A-LEVEL

Mathematics

Statistics MS2 – MS2B
Mark scheme

6360
June 2015

Version/Stage 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Marks	Total	Comments
(a)	Use of Po(2.8)	M1		Stated or table value (0.8477, 0.9349, 0.9756 or 3sf equivalents) seen
	P(≤ 5) = **0.935**	A1	**2**	AWRT
(b)	Use of Po(4.4)	B1		Stated or attempt at method seen
	$e^{-4.4} \times 4.4^2 \div 2$	M1		Correct formula or by calculator
	= **0.119**	A1	**3**	AWRT
(c)	Use of Po(15)	B1		Stated or any 1 of 4 relevant values seen 0.1848, 0.2676, 0.7489, 0.8195
	We require P(≤ 17)	M1		Stated **or** use of 0.7489
	– P(≤ 11)	M1		Indep. Stated **or** use of 0.1848
	= 0.7489 – 0.1848 = **0.564**(1)	A1	**4**	AWRT 0.564
		Total	**9**	

Note: (a) The mark is not awarded for simply 2.8. Some indication of Poisson is needed. Eg. Po(2.8) or $\lambda = 2.8$

(b) As for part (a), not simply 4.4.

(c) If Po(15) and P(≤ 17) – P(≤ 11) are seen, 3 marks have been earned irrespective of later numbers.

Q2	Solution	Marks	Total	Comments
(a)	$k = (b - a)$	B1	**1**	CAO
(b)(i)	$\frac{1}{2}(a + b) = 1$ and $\frac{1}{12}(b - a)^2 = 3$	B1		For both equations (not including k)
	$(b - a)^2 = 36 \rightarrow (b - a) = \pm 6$	M1		6 or ±6 required for this mark
	$b > a$ stated giving $b - a = 6$ only **or** both $b - a = 6$ and $b - a = -6$ used	m1		Consideration of two solutions
	$\boldsymbol{b = 4}$ and $\boldsymbol{a = -2}$	A1	**4**	CAO not dependent on m1
(ii)	$P(X < 0) = \frac{1}{3}$	B1		Stated or used (accept 0.333)
	$4 \times p \times (1 - p)^3$ where p = candidate's stated $P(X < 0)$	M1		$0 < p < 1$
	$= \frac{32}{81}$ **(= 0.395)**	A1	**3**	CAO or AWRT 0.395
		Total	**8**	

(b)(i)	**Alternative solution** $\frac{1}{2}(a + b) = 1$ and $\frac{1}{12}(b - a)^2 = 3$	B1		For both equations (not including k)
	$b = 2 - a \rightarrow 4a^2 - 8a - 32 = 0$ **or** $a = 2 - b \rightarrow 4b^2 - 8b - 32 = 0$	M1		For obtaining one of these quadratics or equivalent
	$a = -2$, $b = 4$ and $a = 4$, $b = -2$	A1		For both correct pairs of solutions or one pair with any justification
	Selection of correct solution $\boldsymbol{b = 4}$ and $\boldsymbol{a = -2}$	A1	**4**	CAO not dependent on previous A1

Note: (b) Integration may be used but must reach the two correct equations to earn any marks.

Many will use $b - a = 6$, ignoring the ±, and obtain the correct values for a and b.
This scores B1 M1 m0 A1.

Q3	Solution	Marks	Total	Comments
(a)(i)	Mean of sample is **909.2**	B1		If wrong here, the B1 here may be earned for a correct value seen in (ii)
	Use of **1.96**	B1		AWRT
	$909.2 \pm 1.96 \times \frac{2.2}{\sqrt{8}}$	M1		Allow for M1 if AWFW 1.64 to 1.65 used instead of 1.96
	907.7, 910.7	A1	**4**	For both. AWRT
	Notes: **1** Seen use of s ⇒ B1 B1 M0 A0 max **2** Seen use of t ⇒ B1 B0 M0 A0 max **3** Seen use of t and s ⇒ B1 B0 M0 A0 max			
(ii)	$t_7 = 2.365$	B1		AWFW 2.36 to 2.37
	$s = 2.39$ or 2.24 (or $s^2 = 5.72$ or $5.00(5)$)	B1		AWRT
	$909.2 \pm (2.36 \text{ to } 2.37) \times se$	M1		Allow for M1 if AWFW 1.89 to 1.90 used instead of (2.36 to 2.37)
	where $se = 2.39/\sqrt{8}$ or $2.24/\sqrt{7}$			OE in terms of s^2
	907.2, 911.2	A1	**4**	For both. AWRT
	Notes: **1** Seen use of 2.2 ⇒ B1 B0 M0 A0 max **2** Seen use of z ⇒ B0 B1 M0 A0 max **3** Seen use of z and 2.2 ⇒ B0 B0 M0 A0			
(b)	**Both** confidence intervals are **above 907** so **mean/average** weight is probably acceptable	Edep1		OE Dependent on A1 in (i) and A1 in (ii). Must specify **both, 907** and **mean/average.**
	One of data values (or 905.6) is below 907 (or underweight)	E1	**2**	
		Total	**10**	

