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A Level maths Card Answers

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Pure

P1

$64^{3x-1} = (4^3)^{3x-1} = 4^{9x-3}$

$\therefore y = 9x - 3$

$\frac{3x^4+4x^3}{\sqrt{x^3}} = \frac{3x^4}{\sqrt{x^3}} + \frac{4x^3}{\sqrt{x^3}} = (3x^4 \times x^{-\frac{3}{2}}) + (4x^3 \times x^{-\frac{3}{2}}) = 3x^{\frac{5}{2}} + 4x^{\frac{3}{2}}$

$\therefore a = \frac{5}{2}$ and $b = \frac{3}{2}$

P2

$(x+4)(2x-8) = 2x^2 - 8x + 8x - 32 = 2x^2 - 32$

$x^3 + 5x^2 + 6x = x(x^2 + 5x + 6) = x(x+2)(x+3)$

P3

$\frac{3}{\sqrt{27}-\sqrt{18}} = \frac{3}{\sqrt{27}-\sqrt{18}} \times \frac{\sqrt{27}+\sqrt{18}}{\sqrt{27}+\sqrt{18}} = \frac{3\sqrt{27}+3\sqrt{18}}{27-18} = \frac{(3\times\sqrt{9}\times\sqrt{3})+(3\times\sqrt{9}\times\sqrt{2})}{9} = \frac{9(\sqrt{3}+\sqrt{2})}{9} = \sqrt{3}+\sqrt{2}$

P4

$5x^2 + 20x + 12$

$= 5(x^2 + 4x) + 12$

$= 5((x+2)^2 - 4) + 12$

$= 5(x+2)^2 - 20 + 12$

$= 5(x+2)^2 - 8$

$\therefore 5x^2 + 20x + 12 = 0 \Rightarrow 5(x+2)^2 - 8 = 0$

$\therefore (x+2)^2 = \frac{8}{5}$

$\therefore x + 2 = \pm\frac{2\sqrt{10}}{5}$

$\therefore x = -2 \pm \frac{2\sqrt{10}}{5}$

$3x^2 + 11x + 10 = (3x+5)(x+2) = 0$

$\therefore x = -2$ or $x = -\frac{5}{3}$

P5

a) $3(x+4)(x-1) - (x+4)^2 = 0$

$\Rightarrow 3x^2 + 9x - 12 - x^2 - 8x - 16 = 0$

$\Rightarrow 2x^2 + x - 28 = 0$

Using the quadratic formula with $a = 2, b = 1$ and $c = -28$:

$x = \frac{-1\pm\sqrt{(-1)^2-4\times2\times-28}}{2\times2} = \frac{-1\pm\sqrt{225}}{4} = \frac{-1\pm15}{4}$

$\therefore x = \frac{7}{2}$ or $x = -4$

b) Draw the function $f(x) = x^2 + 7x + 10$ and label the turning point and coordinates for which the function crosses the axes.

$f(0) = 0^2 + 7(0) + 10 = 10 \Rightarrow f(x)$ crosses the y-axis at $(0,10)$

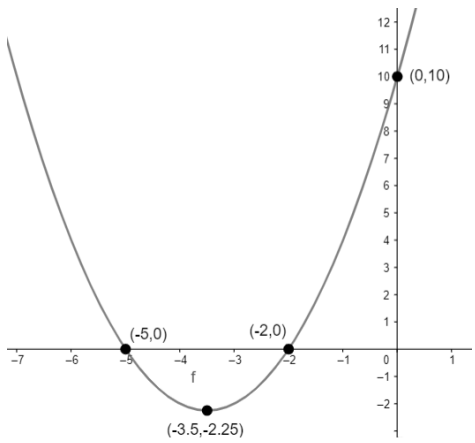
$f(x) = 0 \Rightarrow x^2 + 7x + 10 = 0 \Rightarrow (x+5)(x+2) = 0 \Rightarrow x = -5$ or $x = -2$

$\therefore$ The function crosses the x-axis at coordinates: $(-2,0)$ and $(-5,0)$

By completing the square, we can determine the turning point of the function:

$f(x) = \left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + 10 = \left(x + \frac{7}{2}\right)^2 - \frac{9}{4}$

$\therefore$ The turning point has coordinates $\left(-\frac{7}{2}, -\frac{9}{4}\right)$



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P6

a) $3x^2 + xy - 70 = 0$ and $y = 3 - 2x$

$\therefore 3x^2 + x(3 - 2x) - 70 = 0$

$\therefore 3x^2 + 3x - 2x^2 - 70 = 0$

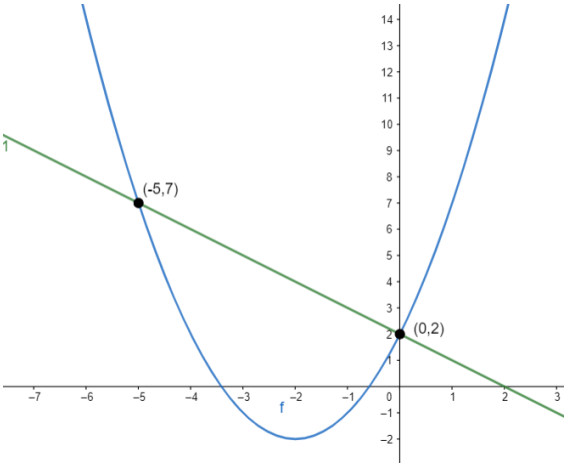
$\therefore x^2 + 3x - 70 = 0$

Factorising, we have: $(x + 10)(x - 7) = 0 \Rightarrow x = 7$ or $x = -10$

When $x = 7, y = 3 - 2(7) = -11$

When $x = -10, y = 3 - 2(-10) = 23$

b)



$$x + y = 2 \Rightarrow y = 2 - x$$

Shown are the graphs of $y = 2 - x$ and $y = x^2 + 4x + 2$

The solutions to the simultaneous equations are the x-coordinates and corresponding y-coordinates of the points of intersection between the 2 functions.

P7

$\frac{3}{x-4} < 2$

You must multiply both sides of the inequality by $(x - 4)^2$ as this is always positive and ensures the inequality sign remains unchanged.

$\frac{3(x-4)^2}{(x-4)} < 2(x - 4)^2$

$3(x - 4) < 2(x - 4)^2$

$3x - 12 < 2(x^2 - 8x + 16)$

$3x - 12 < 2x^2 - 16x + 32$

$2x^2 - 19x + 44 > 0$

Consider: $2x^2 - 19x + 44 = 0$

Using the quadratic formula with $a = 2,\ b = -19$ and $c = 44$:

$x = 4$ or $x = \frac{11}{2}$ (These are the critical values)

Hence, the solution to our inequality is: $x < 4, x > \frac{11}{2}$

In set notation: $\{x\colon\ x < 4, x > \frac{11}{2}\}$

P8

a)

$\frac{x^3-x^2-10x-8}{2x^3-x^2-22x-24}$

Note: $(-1)^3 - (-1)^2 - 10(-1) - 8 = 0$

$\therefore x = -1$ is a root of the numerator $\Rightarrow x + 1$ is a factor of $x^3 - x^2 - 10x - 8$

$$\begin{array}{r|l} & x^2 - 2x - 8 \\ \hline x+1 & x^3 - x^2 - 10x - 8 \\ & \underline{x^3 + x^2} \\ & -2x^2 - 10x \\ & \underline{-2x^2 - 2x} \\ & -8x - 8 \\ & \underline{-8x - 8} \\ & 0 \end{array}$$

$\therefore x^3 - x^2 - 10x - 8 = (x + 1)(x^2 - 2x - 8) = (x + 1)(x - 4)(x + 2)$

$2(-2)^3 - (-2)^2 - 22(-2) - 24 = 0$

$\therefore x = -2$ is a root of the denominator $\Rightarrow x + 2$ is a factor of $2x^3 - x^2 - 22x - 24$

$$\begin{array}{r|l} & 2x^2 - 5x - 12 \\ \hline x+2 & 2x^3 - x^2 - 22x - 24 \\ & \underline{2x^3 + 4x^2} \\ & -5x^2 - 22x \\ & \underline{-5x^2 - 10x} \\ & -12x - 24 \\ & \underline{-12x - 24} \\ & 0 \end{array}$$

$\therefore 2x^3 - x^2 - 22x - 24 = (x + 2)(2x^2 - 5x - 12) = (x + 2)(2x + 3)(x - 4)$

Hence, we can write the cubic equations as a product of their factors and simplify:

$\frac{x^3-x^2-10x-8}{2x^3-x^2-22x-24} = \frac{(x+1)(x-4)(x+2)}{(x+2)(2x+3)(x-4)} = \frac{x+1}{2x+3}$

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b)

$$
\begin{array}{r|l}
 & 2x^2 \quad + 3x - 1 \\
\hline
x^2 - 2x - 3 & 2x^4 - x^3 - 13x^2 - 7x + 2 \\
 & \underline{2x^4 - 4x^3 - 6x^2} \\
 & \quad 3x^3 - 7x^2 - 7x \\
 & \quad \underline{3x^3 - 6x^2 - 9x} \\
 & \qquad -x^2 + 2x + 2 \\
 & \qquad \underline{-x^2 + 2x + 3} \\
 & \qquad\qquad -1
\end{array}
$$

Hence, $\frac{2x^4-x^3-13x^2-7x+2}{(x-3)(x+1)} = 2x^2 + 3x - 1 - \frac{1}{x^2-2x-3} = 2x^2 + 3x - 1 - \frac{1}{(x-3)(x+1)}$

