right) = \frac{1}{20}$	B1	1	AG
(b)(i)	$P(X \ge 7) = 1 - F(7)$ $= 1 - \frac{12}{20}$ $= \frac{2}{5}$ **or** $\left[\frac{8}{20}; \frac{4}{10}; 0.4\right]$	B1	1	**Alternative:** Use of $f(x) = \frac{1}{20}$ ***or*** graph $\Rightarrow$ $P(X \ge 7) = \frac{1}{20} \times (15-7) = \frac{2}{5}$ (oe)
(ii)	$P(X \ne 7) = 1$	B1	1	cao
(iii)	$E(X) = \frac{1}{2}(-5+15) = 5$	B1	1	**Alternative:** $E(X) = \int_{-5}^{15} \frac{x}{20} dx = \left[\frac{x^2}{40}\right]_{-5}^{15}$ $= \frac{1}{40}(225-25)$ $= \frac{1}{40} \times 200$ $= 5$ **B1** (cao)
(iv)	$E(3X^2) = \int_{-5}^{15} \frac{3x^2}{20} dx$ } (ignore limits)	M1		
	$\left[\frac{x^3}{20}\right]_{-5}^{15}$ $\frac{1}{20}(3375+125)$ $168\frac{3}{4} + 6\frac{1}{4}$ }	A1		correct limits seen / used
	$= 175$	A1	3	(cao) (allow $174.\dot{9}$)
	Alternative:			
	$\text{Var}(X) = \frac{1}{12}(15--5)^2 = \frac{400}{12}$ (oe)	(B1)		$E(3X^2) = 3E(X^2)$
	$E(3X^2) = 3 \times \left[\frac{400}{12} + 5^2\right]$	(M1)		$= 3 \times \left[\{\text{their Var}(X) > 0\} + \{\text{their E}(X)\}^2\right]$ ***used*** $(\Rightarrow$ M1$)$
	$= 175$	(A1)		
	Total		**7**	

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Q	Solution	Marks	Total	Comments
4(a)				
	r | 1 | 2 | 3 | 4 | 5 p | .5 | **.24** | **.144** | **.0864** | 0.0296			
	$0.4\times0.6=0.24$ $0.24\times0.6=0.144$ $0.144\times0.6=0.0864$	B2,1	2	B1 for any 1 correct (unsimplified) (B1) B2 all correct and simplified
(b)	$\mathrm{P}(\text{fewer than 3 bedrooms **are** rented})$ $=\mathrm{P}(R=1,2)\Rightarrow$			**Alternative:** P(fewer than 3 not rented) = P(0, 1 or 2 not rented) = P(5, 4 or 3 are rented)
	$\mathrm{P}(\text{fewer than 3 bedrooms **not** rented})$ $=1-\mathrm{P}(R=1,2)$	M1		= P(R = 3, 4, 5) **M1**
	$=1-\mathrm{P}(\text{1 or 2 rooms are rented})$			$p=0.4\times0.6^2+0.4\times0.6^3+0.0296$
	$=1-$ (0.5 + **0.24**) [their $0<p(2)\le0.4704$ value from table **used**]	m1		= **0.144 + 0.0864** + 0.0296 **m1** [**or** their $p(3)+p(4)\le0.4704$ value from table **used**]
	$=1-0.74$ $=0.26$	A1	3	= 0.26 (cao) **A1** [SC 0.74 for B1]
(c)(i)	$\mathrm{E}(R)=0.5\times1+0.4\times0.6\times2$ $+0.4\times0.6^2\times3+0.4\times0.6^3\times4$ $+0.0296\times5$	M1		$\sum_1^5 r_i\times\mathrm{P}(R=r_i)$ from their table
	$=0.5\times1+0.24\times2+0.144\times3+0.0864\times4$ $+0.0296\times5$			
	= 0.5 + 0.48 + 0.432 + 0.3456 + 0.148 $\left[=\frac{1}{2}+\frac{12}{25}+\frac{54}{125}+\frac{216}{625}+\frac{37}{250}\right]$			$(0.5+1.2576+0.148)$
	$\therefore\ \mathrm{E}(R)=1.9056$	A1	2	[awfw 1.9 to 1.91] $\left[1\frac{566}{625}\right]$
(ii)	$\mathrm{E}(R^2)=0.5\times1^2+0.4\times0.6\times2^2$ $+0.4\times0.6^2\times3^2+0.4\times0.6^3\times4^2$ $+0.0296\times5^2$			$[0.5+0.96+1.296+1.3824+0.74]$
	$\mathrm{E}(R^2)=4.8784$	B1		AG
	$\mathrm{Var}(R)=4.8784-1.9056^2$ **(=** 1.24708864)	M1		4.8784 – their $\mathrm{E}^2(R)$
	$=1.25$ **(3sf)**	A1	3	(awfw 1.23 to 1.25)