Note: In both (a)(i) and (ii), **where working is shown**, condone accuracy to more than 4 s.f.
Where working is not shown, if accurate to 4 s.f. allow B4. If not accurate to 4 s.f., award B1 for AWRT 908 – 911 in (i) and another B1 for AWRT 907 – 911 in (ii).

Q4	Solution	Marks	Total	Comments
(a)	(The 100 vehicles can be regarded as a) **random (**sample).	B1	**1**	Must say random and be about the sample. Do not penalise "and independent", but any mention of "normal" anywhere in (a) scores B0
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\bar{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	$z / t = \frac{(43.27 - 44.1)}{\frac{(3.055\text{ to }3.060)}{\sqrt{100}}}$ or $\frac{(43.27 - 44.1)}{\frac{(3.040\text{ to }3.045)}{\sqrt{99}}}$	M1 m1		Denominator is division of candidate's sd by $\sqrt{100}$ or $\sqrt{99}$ Numerator is $\pm(\bar{x} - 44.1$ *or* $40)$
	= –2.71…	A1		AWFW –2.695 to –2.735
	CV: z = **–2.32(63)** or t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	So test statistic in critical region. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1	**8**	Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average).
(c) (i)	Concluding that the mean speed has reduced (or changed) when in fact it has not	E1		Must be in context. Must refer to **mean** speed (μ)
(ii)	Concluding that the mean speed is still 44.1 when in fact it has reduced (or changed)	E1	**2**	Must be in context. Must refer to **mean** speed (μ)
		Total	**11**	

Note: (a) "It is random" is sufficient for B1.
"It is random and normally distributed" scores B0.
"The vehicles arrive at random" scores B0

The final A mark is not awarded for the negative statement "There is no significant evidence that the mean speed is 44.1" or equivalent. There **is** significant evidence of a reduction in the mean. A definite statement "the mean speed has reduced" is accepted for A1.

Alternative method for (b) using critical value for $\overline{x}$

Q4	Solution	Marks	Total	Comments
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\overline{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	CV: z = **–2.32(63)** **or** t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	$\overline{x}_{cv} = 44.1 - CV \times \frac{3.0579}{\sqrt{100}}$ or $\times \frac{3.0425}{\sqrt{99}}$	M1		Division of candidate's sd by $\sqrt{100}$ or $\sqrt{99}$
		m1		Rest of formula
	= 43.37 to 43.395	A1		AWFW 43.37 to 43.395
	43.27 < 43.37 or 43.395 So test statistic in critical region. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1		Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average).
			8	

Alternative method for (b) using confidence interval for $\overline{x}$

Q4	Solution	Marks	Total	Comments
(b)	H_0: $\mu_x = 44.1$ H_1: $\mu_x < 44.1$	B1		Both. Must be "Population mean", μ_x or μ.
	($\overline{x}$ =) **43.27**	B1		CAO
	sd = 3.0579 (var = 9.35 AWRT) or sd = 3.0425 (var = 9.26 AWRT)	B1		AWFW 3.055 to 3.060. AWFW 3.040 to 3.045
	CV: z = **–2.32(63)** or t = **–2.36(46)**	B1		AWFW –2.32 to –2.33 AWFW –2.36 to –2.37
	Upper limit of confidence interval = 43.27 + CV × $\frac{3.0579}{\sqrt{100}}$ or × $\frac{3.0425}{\sqrt{99}}$	M1		Division of candidate's sd by $\sqrt{100}$ or $\sqrt{99}$
		m1		Rest of formula
	= 43.975 to 43.999	A1		AWFW 43.975 to 43.999
	44.1 > 43.975 to 43.999 So previous mean above confidence interval. (Reject H_0), significant evidence that **mean speed has reduced.**	Adep1		Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average)
			8	

<table>
<tr><th>Q5</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>(a)</td><td>H_o:No association (between the age at which they had left education and the rate of income tax that they were paying)
(H_1:Association …..)