Let $\frac{-1}{(x-3)(x+1)} = \frac{D}{x-3} + \frac{E}{x+1}$

$\therefore -1 = D(x+1) + E(x-3)$

$x = 3 \Rightarrow -1 = 4D \Rightarrow D = -\frac{1}{4}$

$x = -1 \Rightarrow -1 = -4E \Rightarrow E = \frac{1}{4}$

$\therefore \frac{2x^4-x^3-13x^2-7x+2}{(x-3)(x+1)} = 2x^2 + 3x - 1 - \frac{1}{4(x-3)} + \frac{1}{4(x+1)}$

$\therefore A = 2, B = 3, C = -1, D = -\frac{1}{4}$ and $E = \frac{1}{4}$

P9

$$
\begin{array}{r|l}
 & 5x + 1 \\
\hline
x^2 + 5x + 6 & 5x^3 + 26x^2 + 51x + 20 \\
 & \underline{5x^3 + 25x^2 + 30x} \\
 & \quad x^2 + 21x + 20 \\
 & \quad \underline{x^2 + \ 5x + \ 6} \\
 & \qquad 16x + 14
\end{array}
$$

$\therefore \frac{5x^3+26x^2+51x+20}{x^2+5x+6} = 5x + 1 + \frac{16x+14}{(x+2)(x+3)}$

Let $\frac{16x+14}{(x+2)(x+3)} = \frac{C}{x+2} + \frac{D}{x+3}$

$\therefore 16x + 14 = C(x+3) + D(x+2)$

$x = -2 \Rightarrow 16(-2) + 14 = C \Rightarrow C = -18$

$x = -3 \Rightarrow 16(-3) + 14 = -D \Rightarrow D = 34$

Hence, $\frac{5x^3+26x^2+51x+20}{x^2+5x+6} = 5x + 1 - \frac{18}{x+2} + \frac{34}{x+3} \Rightarrow A = 5, B = 1, C = -18$ and $D = 34$

P10

a) Let $x = 1$ and $y = -1$

$\therefore\ 1 + (-1) \geq \sqrt{(1)^2 + (-1)^2} \Rightarrow 0 \geq \sqrt{2}$

This is a false statement; hence the statement has been disproven by counterexample.

b) Completing the square on the given expression:

$p^2 - 6p + 10 = (p-3)^2 - 9 + 10 = (p-3)^2 + 1$

given $(p-3)^2$ is always positive, it follows that $(p-3)^2 + 1$ must also be always positive.

Hence, we have proven $p^2 - 6p + 10$ is positive for all real values of p.

P11

Prove by contradiction that if n^2 is odd then n is also odd.

Let us assume that n^2 is odd.

So, we can set $n = 2k$ where k is an integer.

$n^2 = 4k^2$ which is clearly even and therefore contradicts our assumption.

This shows that n^2 is even, however we assumed that n^2 is odd. So, if n^2 is to be odd, n must be odd.

P12

a)

x^2 coefficient is given by $(4C2)\ \times 3^2 \times (\alpha x)^2 = 54\alpha^2 x^2 = 108x^2$

$\therefore 54\alpha^2 = 108$

$\alpha^2 = 2$

$\therefore \alpha = \pm\sqrt{2}$

b)

$$(2+2x)^3 = (2x)^3 + 3(2x)^2(2) + 3(2x)(2)^2 + (2)^3$$
$$= 8x^3 + 24x^2 + 24x + 8$$

$$(x+1)(8x^3 + 24x^2 + 24x + 8)$$
$$= 8x^4 + 8x^3 + 24x^3 + 24x^2 + 24x^2 + 24x + 8x + 8$$

$$= 8x^4 + 32x^3 + 48x^2 + 32x + 8$$

P13

a)

$$\left(4 - \frac{x}{5}\right)^9 = 4^9 + \binom{9}{1}4^8\left(-\frac{x}{5}\right) + \binom{9}{2}4^7\left(-\frac{x}{5}\right)^2 + \binom{9}{3}4^6\left(-\frac{x}{5}\right)^3 + \cdots$$

$$= 262144 - \frac{589824}{5}x + \frac{589824}{25}x^2 - \frac{344064}{125}x^3 + \cdots$$

b)

$4 - \frac{x}{5} = 3.96 \Rightarrow 20 - x = 19.8 \Rightarrow x = 0.2$

To approximate 3.96^9, we can substitute $x = 0.2$ into our expansion from a):

$3.96^9 \approx 262144 - \frac{589824}{5}(0.2) + \frac{589824}{25}(0.2)^2 - \frac{344064}{125}(0.2)^3 = 239473$ (nearest integer)

$[3.96^9 = 239473.0653]$

P14

a)

$(6-12x)^{-\frac{3}{2}} = (6(1-2x))^{-\frac{3}{2}} = 6^{-\frac{3}{2}}(1-2x)^{-\frac{3}{2}}$

$= 6^{-\frac{3}{2}}\left(1 + 3x + \frac{(-1.5)(-2.5)(-2x)^2}{2!} + \frac{(-1.5)(-2.5)(-3.5)(-2x)^3}{3!}\right) + \cdots$

$= 6^{-\frac{3}{2}}\left(1 + 3x + \frac{15}{2}x^2 + \frac{35}{2}x^3\right) + \cdots$

b)

Note: $\frac{1}{(\sqrt{3})^3} = 3^{-\frac{3}{2}}$

$6 - 12x = 3 \Rightarrow x = \frac{1}{4}$

We can substitute this value of x into our approximation from part a) to obtain an estimate for $\frac{1}{(\sqrt{3})^3}$

$\therefore \frac{1}{(\sqrt{3})^3} \approx 6^{-\frac{3}{2}}\left(1 + 3\left(\frac{1}{4}\right) + \frac{15}{2}\left(\frac{1}{4}\right)^2 + \frac{35}{2}\left(\frac{1}{4}\right)^3\right) = 0.17\ (2\ d.p.)$

P15

a)

$(3-x)(3x-1) = -3x^2 + 10x - 3,$

$-15x^2 + 25x - 14 = 5(-3x^2 + 10x - 3) - 25x + 1$

Hence, $\frac{-15x^2+25x-14}{(3-x)(3x-1)} = \frac{5(-3x^2+10x-3)-25x+1}{-3x^2+10x-3} = \frac{5(-3x^2+10x-3)}{-3x^2+10x-3} + \frac{-25x+1}{-3x^2+10x-3} = 5 + \frac{-25x+1}{(3-x)(3x-1)}$

Let $\frac{-25x+1}{(3-x)(3x-1)} = \frac{b}{3-x} + \frac{c}{3x-1}$

$\therefore -25x + 1 = b(3x-1) + c(3-x)$

$x = 3 \Rightarrow\ -74 = 8b \Rightarrow b = -\frac{37}{4}$

$x = \frac{1}{3} \Rightarrow -\frac{22}{3} = \frac{8}{3}c \Rightarrow c = \frac{-11}{4}$

$\Rightarrow \frac{-15x^2+25x-14}{(3-x)(3x-1)} = 5 + \frac{-\frac{37}{4}}{3-x} + \frac{\frac{-11}{4}}{3x-1} = 5 + \frac{-37}{4(3-x)} - \frac{11}{4(3x-1)}$

Hence, $a = 5, b = -\frac{37}{4}$ and $c = -\frac{11}{4}$

b)

You must use the binomial expansion on each fraction separately:

$5 + \frac{-37}{4(3-x)} - \frac{11}{4(3x-1)} = 5 - \frac{37}{4}(3-x)^{-1} - \frac{11}{4}(3x-1)^{-1} + \cdots$

$= 5 - \frac{37}{4}\left(3\left(1 - \frac{x}{3}\right)\right)^{-1} - \frac{11}{4}\left(-1(1-3x)\right)^{-1} + \cdots$

$= 5 - \frac{37}{12}\left(1 - \frac{x}{3}\right)^{-1} + \frac{11}{4}(1-3x)^{-1} + \cdots$

$= 5 - \frac{37}{12}\left(1 + \frac{x}{3} + \frac{(-1)(-2)\left(-\frac{x}{3}\right)^2}{2!}\right) + \frac{11}{4}\left(1 + 3x + \frac{(-1)(-2)(-3x)^2}{2!}\right) + \cdots$

$= \frac{14}{3} + \frac{65}{9}x + \frac{659}{27}x^2 + \cdots$

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P16

To draw the function, note the general shape of a quartic and compute where the function crosses each axis.

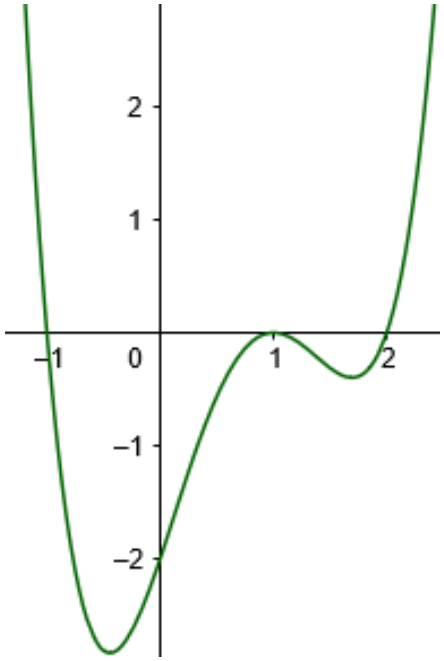
$(x^2 - x - 2)\,(x-1)^2 = (x+1)\,(x-2)\,(x-1)^2$

Hence, we can deduce that the graph crosses the x-axis at $(-1, 0)$, $(2, 0)$ and $(1, 0)$.