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Q	Solution	Marks	Total	Comments
(d)	$\mathrm{E}(M) = 1250\mathrm{E}(R) - 282$ $= 1250 \times 1.9056 - 282$ $= 2100$	B1		cao
	$\mathrm{Var}(M) = 1250^2 \times \left[4.8784 - 1.9056^2 \right]$	M1		$1250^2 \times$ their $\mathrm{Var}(R) > 0$ in (c)(ii) (1 948 473 to 1 953 125)
	$\mathrm{sd}(M) = 1250 \times \sqrt{1.24708864}$ $= 1395.91$	A1	3	$\mathrm{sd}(M) = \sqrt{1948437} = 1395.9$ $\left(\sqrt{1953125} = 1397.5\right)$ (awfw 1395 to 1400)
	Total		**13**	

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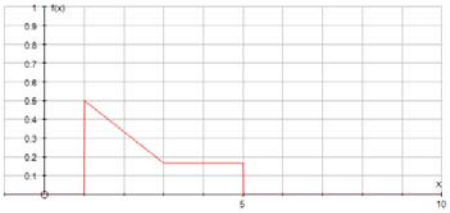
	Solution	Marks	Total	Comments
5(a)(i)	$\mathrm{P}(X \geq 9) = 1 - \mathrm{P}(X \leq 8)$ $= 1 - 0.5231$ $= 0.4769$	B2,1	2	$1 - 0.6530 = 0.347$ (B1) awfw 0.476 and 0.477
(ii)	$\mathrm{P}(5 < X < 10) = \mathrm{P}(X \leq 9) - \mathrm{P}(X \leq 5)$ $= 0.653 - 0.1496$ $= 0.5034$	B3,2,1	3	awfw 0.503 to 0.504 $0.7634 - 0.1496 = 0.613$ to 0.614 (B2) $0.6530 - 0.2562 = 0.397$ to 0.398 (B2) $0.7634 - 0.2562 = 0.507$ to 0.508 (B1) $\alpha - 0.1496$ or $0.653 - \alpha$ (B1) iff $0 < p < 1$
(b)	$\mathrm{P}(Y < 2) = \mathrm{P}(Y \leq 1) = \mathrm{P}(Y = 0 \text{ or } Y = 1)$ $= \mathrm{e}^{-1.5} + \mathrm{e}^{-1.5} \times 1.5$ $[0.2231 + 0.3347]$ $= 0.5578254$ $= 0.558$	M1 A1	2	0.8 to 0.81 (B1) (both) awfw 0.557 to 0.56
(c)(i)	$\lambda = 8.5 + 1.5 = 10$	B1	1	Allow P(10) or $\mathrm{Po}(10)$
(ii)	$\mathrm{P}(T > 16) = 1 - \mathrm{P}(T \leq 16)$ $= 1 - 0.9730$ $= 0.027$	M1 A1	2	
(iii)	$p = {}^3C_2\, 0.027^2 \times 0.973$ $+ 0.027^3$ $= 0.002128 + 0.00001968$ $= 0.0021$ [4 dp]	M1 M1 A1	3	for either term correct for addition of the two correct terms 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Alternative: $p = 1 - \mathrm{P}(X \leq 1)$ $\mathrm{P}(X = 0) + \mathrm{P}(X = 1)$ $= 0.973^3 + 3 \times 0.973^2 \times 0.027$ $= 0.921167 + 0.076685$ $p = 1 - 0.99785$ $= 0.0021$	(M1) (M1) (A1)		for either term correct for 1 – [sum of two correct terms] 0.0021 or 0.0022 [iff M1M1 (+ 4dp)]
	Total		**13**	

MS2B

Q	Solution	Marks	Total	Comments
6(a)	H_0: No association between A level grade and class of degree H_1: Association between A level grade and class of degree	B1		At least H_0 correct
	O_i / E_i: 20/11.6; 9/17.4; 36/36.4; 55/54.6; 22/28; 48/42; 2/4; 8/6; 200/200	M1		For E_i's attempted
	Combine Class 2(ii) and 3	M1		For combining attempted
	20 | 11.6 | 8.4 | 6.0827 9 | 17.4 | –8.4 | 4.0552 36 | 36.4 | –0.4 | 0.0044 55 | 54.6 | 0.4 | 0.0029 24 | 32 | –8 | 2.0 56 | 48 | 8 | 1.3333 200 | 200 | 0 | 13.47	M1 A1		For final column attempted (awrt 13.5)
	$\nu = 2$ $\chi^2_{1\%}(2) = 9.210$	B1 B1		[$\nu = 3$ with $\chi^2 = 11.345$ (B0B1ft)]
	Reject H_0	A1		Dep on B1 M1M1M1 B1B1, not A1
	Fiona's belief justified	E1	9	Dep on B1 M1M1M1 B1B1, not A1
(b)	**Fewer than expected gained a Class 1** degree having gained grade B in A-level Mathematics.	E1		
	More than expected gained a Class 2(ii) degree having gained grade B in A-level Mathematics.	E1	2	correct comments (see below)
	1 / 2(i) / 2(ii) / 3 / comb **9 / 55 / 48 / 8 / 56** **17.6 / 54.6 / 42 / 6 / 48** **A / B / C / D / E**			A: fewer than expected B: as expected C: more than expected D: more or similar than expected E: more than expected
	Total		**11**	