<table><tr><td>29.445</td><td>3.9</td><td>5.655</td></tr><tr><td>98.905</td><td>13.1</td><td>18.995</td></tr><tr><td>22.65</td><td>3</td><td>4.35</td></tr></table>
Combine last two columns

<table><tr><th colspan="2">Observed</th></tr><tr><th>≤ 16</th><th>> 16</th></tr><tr><td>32</td><td>7</td></tr><tr><td>102</td><td>29</td></tr><tr><td>17</td><td>13</td></tr></table> <table><tr><th colspan="2">Expected</th></tr><tr><th>≤ 16</th><th>> 16</th></tr><tr><td>29.445</td><td>9.555</td></tr><tr><td>98.905</td><td>32.095</td></tr><tr><td>22.65</td><td>7.35</td></tr></table>
$\Sigma (O_i - E_i)^2/E_i$ = 0.2217.. + 0.0968.. + 1.4093.. + 0.6832.. + 0.2984.. + 4.3431..

= 7.05

v = (3 – 1)(2 – 1) = 2

Crit val = 5.99(1)

(Reject H_o)
Significant evidence that there is an association between age at leaving education and rate of income tax paid.</td><td>B1

M1

M1

A1

m1

A1

B1

B1

Adep1</td><td>9</td><td>Allow “rate of tax independent of age of leaving” but no other words.

For at least H_o stated correctly.

Expected values attempted, seen here or after combining
(at least 4 correct (at least 2dp in 1st and 3rd columns))

Attempt at combining columns 2 & 3 (not just individual cells)

Combined columns numerically correct (six values)

Attempt at $\Sigma(O_i - E_i)^2/E_i$ dep on first M1 (at least 2 values correct to 3sf)
Can be implied by correct answer.

AWFW 7.0 to 7.1

Can be implied by correct answer.
Correct v or v = 4 from no combining

AWRT 5.99 or 9.488 from no combining

Dep on A1 for 7.05, B1 for 5.99.
For conclusion in context.</td></tr>
<tr><td>(b)</td><td>Belief supported (or equivalent).</td><td>E1</td><td>1</td><td>Must be supported by reference to stated O and E values, comparing 8 with 4.35 or 13 with 7.35, or other numerical justification, comparing $^{8}/_{29}$ (27.6%) with $^{17}/_{151}$ (11.3%) or $^{13}/_{49}$ (26.5%) with $^{17}/_{151}$ (11.3%).</td></tr>
<tr><td></td><td></td><td>Total</td><td>10</td><td></td></tr>
<tr><td></td><td colspan="4">No combining can score B1 M1 M0 A0 m1 A0 B1 B1 Adep0 = max of 5 out of 9 (gives 7.118)
Combining first and third rows can also score B1 M1 M0 A0 m1 A0 B1 B1 Adep0 = max of 5 out of 9 (gives 1.156). Use of Yates automatically loses m1 A1 and Adep1.</td></tr>
</table>

Note: (a) Final A mark is not awarded for the double negative statement “No significant evidence that there is no association ….”. There **is** significant evidence of an association. A definite conclusion “there is an association between age at leaving education and rate …” is accepted for A1 “**Association**” is the expected word. Use of **independent** must say “tax rate is not independent of age …”. No other words are accepted.

Q6	Solution	Marks	Total	Comments
(a)	$F(0.4) = \frac{0.4}{2} - \frac{0.16}{16} = 0.2 - 0.01 = \mathbf{0.19}$ $F(0.8) = \frac{0.8}{2} - \frac{0.64}{16} = 0.4 - 0.04 = \mathbf{0.36}$ $P(0.4 < X < 0.8) = 0.36 - 0.19 = \mathbf{0.17}$	M1 A1	**2**	For either, can be implied by correct answer. CAO
(b)	Clear correct use of differentiation of $F(x)$.	B1	1	AG Sight of $F'(x)$, $\frac{d}{dx}$, $\frac{dy}{dx}$ etc. = correct answer
(c)(i)	$E(X) = \int_0^4 (\frac{1}{2}x - \frac{1}{8}x^2)\,dx$ $= [\frac{1}{4}x^2 - \frac{1}{24}x^3]_0^4$ $= 4 - \frac{8}{3} = \mathbf{\frac{4}{3}}$	M1 A1 A1	**3**	Attempt at integrating $xf(x)$ (condone omission of limits and dx) Integration completed **correctly with limits** OE exact form
(ii)	$E(X^2) = \int_0^4 (\frac{1}{2}x^2 - \frac{1}{8}x^3)\,dx$ $= [\frac{1}{6}x^3 - \frac{1}{32}x^4]_0^4$ $= \frac{32}{3} - 8 = \mathbf{\frac{8}{3}}$ $Var(X) = E(X^2) - E(X)^2 = \frac{8}{3} - (\frac{4}{3})^2 \quad (= \mathbf{\frac{8}{9}})$	M1 A1 A1 A1	**4**	Attempt at integrating $x^2f(x)$ (condone omission of limits and dx) Integration completed **correctly with limits** OE exact form AG
(d)	$E(Y) = 3E(X) - 2 = 3 \times \frac{4}{3} - 2 = \mathbf{2}$ $Var(Y) = 3^2 \times Var(X) = 9 \times \frac{8}{9} = \mathbf{8}$	B1F B1	**2**	FT their (c)(i) provided $0 < E(X) < 4$ CAO
		Total	**12**	