To deduce the y-intercept, we can multiply all constant terms in each factor together:

$1 \times -2 \times -1 \times -1 = -2$

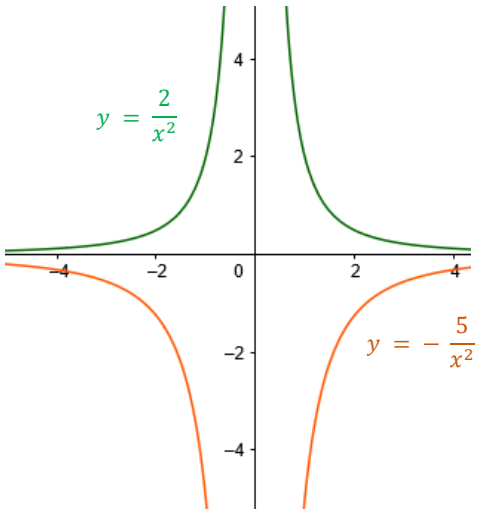
Therefore, the function crosses the y-axis at $(0, -2)$



The intersection points with the axes are:
$(-1, 0), (1, 0), (2, 0)$ and $(0, -2)$

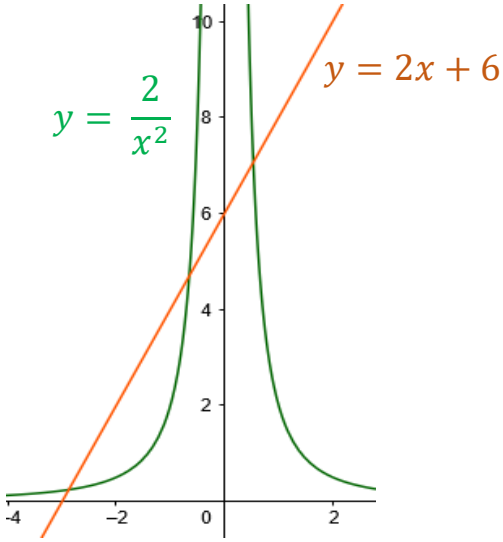
P17

Note: the graphs of $y = \frac{2}{x^2}$ and $y = -\frac{5}{x^2}$ have asymptotes at $x = 0$ and $y = 0$



P18

a)

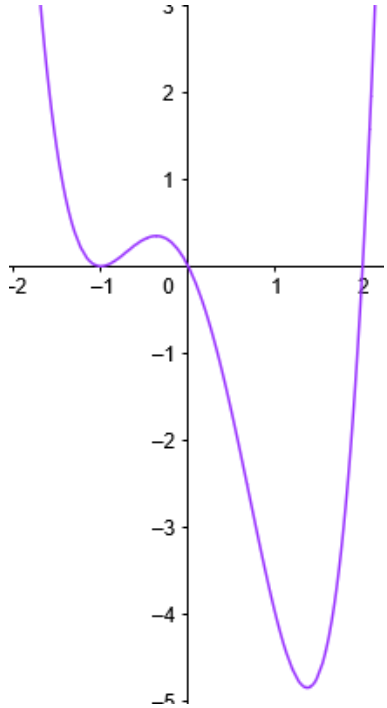


b) Since the two graphs cross each other at 3 points, there are 3 solutions to the equation: $\frac{2}{x^2} = 2x + 6$

P19

a)

When sketching graphs remember to find where the graph intersects each axis.



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b)

$y = (x+b)\,(x-2+b)\,(x+1+b)^2$ is a translation of $y = x(x-2)(x+1)^2$ by vector $\begin{pmatrix}-b\\0\end{pmatrix}$

$y = x\,(x-2)\,(x+1)^2$ intersects the x-axis at points $(-1,0), (0,0)$ and $(2,0)$

If $b = -2$, then the graph will be translated 2 units to the right and the point $(-1,0)$ will translate to $(1,0)$

If $b = -1$, then the graph will be translated 1 unit to the right and the point $(0,0)$ will translate to $(1,0)$

If $b = 1$, then the graph will be translated 1 unit to the left and the point $(2,0)$ will translate to $(1,0)$

Hence, $b = -2, -1$ or 1

P20

a)

Gradient of line segment $DE = \frac{\frac{7}{2}-1}{3--2} = \frac{1}{2}$

Gradient of line segment $DF = \frac{6-1}{8--2} = \frac{1}{2}$

Both line segments DE and DF have the same gradient and they have a common point E

Hence D, E and F are collinear.

b)

$y - 1 = \frac{1}{2}(x+2)$

$y - 1 = \frac{x}{2} + 1$

$2y - 2 = x + 2$

$x - 2y + 4 = 0$

Hence, $a = 1$, $b = -2$ and $c = 4$

P21

a)

$4y - x + 8 = 0$

At point A, $y = 0$:

$4(0) - x + 8 = 0 \Rightarrow 8 = x \Rightarrow A$ has coordinates $(8,0)$

b)

$L_1: 4y - x + 8 = 0$

$\therefore 4y = x - 8$

$\therefore y = \frac{x}{4} - 2$

gradient of L_1 is $\frac{1}{4}$ and since L_2 is perpendicular, L_2 has gradient $-\frac{1}{\left(\frac{1}{4}\right)} = -4$

L_2 also passes through $(8,0)$, hence we can find L_2 using $y - y_1 = m(x - x_1)$:

$L_2: \quad y - 0 = -4(x-8)$

$\therefore y = -4x + 32$

c)

$y = -4x + 6$

P22

$\sqrt{(6-2)^2 + (8-y)^2} = 4\sqrt{10}$

$\sqrt{16 + y^2 - 16y + 64} = 4\sqrt{10}$

$\sqrt{y^2 - 16y + 80} = 4\sqrt{10}$

$y^2 - 16y + 80 = 160$

$y^2 - 16y - 80 = 0$

$(y-20)(y+4) = 0$

$y = 20$ or $y = -4$

P23

a) $(21, 69.8)$ $(8, 46.4)$

$\frac{69.8 - 46.4}{21 - 8} = 1.8$

$F = 1.8C + b$

$69.8 = 1.8(21) + b$

$32 = b$

$F = 1.8C + 32$

b) a is the gradient, for every increase of 1°C, there is an increase of 1.8°F

b is the y intercept of the graph, so when it is 0°C, it is 32°F

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P24

Midpoint of XY is $(5, \frac{4+q}{2})$ gradient of $-\frac{1}{2}$

$$\frac{q-4}{3-7} = -\frac{1}{2} \Rightarrow q - 4 = 2 \Rightarrow q = 6$$

Therefore the midpoint of XY is $(5, 5)$

$y = 2x + b$

$5 = 2(5) + b$

$-5 = b$

$b = -5$ and $q = 6$

P25

$(x-k)^2 + y^2 = 72$

$(3-k)^2 + 6^2 = 72$

$k^2 - 6k + 9 + 36 = 72$

$k^2 - 6k - 27 = 0$

$(k-9)(k+3) = 0$

$k = 9$ or -3

P26

a) $A(1, 12)$ $B(5, 10)$

The gradient of AB is $\frac{10-12}{5-1} = -\frac{1}{2}$

Therefore the gradient of l is 2

The midpoint of AB is $(3, 11)$

The equation of the line l is

$y - 11 = 2(x - 3)$
$y - 11 = 2x - 6$

$y = 2x + 5$

b) If C is the centre of the circle, then the equation of the circle is

$(x-1)^2 + (y-7)^2 = r^2$

$(CB)^{\rightarrow} = \sqrt{4^2 + 3^2} = 5$

Therefore, the equation of the circle is

$(x-1)^2 + (y-7)^2 = 25$

c)

Length $CM = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

Length $AM = \sqrt{2^2 + 1^2} = \sqrt{5}$

Area of ABC = 2 x Area of AMC = $2\sqrt{5} \times \sqrt{5} = 10$

P27

$\overrightarrow{AB} = \sqrt{4^2 + 8^2} = 4\sqrt{5}$

$\overrightarrow{AC} = \sqrt{8^2 + 6^2} = 10$

$\overrightarrow{BC} = \sqrt{4^2 + 2^2} = 2\sqrt{5}$

$(4\sqrt{5})^2 + (2\sqrt{5})^2 = 10^2$, therefore, AC is the diameter of the circle

The midpoint of AC is the centre of the circle.

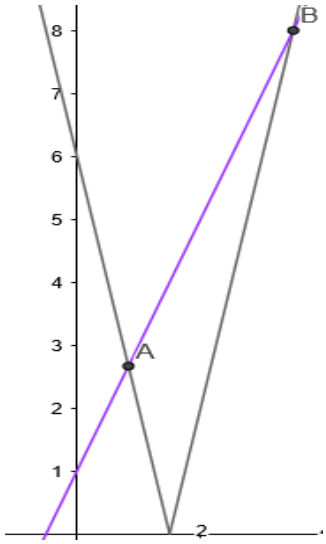
Midpoint of AC is $(1, 2)$ and the diameter of the circle is 10, meaning the radius is 5

Therefore, the equation of the circle is

$(x-1)^2 + (y-2)^2 = 25$

P28

a) When sketching $|4x - 6|$, first you must sketch 4x – 6 and then reflect the section below the x axis into the x axis



b) $4x - 6 = 2x + 1$

$2x = 7$

$x = \frac{7}{2}$

$6 - 4x = 2x + 1$

$5 = 6x$

$\frac{5}{6} = x$

$\frac{5}{6} < x < \frac{7}{2}$

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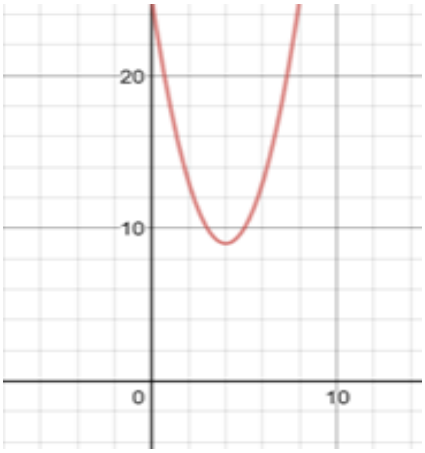
P29



From the graph, the minimum is $x = 4$, therefore for $h(x)$ to be a one to one function, the smallest value in the domain is 4.