MS2B

Q	Solution	Marks	Total	Comments
7(a)		B2,1	2	Straight line from (1 , 0.5) to (3 , $\frac{1}{6}$). Horizontal straight line from (3 , $\frac{1}{6}$) to (5 , $\frac{1}{6}$).
(b)	$\mathrm{E}(X)=\frac{1}{6}\int_1^3 x(4-x)\,\mathrm{d}x+\frac{1}{6}\int_3^5 x\mathrm{d}x$	M1		ignore limits (**both** parts attempted)
	$=\frac{1}{6}\left[2x^2-\frac{x^3}{3}\right]_1^3+\frac{1}{6}\left[\frac{x^2}{2}\right]_3^5$	A1		ignore limits (**both** correct)
	$=\frac{1}{6}\left[(18-9)-\left(2-\frac{1}{3}\right)\right]+\frac{1}{6}\left[\frac{25}{2}-\frac{9}{2}\right]$			
	$=\frac{1}{6}\left[7\frac{1}{3}+8\right]$	m1		use of correct limits. dep on M1A1
	$=2\frac{5}{9}$	A1	4	(AG)
(c)(i)	$\mathrm{P}(X>2.5)=\frac{1}{3}+\frac{1}{2}\times\left(0.25+\frac{1}{6}\right)\times\frac{1}{2}$	M1		Or $1-\int_1^{2.5}\frac{1}{6}(4-x)\mathrm{d}x=1-\left[\frac{1}{6}\left(4x-\frac{x^2}{2}\right)\right]_1^{2.5}$
	$=\frac{7}{16}$	A1	2	cao (0.4375)
(ii)	$\mathrm{P}(1.5<X<4.5)=\frac{1}{2}\times\left(\frac{5}{12}+\frac{1}{6}\right)\times1.5$ $+(4.5-3)\times\frac{1}{6}$	M1		Or $\int_{1.5}^{3}\frac{1}{6}(4-x)\mathrm{d}x$ + $\int_3^{4.5}\frac{1}{6}\mathrm{d}x$
	$=\frac{7}{16}+\frac{1}{4}$	A1		
	$=\frac{11}{16}$	A1	3	cao ($=\frac{11}{16}$ or 0.6875)
(iii)	$\mathrm{P}(X>2.5$ **and** $1.5<X<4.5)$ $=\mathrm{P}(2.5<X<4.5)$			$\int_{2.5}^{3}\frac{1}{6}(4-x)\mathrm{d}x=\left[\frac{1}{6}\left(4x-\frac{x^2}{2}\right)\right]_{2.5}^{3}=\frac{5}{48}$
	$=\frac{1}{2}\times\left(0.25+\frac{1}{6}\right)\times0.5+\frac{1}{4}$	M1		
	$=\frac{5}{48}+\frac{1}{4}$			
	$=\frac{17}{48}$	A1	2	cao $\left(0.35416\dot{}\right)$
(iv)	$\mathrm{P}(X>2.5\|1.5<X<4.5)=\frac{17/48}{11/16}$	M1		their $\frac{\text{(iii)}}{\text{(ii)}}$ iff 0 < p's < 1
	$=\frac{17}{33}$	A1	2	cao $\left(\text{allow } 0.5\dot{1}\right)$

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Q	Solution	Marks	Total	Comments
7(c)	**Alternative Solution**			
	$F(x)=\begin{cases}0 & x<1\\ \frac{1}{12}(x-1)(7-x) & 1\le x<3\\ \frac{1}{6}(x+1) & 3\le x<5\\ 1 & x\ge 5\end{cases}$			
(i)	$P(X>2.5)=1-F(2.5)$			
	$=1-\frac{1}{12}(2.5-1)(7-2.5)$	(M1)		
	$=1-\frac{1}{12}\times 1.5\times 4.5$			
	$=1-0.5625$			
	$=0.4375$ **or** $\frac{7}{16}$	(A1)		cao
(ii)	$P(1.5<X<4.5)=F(4.5)-F(1.5)$			
	$=\frac{1}{6}(4.5+1)-\frac{1}{12}(1.5-1)(7-1.5)$	(M1)		
	$=\frac{11}{12}-\frac{11}{48}$	(A1)		
	$=\frac{11}{16}$ **or** 0.6875	(A1)		cao
(iii)	$P(X>2.5$ **and** $1.5<X<4.5)$			
	$=P(2.5<X<4.5)$			
	$=F(4.5)-F(2.5)$			
	$=\frac{11}{12}-\frac{9}{16}$	(M1)		
	$=\frac{17}{48}$	(A1)		cao
(iv)	$P(X>2.5\|1.5<X<4.5)$			
	$=\frac{F(4.5)-F(2.5)}{F(4.5)-F(1.5)}$ **or** $\frac{\text{their (iii)}}{\text{their (ii)}}$	(M1)		
	$=\frac{17/48}{11/16}$			
	$=\frac{17}{33}$ **or** $(\text{allow } 0.\dot{5}\dot{1})$	(A1)		cao
	Total		**15**	
	TOTAL		**75**	