Q7	Solution	Marks	Total	Comments
(a)	(I) a requires the "= 3" value using Po(2) $= (e^{-2} \times 2^3) \div 3!$ or 0.8571 – 0.6767 or 0.1804… from calculator = **0.180** (II) $b = 1 - P(\text{demand} \leq 3) = 1 - 0.8571 =$ **0.143** (III) $b = 1 - (0.135+0.271+0.271+ 0.180) =$ **0.143** (IV) $a = 1 - (0.135+ 0.271+ 0.271+ 0.143) =$ **0.180** (I) & (II) or (I) & (III) or (II) & (IV) SC If M0 can award B1 for $a + b = 0.323$ derived from sum of probabilities = 1	M1 m1 A1	**3**	One M1 for correct use of correct Poisson for either a or b. A dependent m1 for use of Poisson again for b or a or for subsequent use of probability sum = 1 A1 for both correct calculations AG
(b)	$E(X) =$ $1\times0.135+2\times0.271+3\times0.271+4\times0.180+5\times0.143$ $(= 0.135 + 0.542 + 0.813 + 0.72 + 0.715)$ = **2.925** $E(X^2) = 1^2\times0.135 + 2^2\times0.271 + 3^2\times0.271 +$ $4^2\times0.180 + 5^2\times0.143$ $(= 0.135 + 1.084 + 2.439 + 2.88 + 3.575)$ = **10.113** S.D. $= \sqrt{(10.113 - 2.925^2)} =$ **1.25**	M1 A1 M1 A1 B1	**5**	Evidence of at least two of the five products added OE AWFW 2.92 to 2.93 Evidence of at least two of the five products added AWRT 10.1 AWRT
(c)	$1 \times E(X) - 0.5 \times (5 - E(X))$ = £**1.89** **or profit/loss table** E(Profit) = –0.135 + 0.135 + 0.542 + 0.630 + 0.715 = £**1.89**	M1 A1 (M1) (A1)	**2**	Candidate's $E(X)$ AWRT Condone omission of '£' AWRT Condone omission of '£'

Profit	-1	0.5	2	3.5	5
$P(X = x)$	0.135	0.271	0.271	0.180	0.143

Note: (a) One of the three methods of getting 0.180 – formula, subtraction of two figures from tables, or direct calculation showing fourth decimal place (4) – must be seen before the M1 for use of Poisson is awarded. Similarly for 0.143 (0.1429) done using Poisson.

If value of $E(X)$ calculated in part (b) is used retrospectively in part (a) to calculate a and b, then only the SC B1 can be earned.

Q7	Solution	Marks	Total	Comments
(d)	New distribution x | 1 | 2 | 3 | 4 $P(X = x)$ | 0.135 | 0.271 | 0.271 | 0.323	M1 A1		P(demand ≥ 3)= P($X = 4$) Complete distribution (not necessarily in a table)
	$E(X) = 1\times0.135+2\times0.271+3\times0.271+4\times0.323$ $= 2.782$	A1		E(X) = 2.78(2) without shown working scores B3
	E(Profit) = $1 \times E(X) – 0.5 \times (4 – E(X))$ = £**2.17** (which is more than £1.89)	M1 A1		AWRT Condone omission of '£'
	or profit/loss table Profit | -0.5 | 1 | 2.5 | 4 $P(X = x)$ | 0.135 | 0.271 | 0.271 | 0.323	(M1) (A1) (A1)		Any two profit values correct P(demand ≥ 3)= P($X = 4$) Complete table
	E(Profit) = $–0.5\times0.135 + 1\times0.271 + 2.5\times0.271 + 4\times0.323$ = £**2.17** (which is more than £1.89)	(M1) (A1)	**5**	AWRT Condone omission of '£'
		Total	**15**	