Therefore $a = 4$

P30

a) $g(x) = 4x + 3$

$f(4x+3) = 3(4x+3)^2 = 3(16x^2 + 24x + 9) = 48x^2 + 72x + 27$

b) $y = 48x^2 + 72x + 27$

$y = 48\left(x^2 + \frac{3}{2}x\right) + 27 = 48\left(\left(x + \frac{3}{4}\right)^2 - \frac{9}{16}\right) + 27 = 48\left(x + \frac{3}{4}\right)^2 - 27 + 27$

$y = 48\left(x + \frac{3}{4}\right)^2$

$\frac{y}{48} = \left(x + \frac{3}{4}\right)^2$

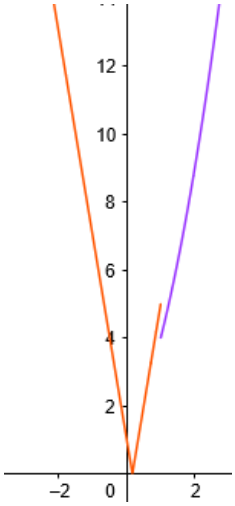
$\sqrt{\frac{y}{48}} = x + \frac{3}{4}$

$\sqrt{\frac{y}{48}} - \frac{3}{4} = x$

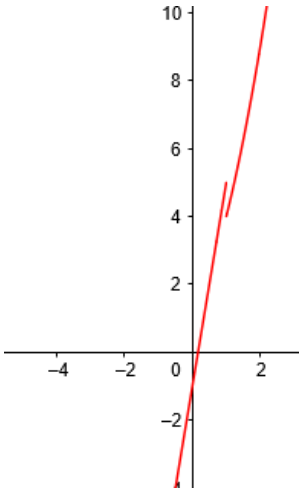
Therefore $(fg)^{-1}(x) = \sqrt{\frac{x}{48}} - \frac{3}{4}$

P31

a)



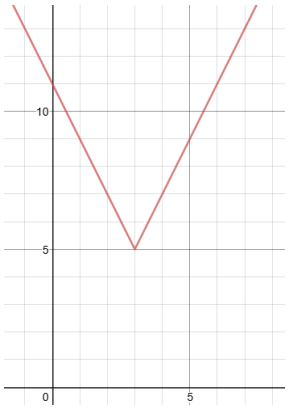
b)



Remember y = $|f(x)|$ affects the y values, any negative y value will be made positive

P32

a)



b) The range of a function is the complete set of all possible values for the variable y.
So, the range is: $g(x) \geq 5$

c) $2(x-3) + 5 = x + 3$
$2x - 1 = x + 3$
$x = 4$
$-2(x+3) + 5 = x + 3$
$-2x - 1 = x + 3$
$8 = 3x$
$\frac{8}{3} = x$
$x = 4 \text{ or } \frac{8}{3}$

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P33

a) $x = \ln t$

$e^x = t$

$y = (e^x)^2 - 2$

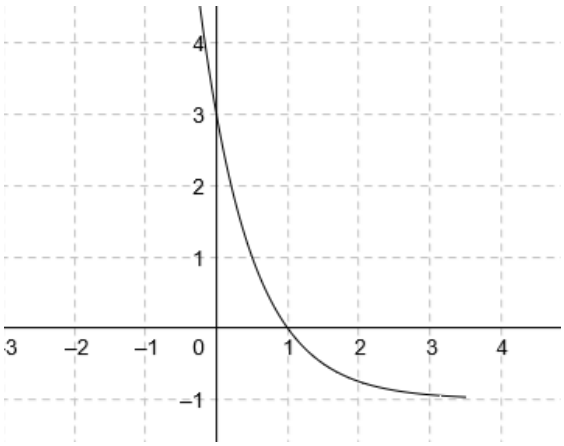
$y = e^{2x} - 2$

b) $x - 1 = \sin t$
$y + 2 = \cos t$

$\sin^2 t + \cos^2 t = 1$

$(x-1)^2 + (y+2)^2 = 1$

P34



a)

b) $0 = 1 - \frac{1}{2}t$
$2 = t$
$y = 2^2 - 1 = 3$
Therefore point A has coordinates of $(0, 3)$

c) $0 = 2^t - 1$
$1 = 2^t \quad t = 0$
$x = 1 - \frac{1}{2}(0)$
$x = 1$
Point B has coordinates of $(1, 0)$

P35

a) $\tan\theta = \frac{y}{x} = \frac{180}{800}$

$\theta = \tan^{-1}(\frac{180}{800}) = 12.7°\ (1\ d.p)$

b) $x = (v\cos\theta)t$

$= 45\cos(12.7) \times t = 43.9t\ \ (3sf)$

$y = (v\sin\theta) \times t$

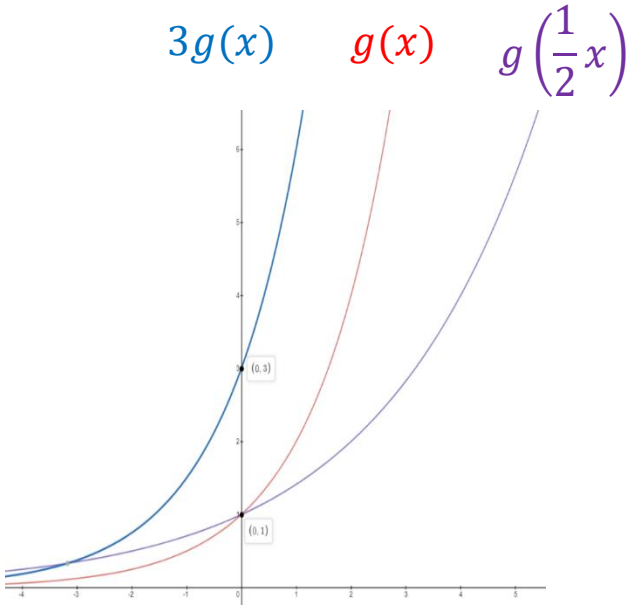
$= 45\sin(12.7) \times t = 9.89t\ (3sf)$

c) $y = 9.88t \qquad t = 10$

The vertical distance is 9.88×10, which equals 98.8 metres

So, the plane has climbed 98.8 metres after 10 seconds

P36



P37

$f(x) = e^{0.1x}$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$

$\therefore f'(x) = 0.1e^{0.1x}$

$f'(10) = 0.1e^{0.1(10)} = 0.1e$

$\therefore$ The tangent to $f(x)$ at $(10,\ e)$ has gradient $0.1e$

Our tangent has equation: $y - e = 0.1e(x - 10)$

$\Rightarrow y = (0.1e)x$

$x = 0 \Rightarrow y = 0.1e(0) = 0$

Hence, we have shown the tangent to pass through the origin as required.

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P38

a) $560m = 0.56km$

$\therefore y = 0.56 \Rightarrow x = e^{-0.23(0.56)} = 0.879\ (3\ d.p.)$

b) $\frac{d}{dx}(e^{kx}) = ke^{kx}$

$x = e^{-0.23y}$, x is the 'pressure' and y is the 'height'.

$\therefore \frac{dx}{dy} = -0.23e^{-0.23y}$

$= -0.23x$

$\therefore$ The rate of change of atmospheric pressure with respect to the height above the planet is proportional to the atmospheric pressure.

c)As $y \to \infty$, $x \to 0$

This is because as $y \to \infty$, $e^{-0.23y} \to 0$

Hence, as the distance from the planet's surface increases, the pressure decreases.

P39

a)$log_{10} \sqrt[3]{100} = log_{10} \sqrt[3]{10^2} = log_{10} 10^{\frac{2}{3}} = \frac{2}{3}$

b)$x = log_3 56$

$3^3 = 27$ and $3^4 = 81$

$27 < 56 < 81$

$\therefore 3 < x < 4$ as required.

c)Using a calculator, $x = 3.66\ (3\ s.f.)$

d)$log_3(t^2 + 5t + 15) = 2$

$\therefore 3^2 = t^2 + 5t + 15$

$\therefore t^2 + 5t + 6 = 0$

$\therefore (t+3)(t+2) = 0 \Rightarrow t = -3$ or $t = -2$

P40

$$log_a\left(\frac{pr^3}{\sqrt[3]{q^5}}\right) = log_a\left(pr^3q^{-\frac{5}{3}}\right)$$

$$= log_a(p) + log_a(r^3) + log_a(q^{-\frac{5}{3}})$$

$$= log_a(p) + 3log_a(r) - \frac{5}{3}log_a(q)$$

P41

$2(5^{2y}) - 12(5^y) - 14 = 0$

This is a quadratic equation in terms of 5^y.

This can be seen more clearly by writing the equation as:

$2(5^y)^2 - 12(5^y) - 14 = 0$

$\therefore (5^y)^2 - 6(5^y) - 7 = 0$

$\therefore (5^y - 7)(5^y + 1) = 0$

$\therefore 5^y = 7$ or $5^y = -1$

We can dismiss $5^y = -1$ as this has no real solutions.

