	Proofs Worksheet (Higher) Mark Scheme	
1(a)	$4(2x-3)-2(2x+9)=8x-12-4x-18$	**[1]** Expanding brackets
	$=8x-4x-12-18$ $\equiv 4x-30$	**[1]** Grouping similar terms
1(b)	$(n-1)^2-(n-2)^2$ $(n^2-2n+1)-(n^2-4n+4)$	**[1]** Expanding brackets
	$=n^2-n^2-2n+4n+1-4$ $=2n-3$	**[1]** Grouping similar terms
1(c)	$(n+2)(n+2)-3(n+4)$ $=n^2+2n+2n+4-3n-12$	**[1]** Expanding brackets
	$=n^2+n-12+4$ $\equiv(n+4)(n-3)+4$	**[1]** Grouping similar terms
1(d)	$=3(n^2+2n-3)-3+3n$	**[1]** Expanding brackets
	$=3n^2+9n-12$ $\equiv(3n-3)(n+4)$	**[1]** Grouping and cancelling similar terms
2(a)	$=3n^2+9n+n+3-3n^2-7n$	**[1]** Expanding brackets
	$=3n+3$ $=3(n+1)$	**[1]** Grouping and cancelling similar terms
2(b)	$n^2+6n+9-3n-4$	**[1]** Expanding brackets
	$=n^2+3n+5$ $(n-3)^2-(2n+1)$	**[1]** Grouping and cancelling similar terms
2(c)	$=n^2-6n+9-2n-1$	**[1]** Expanding brackets
	$=n^2-8n+8$ $=(n-4)^2-8$	**[1]** Grouping and cancelling similar terms
3(a)	$2n\times 2m=4nm$	**[1]** Let n and m be any integers so that 2n and 2m are both even numbers.
	$=2(2nm)$ which is even	**[1]** Answer as required
3(b)	$(2n+1)(2m+1)=4nm+2n+2m+1$	**[1]** Let n and m be any integers so that 2n+1 and 2m+1 are both odd numbers.
	$=2(2nm+n+m)+1$ which is odd	**[1]** Answer as required
3(c)	$(2n+1)(2n+3)(2n+5)$ $=(4n^2+2n+6n+3)(2n+5)$	**[1]** Creation of correct algebraic expression
	$(4n^2+2n+6n+3)(2n+5)$ $=8n^3+36n^2+46n+15$	**[1]** Expanding brackets
	$=2(4n^3+18n^2+23n+7)+1$ simplifies to $2(n)+1$	**[1]** Factorising to show its always odd

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4(a)	$(2n+1)+(2m+1)+(2p+1)$	**[1]** Creation of correct algebraic expression
	$=2(n+m+p+1)+1$ $=2(x)+1$	**[1]** Factorising to show its always odd
4(b)	$(2n+1)^2+(2m+1)^2=4n^2+4n+1+4m^2+4m+1$	**[1]** Creation of correct algebraic expression and expanding brackets
	$2(2n^2+2m^2+2n+2m+1)$	**[1]** Factorising to show its always odd
4(c)	$(n+1)^2-n^2=n^2+2n+1-n^2$	**[1]** Creation of correct algebraic expression and expanding brackets
	$=2n+1$	**[1]** Simplifying to final answer
5(a)	$=n^2+3n+3n+9+3n-n^2-3n-12$	**[1]** Expanding brackets
	$=6n-3$ $=3(2n-1)$	**[1]** Grouping similar terms
5(b)	If a number is n, then the next number is $n+1$ The sum is therefore $n+n+1=2n+1$	**[1]** Demonstration of logic
	By definition , $2n$ is even, and so $2n+1$ must be odd.	**[1]** Final explanation
5(c)	$(5n)^2+(5n+5)^2=25n^2+25n^2+25n+25n+25$	**[1]** Expanding brackets
	$=50n^2+50n+25$	**[1]** Grouping similar terms
6(a)	$2n+(2n+2)+(2n+4)$	**[1]** Expanding brackets
	$=6n+6=6(n+1)$	**[1]** Grouping similar terms and factorisation to show it is divisible by 6
6(b)	$=16n^2+16n+4-4n^2-8n-4$	**[1]** Expanding brackets
	$=12n^2+8n=4(3n^2+2n)$	**[1]** Grouping similar terms and factorisation to show multiple of 4
6(c)	$=4n^2+12n+9-4n^2+12n-9$	**[1]** Expanding brackets
	$=24n=\ 8(3n)$	**[1]** Grouping similar terms and factorisation to show multiple of 8
7(a)	$(2n)^2+(2n+2)^2$	**[1]** Creation of correct algebraic expression
	$=4n^2+4n^2+8n+4$	**[1]** Expanding brackets
	$=4(2n^2+2n+1)$	**[1]** Grouping similar terms and factorisation to show multiple of 4

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<table>
<tr><td>7(b)</td><td>$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2$</td><td>[1] Creation of correct algebraic expression</td></tr>
<tr><td></td><td>$= 2n + 1$</td><td>[1] Accept same proof with $2n - 1$</td></tr>
<tr><td>8(a)</td><td>$7x - (2x+3)(x+2) = 7x - (2x^2 + 4x + 3x + 6)$
$= -2x^2 - 6 = -(2x^2 + 6)$</td><td>[1] Expanding brackets and grouping similar terms</td></tr>
<tr><td></td><td>$= -(2x^2 + 6)$</td><td>[1] Factorisation</td></tr>
<tr><td></td><td>$-(2x^2 + 6)$
is always positive, so multiplying by a negative means that the answer is always negative, so Tom is correct.</td><td>[1] Suitable explanation of logic</td></tr>
<tr><td>8(b)</td><td>Changing the +3 into a -3 or +2 into a -2 would give a negative number when expanding the brackets but would then be made positive when multiplying by -1.</td><td>[1]</td></tr>
<tr><td>9(a)</td><td>$= (7 \times 2)^{20} - (7 \times 3)^2 = 7^{20} \times 2^2 - 7^2 \times 3^2$</td><td>[1] Changing of powers</td></tr>
<tr><td></td><td>$= 7(7^{19} \times 2^2 - 7 \times 3^2)$</td><td>[1] Factorisation with 7 taken out</td></tr>
<tr><td></td><td>A factor of 7 can be taken out, so the answer must be divisible by 7, and therefore a multiple of 7.</td><td>[1] Suitable explanation of logic</td></tr>
<tr><td>9(b)</td><td>3^{60} is always odd, because you are multiplying odd numbers. Also 25 is odd. So the difference between two odd numbers, i.e. subtracting them, is always even.</td><td>[1] Suitable explanation of logic</td></tr>
<tr><td></td><td>$3^{60} - 25$ is going to be even. All even numbers are divisible by 2, so $3^{60} - 25$ is not prime.</td><td>[1] Suitable explanation of logic</td></tr>
<tr><td>10(a)</td><td>
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<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr>
<tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td></tr>
<tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td></tr>
<tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td></tr>
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Consider the new square shown,
$(44 - 33) \times (43 - 34) = 11 \times 9 = 99$</td><td>[1] Any example shown</td></tr>
<tr><td>10(b)</td><td>Taking the top left number as n, the other numbers can be written in terms of n.
$(n + 11 - n) \times (n + 10 - (n+1))$
$= 11 \times (n + 10 - n - 1) = 11 \times 9$
$= 99$</td><td>[1]
[1]
[1]
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<tr><td>n</td><td>n+1</td></tr>
<tr><td>n+10</td><td>n+11</td></tr>
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