Consider: $5^y = 7$

Hence, $y = \log_5 7 = 1.21\ (3\ s.f.)$

P42

a)$g(x) = ln(3 + x) - 2$

$g(0) = ln(3) - 2$

Hence, $g(x)$ crosses the y-axis at coordinates: $(0, \ln(3) - 2)$

b)Let $g(x) = 0$

$\therefore ln(3 + x) - 2 = 0$

$\therefore ln(3 + x) = 2$

$\therefore e^2 = 3 + x$

$\therefore x = e^2 - 3$

Hence, $g(x)$ crosses the x-axis at coordinates: $(e^2 - 3,\ 0)$

P43

a)The equation is a straight line so takes the form: $y = mx + c$

Hence, $log\ a = 2\ log\ b - 0.234$

b)From our equation, we can deduce:

$a = 10^{(2\ log\ b) - 0.234}$

$\therefore a = 10^{(\log b^2) - 0.234}$

$\therefore a = 10^{-0.234}b^2$

$\therefore a = 0.583b^2$

Hence, $p = 0.583$ and $q = 2$

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P44

$g(x) = px^3$

$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$

$\therefore g'(x) = \lim_{h\to 0} \frac{p(x+h)^3-px^3}{h}$

$= \lim_{h\to 0} \frac{p(x^3+3x^2h+3xh^2+h^3)-px^3}{h}$

$= \lim_{h\to 0} \frac{px^3+3px^2h+3pxh^2+ph^3-px^3}{h}$

$= \lim_{h\to 0} \frac{h(3px^2+3pxh+ph^2)}{h}$

$= \lim_{h\to 0} 3px^2 + 3pxh + ph^2$

$= 3px^2$ as required.

P45

a)$f(x) = x^2 + 6x - 9$

- If $f(x) = x^n$, then $f'(x) = nx^{n-1}$
- If $f(x) = ax$, then $f'(x) = a$
- If $f(x) = a$, then $f'(x) = 0$

$\therefore f'(x) = 2x + 6$

b)$f'(5) = 2(5) + 6 = 16$

Hence, the gradient of the tangent to $f(x)$ at $x = 5$ is 16.

c) Let $f'(x) = 8$

$\therefore 2x + 6 = 8$

$\therefore x = 1$

$f(1) = 1^2 + 6(1) - 9 = -2$

Hence, at coordinates $(1, \ -2)$, the gradient is 8.

P46

$y = 4x^2$

$\therefore \frac{dy}{dx} = 8x$

When $x = 2, \frac{dy}{dx} = 8(2) = 16$

$\therefore$ The normal to the curve at $x = 2$ has gradient $-\frac{1}{16}$

$x = 2 \Rightarrow y = 4(2)^2 = 16$

Hence, the normal has equation:

$y - 16 = -\frac{1}{16}(x-2) \Rightarrow y = -\frac{x}{16} + \frac{129}{8}$

Equating the normal and curve:

$4x^2 = -\frac{x}{16} + \frac{129}{8}$

$\therefore 4x^2 + \frac{x}{16} - \frac{129}{8} = 0$

Using our calculator: $x = 2$ or $x = -2.016\ (3\ d.p.)$

When $x = -2.016 \Rightarrow y = 4(-2.016)^2 = 16.251\ (3\ d.p.)$

Hence B has coordinates: $(-2.016,\ 16.251)$

P47

$f(x) = 6 - 3x(1 + x^2)$

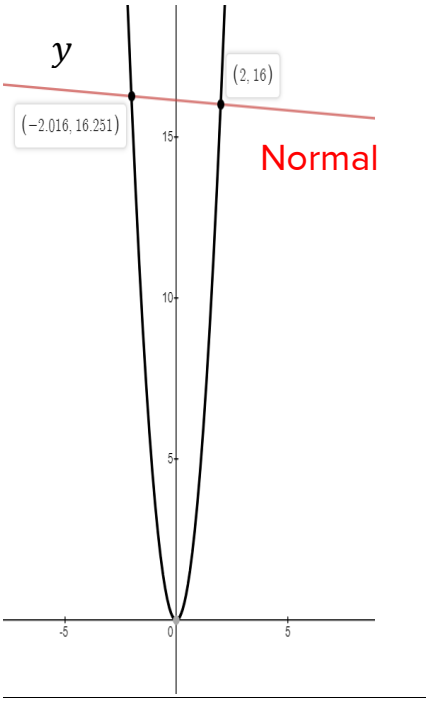
If $f'(x) \leq 0$ for all the values of x such that $a < x < b$, then $f(x)$ is decreasing on $[a, b]$

$f(x) = 6 - 3x - 3x^3$

$\therefore f'(x) = -3 - 9x^2$

$= -3(1 + 3x^2)$

Hence $f'(x) < 0$ for all $x \Rightarrow f(x)$ is a decreasing function for all x.



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P48

a) $y = 11 - 3x - \frac{4}{x^3}$

$\therefore y = 11 - 3x - 4x^{-3}$

$\therefore \frac{dy}{dx} = -3 + 12x^{-4}$

$= -3 + \frac{12}{x^4}$

$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0$

$\therefore -3x^4 + 12 = 0$

$\therefore x^4 = 4 \Rightarrow x = \pm\sqrt{2}$

$x = \sqrt{2} \Rightarrow y = 11 - 3(\sqrt{2}) - \frac{4}{(\sqrt{2})^3} = 11 - 4\sqrt{2}$

$x = -\sqrt{2} \Rightarrow y = 11 - 3(-\sqrt{2}) - \frac{4}{(-\sqrt{2})^3} = 11 + 4\sqrt{2}$

Hence the coordinates of the turning points are:

$(\sqrt{2},\ 11 - 4\sqrt{2})$ and $(-\sqrt{2},\ 11 + 4\sqrt{2})$

b) $\frac{dy}{dx} = -3 + 12x^{-4}$

$\therefore \frac{d^2y}{dx^2} = -48x^{-5}$

c) $x = \sqrt{2} \Rightarrow \frac{d^2y}{dx^2} = -48(\sqrt{2})^{-5} = -8.48 \ldots < 0$

Hence $(\sqrt{2},\ 11 - 4\sqrt{2})$ is a local maximum.

$x = -\sqrt{2} \Rightarrow \frac{d^2y}{dx^2} = -48(-\sqrt{2})^{-5} = 8.48 \ldots > 0$

Hence $(-\sqrt{2},\ 11 + 4\sqrt{2})$ is a local minimum.

P49

Let $f(x) = \cos x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$

$= \lim_{h \to 0} \left(\frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h} \right)$

$= \lim_{h \to 0} \left(\cos(x) \left(\frac{\cos(h) - 1}{h} \right) - \sin(x) \frac{\sin(h)}{h} \right)$

Assuming: $\lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$ and $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$,

We can conclude $f'(x) = -\sin x$ as required.

P50

a) $g(x) = 2e^{3x} - \ln x^3 - 14$

$\therefore g(x) = 2e^{3x} - 3\ln x - 14$

- $\frac{d}{dx}(e^{kx}) = ke^{kx}$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\therefore g'(x) = 6e^{3x} - \frac{3}{x}$

b) There is a stationary point at $x = a$

Hence $g'(a) = 0$ must be a true statement.

$\therefore 6e^{3a} - \frac{3}{a} = 0$

$\therefore 6ae^{3a} - 3 = 0$

$\therefore 6ae^{3a} = 3$

$\therefore ae^{3a} = \frac{1}{2}$ as required.

P51

$f(x) = \frac{3}{(1-6x)^3} = 3(1-6x)^{-3}$

$\therefore f'(x) = 3(-3)(1-6x)^{-4}(-6)$

$= \frac{54}{(1-6x)^4}$

$f'(0) = \frac{54}{(1-6(0))^4} = 54$

Hence, the gradient of the tangent to $f(x)$ at $x = 0$ is 54

$f(0) = \frac{3}{(1-6(0))^3} = 3 \Rightarrow A$ has coordinates: $(0,\ 3)$

Hence, the tangent to A has equation:

$y - 3 = 54(x - 0) \Rightarrow y = 54x + 3 \Rightarrow 54x - y + 3 = 0$

$\therefore a = 54,\ b = -1$ and $c = 3$

P52

$y = 4e^{3t}\sqrt[3]{1+4t} = 4e^{3t}(1+4t)^{\frac{1}{3}}$

The product rule states: $y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$, where u and v are functions of x

$\therefore \frac{dy}{dt} = 12e^{3t}(1+4t)^{\frac{1}{3}} + 4e^{3t}\left(\frac{1}{3}\right)(1+4t)^{-\frac{2}{3}}(4)$

$= e^{3t}\left(12(1+4t)^{\frac{1}{3}} + \frac{16}{3}(1+4t)^{-\frac{2}{3}}\right)$

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P53

$f(x) = \frac{3\sin 5x}{e^{2+x}}$

The quotient rule states: If $f(x) = \frac{g(x)}{h(x)} \Rightarrow f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}$

Let $g(x) = 3\sin 5x \Rightarrow g'(x) = 15\cos 5x$

Let $h(x) = e^{2+x} = e^2e^x \Rightarrow h'(x) = e^2e^x$

Hence, $f'(x) = \frac{(15\cos 5x)(e^2e^x) - (3\sin 5x)(e^2e^x)}{[e^2e^x]^2}$

$f'(x) = 0 \Rightarrow \frac{(15\cos 5x)(e^2e^x) - (3\sin 5x)(e^2e^x)}{[e^2e^x]^2} = 0$

$\therefore (15\cos 5x)(e^2e^x) - (3\sin 5x)(e^2e^x) = 0$

$\therefore 15\cos 5x - 3\sin 5x = 0$

$\therefore 15\cos 5x = 3\sin 5x$

$\therefore \tan 5x = 5 \Rightarrow 5x = \arctan(5) = 1.373\ldots \Rightarrow x = 0.275$ $(3\ s.f.)$

$f(0.275) = \frac{3\sin 5(0.275)}{e^{2+(0.275)}} = 0.302\ (3\ s.f.)$

Hence, the coordinates of the given turning point are: $(0.275,\ 0.302)$

P54

$y = \arccos\left(\frac{x}{x-1}\right) = \arccos(x(x-1)^{-1})$

$\frac{d}{dt}(\arccos t) = \frac{-1}{\sqrt{1-t^2}}$

$\therefore \frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{x}{x-1}\right)^2}} \times [(x-1)^{-1} + x(-1)(x-1)^{-2}]$

$= \frac{-1}{\sqrt{1-\left(\frac{x}{x-1}\right)^2}} \times -\frac{1}{(x-1)^2}$

$= \frac{1}{(x-1)^2\sqrt{1-\left(\frac{x}{x-1}\right)^2}}$

Note: The question only specified you to find $\frac{dy}{dx}$ and this is a hard derivative to determine. The question is testing your ability to differentiate, not the algebraic manipulation at the end!

P55

a) $x = \sin\theta,\ y = \sin\left(\theta + \frac{\pi}{6}\right),\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

If x and y are given as functions of the parameter θ, then $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$\frac{dy}{d\theta} = \cos\left(\theta + \frac{\pi}{6}\right)$

$\frac{dx}{d\theta} = \cos\theta$

Hence, $\frac{dy}{dx} = \frac{\cos\left(\theta + \frac{\pi}{6}\right)}{\cos\theta}$

When $\theta = \frac{\pi}{6}$:

- $\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\sqrt{3}}{3}$
- $x = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$
- $y = \sin\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Hence, the equation of our normal is:

$y - \frac{\sqrt{3}}{2} = -\frac{1}{\left(\frac{\sqrt{3}}{3}\right)}\left(x - \frac{1}{2}\right)$

$\therefore y = -x\sqrt{3} + \sqrt{3}$

b) $y = \sin\left(\theta + \frac{\pi}{6}\right)$

$\therefore y = (\sin\theta)\cos\left(\frac{\pi}{6}\right) + (\cos\theta)\sin\left(\frac{\pi}{6}\right)$

$\therefore y = \frac{x\sqrt{3}}{2} + \frac{\sqrt{1-x^2}}{2} \Rightarrow a = \frac{\sqrt{3}}{2}$ and $b = 2$

P56

a) $\left(\frac{3}{2}\right)^x + y^2 = 4xy$

$\frac{d}{dx}(a^x) = \ln a\, a^x$

$\therefore \ln\frac{3}{2}\left(\frac{3}{2}\right)^x + 2y\frac{dy}{dx} = 4x\frac{dy}{dx} + 4y$

$\therefore \frac{dy}{dx}(4x - 2y) = \ln\frac{3}{2}\left(\frac{3}{2}\right)^x - 4y$

$\therefore \frac{dy}{dx} = \frac{\ln\frac{3}{2}\left(\frac{3}{2}\right)^x - 4y}{4x - 2y}$

b) Consider $\left(\frac{3}{2}\right)^x + y^2 = 4xy$

$x = 2 \Rightarrow \left(\frac{3}{2}\right)^2 + y^2 = 4(2)y$

$\therefore y^2 - 8y + 2.25 = 0$

Using our calculator: $y = \frac{8 \pm \sqrt{55}}{2}$

$x = 2,\ y = \frac{8+\sqrt{55}}{2} \Rightarrow \frac{dy}{dx} = 4.03\ (3\ s.f.)$

$x = 2,\ y = \frac{8-\sqrt{55}}{2} \Rightarrow \frac{dy}{dx} = -0.0344\ (3\ s.f.)$

c) There are two normals to the curve when $x = 2$. For simplicity, we shall only consider the normal for $\left(2, \frac{8+\sqrt{55}}{2}\right)$

Using our answer to part b) we can deduce the normal has equation:

$y - \left(\frac{8+\sqrt{55}}{2}\right) = -\frac{1}{4.03}(x-2) \Rightarrow y = -0.25x + 8.20$ (all numbers are rounded to 2 $d.p.$)

P57

$g(x) = 3x^2 \ln x$

$\therefore g'(x) = 6x \ln x + \frac{3x^2}{x} = 6x \ln x + 3x$

$\therefore g''(x) = 6 \ln x + \frac{6x}{x} + 3 = 6 \ln x + 9$

Let $g''(x) = 0 \Rightarrow 6 \ln x + 9 = 0$

$\therefore \ln x = -\frac{3}{2} \Rightarrow x = e^{-\frac{3}{2}}$

$g\left(e^{-\frac{3}{2}}\right) = 3\left(e^{-\frac{3}{2}}\right)^2 \left(\ln e^{-\frac{3}{2}}\right) = -\frac{9}{2e^3}$

Hence, the coordinates of the point of inflection are: $\left(e^{-\frac{3}{2}}, -\frac{9}{2e^3}\right)$

There must also only be one point of inflection as there exists only one value of x such that $g''(x)$ changes from negative to positive as $g''(x) = 6 \ln x + 9$ is a continuous function.

P58

$\frac{dV}{dt} \propto -\sqrt{V} \Rightarrow \frac{dV}{dt} = -x\sqrt{V}$ for some constant x.

$V = \pi r^2 a$, for another constant r.

$\therefore \frac{dV}{da} = \pi r^2 \Rightarrow \frac{da}{dV} = \frac{1}{\pi r^2}$

$\frac{da}{dt} = \frac{da}{dV} \times \frac{dV}{dt}$

$\therefore \frac{da}{dt} = \frac{1}{\pi r^2} \times -x\sqrt{V} = \frac{-x}{\left(\frac{1}{\pi r^2}\right)}\sqrt{V}$

But $V = \pi r^2 a$,

$\therefore \frac{da}{dt} = \frac{-x}{\left(\frac{1}{\pi r^2}\right)}\sqrt{\pi r^2 a}$

$= -x\, \pi r^2 \sqrt{\pi r^2}\sqrt{a}$

$= -x(\pi r^2)^{\frac{3}{2}}\sqrt{a}$

x is a constant and r is a constant.

Hence, $-x(\pi r^2)^{\frac{3}{2}}$ is also a constant.

Thus, we have shown $\frac{da}{dt} = -k\sqrt{a}$ for some constant k as required.

P59

$\int \frac{a}{4x^2} + ab\, dx = -\frac{12}{x} + 12x + C$

Consider the LHS:

$\int \frac{a}{4x^2} + ab\, dx$

$= \frac{a}{4}\int \frac{1}{x^2}\, dx + \int ab\, dx$

$= \frac{a}{4}\int x^{-2}\, dx + \int ab\, dx$

$= \left(\frac{a}{4} \times \frac{x^{-1}}{-1}\right) + (ab)x + C$

$= -\frac{a}{4x} + (ab)x + C$

Comparing to the RHS:

$-\frac{a}{4x} = -\frac{12}{x} \Rightarrow \frac{a}{4} = 12 \Rightarrow a = 48$

$(ab)x = 12x \Rightarrow 48b = 12 \Rightarrow b = \frac{1}{4}$

P60

$\frac{dy}{dx} = 9x^{-\frac{1}{2}} + 6x\sqrt{x}$

Consider $\int 9x^{-\frac{1}{2}} + 6x\sqrt{x}\, dx$

$= \int 9x^{-\frac{1}{2}} + 6x^{\frac{3}{2}}\, dx$

$= \frac{9x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + \frac{6x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + C$

$18x^{\frac{1}{2}} + \frac{12}{5}x^{\frac{5}{2}} + C$

Let $y = 18x^{\frac{1}{2}} + \frac{12}{5}x^{\frac{5}{2}} + C$

$y = 20, x = 4 \Rightarrow 20 = 18(4)^{\frac{1}{2}} + \frac{12}{5}(4)^{\frac{5}{2}} + C \Rightarrow C = -\frac{464}{5}$

Hence, $y = 18x^{\frac{1}{2}} + \frac{12}{5}x^{\frac{5}{2}} - \frac{464}{5}$

P61

$\int_1^k \frac{1}{\sqrt[3]{t}}\, dt = \frac{9}{2}$

Consider the LHS:

$\int_1^k \frac{1}{\sqrt[3]{t}}\, dt$

$= \int_1^k t^{-\frac{1}{3}}\, dt$

$= \left[\frac{3}{2}t^{\frac{2}{3}}\right]_1^k$

$= \frac{3}{2}\left(k^{\frac{2}{3}} - 1^{\frac{2}{3}}\right)$

$= \frac{3}{2}k^{\frac{2}{3}} - \frac{3}{2}$

Comparing to the RHS:

$\frac{3}{2}k^{\frac{2}{3}} - \frac{3}{2} = \frac{9}{2}$

$\therefore k^{\frac{2}{3}} = 4 \Rightarrow k = 8$

P62

$f(x) = \sqrt{x} + \frac{x}{5} - 5$

We must determine where the function crosses the x-axis.

Let $f(x) = 0 \Rightarrow \sqrt{x} + \frac{x}{5} - 5 = 0$

$\therefore \sqrt{x} + \frac{x}{5} = 5$

$\therefore 5\sqrt{x} + x = 25$

$\therefore x = 25 - 5\sqrt{x}$

Using the iterative formula: $x_{n+1} = 25 - 5\sqrt{x_n}$

$f(x)$ crosses the x-axis at $x = 9.549\ (3\ d.p.)$

Hence the area is given by $-\int_0^{9.549} \sqrt{x} + \frac{x}{5} - 5\ dx + \int_{9.549}^{15} \sqrt{x} + \frac{x}{5} - 5\ dx$

Consider: $\int \sqrt{x} + \frac{x}{5} - 5\ dx$

$= \int x^{\frac{1}{2}} + \frac{x}{5} - 5\ dx$

$= \frac{2}{3}x^{\frac{3}{2}} + \frac{x^2}{10} - 5x + C$

Hence, our area is given by: $-\left[\frac{2}{3}x^{\frac{3}{2}} + \frac{x^2}{10} - 5x\right]_0^{9.549} + \left[\frac{2}{3}x^{\frac{3}{2}} + \frac{x^2}{10} - 5x\right]_{9.549}^{15}$

$= -\left(\frac{2}{3}(9.549)^{\frac{3}{2}} + \frac{(9.549)^2}{10} - 5(9.549)\right) + \left(\frac{2}{3}(15)^{\frac{3}{2}} + \frac{(15)^2}{10} - 5(15)\right) - \left(\frac{2}{3}(9.549)^{\frac{3}{2}} + \frac{(9.549)^2}{10} - 5(9.549)\right)$

$= 24.1\ (3\ s.f.)$

Note: There was no need to substitute in the lower limit of 0 as we can see this will result in 0.

P63

a) $f(x) = 3 - x$ and $g(x) = x^2 + 4x + 3$

Let $f(x) = g(x) \Rightarrow 3 - x = x^2 + 4x + 3$

$\therefore x^2 + 5x = 0 \Rightarrow x(x+5) = 0 \Rightarrow x = 0$ or $x = -5$

$f(0) = 3 - 0 = 3$

$f(-5) = 3 - (-5) = 8$

Hence the coordinates of intersection are: $(0,\ 3)$ and $(-5,\ 8)$

b) $g(x) = (x+3)(x+1) \Rightarrow g(x)$ cuts the x-axis at $x = -3$ and $x = -1$

Let the smaller area under the x-axis be called R_1

Hence, $R_1 = -\int_{-3}^{-1} x^2 + 4x + 3\ dx$

$= -\left[\frac{x^3}{3} + 2x^2 + 3x\right]_{-3}^{-1}$

$= -\left[\frac{(-1)^3}{3} + 2(-1)^2 + 3(-1) - \left(\frac{(-3)^3}{3} + 2(-3)^2 + 3(-3)\right)\right]$

$= \frac{4}{3}$

Let the larger area above the x-axis be called R_2

Hence, $R_2 = \int_{-5}^{0} 3 - x\ dx - \int_{-5}^{-3} x^2 + 4x + 3\ dx - \int_{-1}^{0} x^2 + 4x + 3\ dx$

$= \left[3x - \frac{x^2}{2}\right]_{-5}^{0} - \left[\frac{x^3}{3} + 2x^2 + 3x\right]_{-5}^{-3} - \left[\frac{x^3}{3} + 2x^2 + 3x\right]_{-1}^{0}$

Substituting in our limits $\Rightarrow R_2 = \frac{55}{2} - \frac{20}{3} - \frac{4}{3} = \frac{39}{2}$

The total area $= R_1 + R_2 = \frac{4}{3} + \frac{39}{2} = \frac{125}{6}$

Note: This required a lot of problem-solving. Do not worry if you found this difficult!

P64

$\int_{ln(1)}^{ln(a)} 6e^x - 5e^{-x}\, dx = \frac{222}{7}$

$\therefore [6e^x + 5e^{-x}]_{ln(1)}^{ln(a)} = \frac{222}{7}$

$\therefore [6e^x + 5e^{-x}]^{ln(a)} - [6e^x + 5e^{-x}]^{ln(1)} = \frac{222}{7}$

$\therefore 6e^{ln\, a} + 5e^{-ln\, a} - 6e^{ln\, 1} - 5e^{-ln\, 1} = \frac{222}{7}$

$\therefore 6a + 5a^{-1} - 6 - 5 = \frac{222}{7}$

$\therefore 6a + \frac{5}{a} = \frac{222}{77} + 11$

$6a^2 - \frac{299}{7}a + 5 = 0$

$a = 7 \text{ or } a = \frac{5}{42}$

P65

$\int_b^3 (3x-1)^6\, dx = \frac{699\,008}{7}$

Consider the LHS:

$\int_b^3 (3x-1)^6\, dx$

$= \frac{1}{3}\int_b^3 3(3x-1)^6\, dx$

$= \frac{1}{3}\left[\frac{(3x-1)^7}{7}\right]_b^3$

$= \frac{1}{21}(8^7 - (3b-1)^7)$

Comparing to the RHS:

$\frac{1}{21}(8^7 - (3b-1)^7) = \frac{699\,008}{7}$

$\therefore 8^7 - (3b-1)^7 = 2\,097\,024$

$\therefore (3b-1)^7 = 128$

$\therefore 3b - 1 = 2$

$\therefore b = 1$

P66

a) $sin^4\, y = (sin^2\, y)(sin^2\, y)$

$[cos2x = 1 - 2\, sin^2\, x]$

$\therefore sin^4\, y = \left(\frac{1}{2} - \frac{cos\, 2y}{2}\right)\left(\frac{1}{2} - \frac{cos\, 2y}{2}\right)$

$= \frac{1}{4} - \frac{cos\, 2y}{2} + \frac{cos^2\, 2y}{4}$

$[cos2x = 1 - 2\, sin^2\, x]$

$\therefore sin^4\, y = \frac{1}{4} - \frac{cos\, 2y}{2} + \frac{\left(\frac{1}{2} + \frac{cos\, 4y}{2}\right)}{4}$

$= \frac{3}{8} - \frac{cos\, 2y}{2} + \frac{cos\, 4y}{8}$ as required.

b) Hence, $\int sin^4\, y\, dy = \int \frac{3}{8} - \frac{cos\, 2y}{2} + \frac{cos\, 4y}{8}\, dy$

$= \frac{3y}{8} - \frac{\sin 2y}{4} + \frac{\sin 4y}{32} + C$

P67

$\int 2(15x^2 + 9x)\sqrt{5x^3 + \frac{9}{2}x^2}\, dx$

$= 2\int (15x^2 + 9x)\left(5x^3 + \frac{9}{2}x^2\right)^{\frac{1}{2}}\, dx$

$= \frac{2\left(5x^3 + \frac{9}{2}x^2\right)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$

$= \frac{4\left(5x^3 + \frac{9}{2}x^2\right)^{\frac{3}{2}}}{3} + C$

P68

$\int \frac{x^2}{\sqrt{x-1}}\, dx$

$u^2 = x - 1 \Rightarrow x^2 = (u^2+1)^2$

$\therefore 2u\frac{du}{dx} = 1 \Rightarrow \frac{du}{dx} = \frac{1}{2u} \Rightarrow dx = 2u\, du$

Hence, our integral becomes:

$\int \frac{(u^2+1)^2}{\sqrt{u^2}}\, 2u\, du$

$= 2\int (u^2+1)^2\, du$

$= 2\int u^4 + 2u^2 + 1\, du$

$= \frac{2u^5}{5} + \frac{4u^3}{3} + 2u + C$

Note: $u = (x-1)^{\frac{1}{2}}$

Hence, we have:

$\frac{2[(x-1)^{\frac{1}{2}}]^5}{5} + \frac{4[(x-1)^{\frac{1}{2}}]^3}{3} + 2(x-1)^{\frac{1}{2}} + C$

$= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}} + C$

P69

Consider: $\int x \ln x \ dx$

Let $u = \ln x$ and $v' = x$

$\therefore \frac{du}{dx} = \frac{1}{x}$ and $v = \frac{x^2}{2}$

$\therefore \int x \ln x \ dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x} \ dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x \, dx$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

Hence, $\int_1^6 x \ln x \, dx = \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^6$

$= \left(\frac{6^2}{2}\ln 6 - \frac{6^2}{4}\right) - \left(\frac{1^2}{2}\ln 1 - \frac{1^2}{4}\right)$

$= 18 \ln 6 - \frac{35}{4}$

$= 18 \ln 2 + 18 \ln 3 - \frac{35}{4}$

Hence, $a = 18$ and $b = -\frac{35}{4}$

P70

a) $h(x) = \frac{12+6x-2x^2}{x^3+2x^2} = \frac{12+6x-2x^2}{x^2(x+2)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x+2}$

$\therefore 12 + 6x - 2x^2 = A(x+2) + Bx(x+2) + Cx^2$

$x = 0 \Rightarrow 12 = 2A \Rightarrow A = 6$

$x = -2 \Rightarrow -8 = 4C \Rightarrow C = -2$

Comparing x^2 coefficients, we can see that $B = 0$

Hence, $h(x) = \frac{6}{x^2} - \frac{2}{x+2}$

b) $\int_1^4 h(x) \, dx = \int_1^4 \frac{6}{x^2} - \frac{2}{x+2} \ dx$

$= \int_1^4 6x^{-2} - 2\left(\frac{1}{x+2}\right) dx$

$= \left[-\frac{6}{x} - 2\ln|x+2|\right]_1^4$

$= \left(-\frac{6}{4} - 2\ln 6\right) - (-6 - 2\ln 3)$

$= \frac{9}{2} + 2(\ln 3 - \ln 6)$

$= \frac{9}{2} + 2\ln\frac{1}{2}$

$= \frac{9}{2} - 2\ln 2$

P71

$g(x) = x\cos x$

Let $g(x) = 0 \Rightarrow x\cos x = 0$

$\therefore x = 0$ or $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

From the graph, we can see we must only consider: $x = 0, \frac{\pi}{2}$ and $\frac{3\pi}{2}$

Consider: $\int x\cos x \, dx$

Let $u = x$ and $\frac{dv}{dx} = \cos x$

$\therefore \frac{du}{dx} = 1$ and $v = \sin x$

Hence $\int x\cos x \, dx = x \sin x - \int \sin x \ dx$

$= x\sin x + \cos x + C$

Our area is hence given by:

$[x\sin x + \cos x]_0^{\frac{\pi}{2}} - [x\sin x + \cos x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$

$= \left[\left(\frac{\pi}{2} + 0\right) - (0+1)\right] - \left[\left(-\frac{3\pi}{2} + 0\right) - \left(\frac{\pi}{2} + 0\right)\right] = \frac{5\pi}{2} - 1$

P72

a) $f(x) = \frac{12x-15}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}$

$\therefore 12x - 15 = A(x-3) + B(2x+1)$

$x = 3 \Rightarrow 21 = 7B \Rightarrow B = 3$

$x = -\frac{1}{2} \Rightarrow -21 = -\frac{7}{2}A \Rightarrow A = 6$

Hence, $f(x) = \frac{6}{2x+1} + \frac{3}{x-3}$

b) $f\left(\frac{5}{4}\right) = \frac{6}{2\left(\frac{5}{4}\right)+1} + \frac{3}{\left(\frac{5}{4}\right)-3} = \frac{12}{7} - \frac{12}{7} = 0$

Hence, we can conclude the function crosses the x-axis at $x = \frac{5}{4}$

c) $\int_0^{\frac{5}{4}} f(x) \ dx = \int_0^{\frac{5}{4}} \frac{6}{2x+1} + \frac{3}{x-3} \ dx$

$= \int_0^{\frac{5}{4}} 3\left(\frac{2}{2x+1}\right) + 3\left(\frac{1}{x-3}\right) \ dx$

$= [3\ln|2x+1| + 3\ln|x-3|]_0^{\frac{5}{4}}$

$= [3\ln|(2x+1)(x-3)|]_0^{\frac{5}{4}}$

$= 3(\ln -\frac{49}{8} - \ln -3)$

$= 3\ln\frac{49}{24}$

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d)

x	0	0.25	0.5	0.75	1	1.25
y	5	2.91	1.8	1.07	0.5	0

e) Using the table:

$\int_0^{\frac{5}{4}} f(x)\ dx \approx \frac{0.25}{2}(5 + 2(2.91 + 1.8 + 1.07 + 0.5) + 0)$

$\approx 2.2\ (2\ s.f)$

f) Percentage error $= \frac{|2.2 - (3\ \ln\frac{49}{24})|}{(3\ \ln\frac{49}{24})} \times 100$

$= 2.7\%$

P73

a) $x = 4\sin^2 u,\ y = 2\cos u\ ,\ 0 \le u \le \frac{\pi}{2}$

$y = 0 \Rightarrow 2\cos u = 0 \Rightarrow u = \frac{\pi}{2}$ (upper limit)

$y = 2 \Rightarrow 2\cos u = 2 \Rightarrow \cos u = 1 \Rightarrow u = 0$ (lower limit)

$\frac{dx}{du} = 8\sin u\cos u$

Area $= \int y\frac{dx}{du}\ du$

$\therefore$ Area $= \int_0^{\frac{\pi}{2}}(2\cos u)(8\sin u\cos u)\ du$

$= 16\int_0^{\frac{\pi}{2}}\sin u\cos^2 u\ du$ as required.

b) Area $= 16\int_0^{\frac{\pi}{2}}\sin u\cos^2 u\ du = \left[\frac{-16\cos^3 u}{3}\right]_0^{\frac{\pi}{2}} =$
$-\frac{16}{3}(0)^3 - -\frac{16}{3}(1)^3 = \frac{16}{3}$

P74

$\frac{dy}{dx} = 3xe^{-y}$

$\therefore \int e^y\ dy = \int 3x\ dx$

$\therefore e^y = \frac{3x^2}{2} + C$

$x = 2,\ y = ln\,4$

$\therefore e^{\ln 4} = \frac{3}{2}(2)^2 + C \Rightarrow C = -2$

$\therefore e^y = \frac{3x^2}{2} - 2$

$\therefore y = \ln\left|\frac{3x^2}{2} - 2\right|$

P75

a) $\frac{dx}{dT} = \frac{4-x}{125} = \frac{x-4}{-125}$

$\therefore \int \frac{1}{x-4}\ dx = -\frac{1}{125}\ dT$

$\therefore \ln|x-4| = -\frac{1}{125}T + C$

$\therefore x - 4 = e^{-\frac{1}{125}T+C}$

$\therefore x - 4 = e^C e^{-0.008T}$

$\therefore x - 4 = ke^{-0.008T}$

$\therefore x = ke^{-0.008T} + 4$ as required.

$x = 14,\ T = 0 \Rightarrow 14 = k + 4 \Rightarrow k = 10$

$\therefore x = 10e^{-0.008T} + 4$

b) When the temperature is **7**°C, we have:

$7 = 10e^{-0.008T} + 4$

$\therefore \frac{3}{10} = e^{-0.008T}$

$\therefore -0.008T = \ln\frac{3}{10}$

$\therefore T = \frac{\ln\frac{3}{10}}{-0.008} = 150.496\ldots$

Hence it takes 150 minutes for the beverage to cool down to 7°C.

c) $\lim_{T\to\infty}(10e^{-0.008T} + 4) = 10(0) + 4 = 4$

Because the refrigerator is set to 4°C, we have shown that over time, the temperature of the beverage will tend towards that of the refrigerator.

P76

$f(1) = (1)^3 + 3(1)^2 - 5(1) - 2 = -3$

$f(2) = (2)^3 + 3(2)^2 - 5(2) - 2 = 8$

As $f(1)$is negative and $f(2)$is positive and the function $f(x)$ is continuous, there must exists at least one within this interval.

Showing there exists a root in the interval [1.4385,1.4395] verifies that $\alpha = 1.439$ is correct to 3 decimal places

$f(1.4385) = (1.4385)^3 + 3(1.4385)^2 - 5(1.4385) - 2 = -0.00799$

$f(1.4395) = (1.4395)^3 + 3(1.4395)^2 - 5(1.4395) - 2 = 0.00186$

As $f(1.4385)$ is negative and $f(1.4295)$ is positive and the function $f(x)$ is continuous, $\alpha = 1.439$ is correct to 3 decimal places.

P77

a) $xe^{-x} + x - 3 = 0$

$e^{-x} = \frac{3-x}{x}$

$e^{x} = \frac{x}{3-x}$

$x = \ln\left|\frac{x}{3-x}\right|$

b) $x_1 = -1$

$x_2 = -1.38629$

$x_3 = -1.15185$

$x_4 = -1.28218$

$x_5 = -1.20590$

$x_6 = -1.24926$

$x_7 = -1.22419$

$x_8 = -1.23855$

$x_9 = -1.23028$

$x_{10} = -1.23503$

$x_{11} = -1.23230$

$x_{12} = -1.23386$

$x = -1.23\ (2d.p)$

P78

$f(x_{n=1}) = x_n - \frac{x_n^3 - 2x_n^2 - x_n + 1}{3x_n^2 - 4x_n - 1}$

$x_1 = 3$

$x_2 = 2.5$

$x_3 = 2.29032$

$x = 2.290$

P79

We know that $a = k^2$ and that the second term is k so given the second term is $a + d$

$k^2 + d = k \Rightarrow d = k - k^2$

$k + 2d = -10 \Rightarrow d = -5 - \frac{k}{2}$

$\therefore -5 - \frac{k}{2} = k - k^2$

$\therefore -10 - k = 2k - 2k^2$

$\therefore 2k^2 - 3k - 10 = 0$

$\therefore k = \frac{3 \pm \sqrt{89}}{4} \Rightarrow a = 3,\ b = 4$ and $c = 89$

P80

Let $S_n = 1 + 3 + 5 + \cdots + (2n + 1)$

From this we get that $a = 1$ and $d = 2$

In general, $S_n = \frac{n}{2}(2a + (n-1)d)$

$\therefore S_n = \frac{n}{2}\big(2(1) + (n-1)(2)\big)$

$= \frac{n}{2}(2 + 2n - 2)$

$= n^2$ as required.

P81

a) $u_6 = ar^5 = a\left(\sqrt{5}\right)^5 = a(5)^{\frac{5}{2}} = 25\sqrt{5}a$ or equivalent

b) In general, $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore\ S_{10} = \frac{a\left(1-\left(\sqrt{5}\right)^{10}\right)}{1-\sqrt{5}}$

$= \frac{-3124a}{1-\sqrt{5}} \times \frac{1+\sqrt{5}}{1+\sqrt{5}} = \frac{-3125a(1+\sqrt{5}a)}{(1-\sqrt{5})(1+\sqrt{5})} = \frac{-3125a(1+\sqrt{5})}{1-5}$

$= \frac{3125a(1+\sqrt{5})}{4} \Rightarrow p = 3125$ and $q = 1$

P82

Let S denote the total distance the ball travels.

After the first bounce, note that the ball must travel up AND down so we must double our summation.

Consider the ball's motion after it has hit the ground initially.

It achieves a height of $2 \times 0.75 = 1.5m$

This is our first term of our series, a.

Our common ratio, $r = 0.75$

Hence, the total distance travelled, is the $2m$ it initially falls plus $2 \times$ our summation.

$\therefore S = 2 + 2\left(\frac{a}{1-r}\right)$

$\therefore S = 2 + 2\left(\frac{1.5}{1-0.75}\right) = 14m$

P83

a) $\sum_{r=2}^{k} 3(2^r) = 1524$

$\therefore \sum_{r=2}^{k}(2^r) = 508$

$\therefore 2^2 + 2^3 + \cdots + 2^k = 508$

This is a geometric series with first term 4 and common ratio 2.

In general, $S_n = \frac{a(1-r^n)}{1-r}$

Where $a = 4, r = 2$ and $n = k - 1$

$\therefore \frac{4(1-2^{k-1})}{1-2} = 508$

$\therefore 1 - 2^{k-1} = -127$

$\therefore 2^{k-1} = 128 \Rightarrow k = 8$

b) $\sum_{r=k+1}^{10} 3(2^r) = \sum_{r=9}^{10} 3(2^r)$

$= 3(2^9) + 3(2^{10})$

$= 4608$

P84

a) $u_1 = 1,\ u_{n+1} = au_n + {u_n}^2$

$\therefore u_2 = a(1) + 1^2 = 1 + a$

b) $u_3 = au_2 + {u_2}^2$

$\therefore u_3 = a(1+a) + (1+a)^2$

$= a + a^2 + 1 + 2a + a^2$

$= 1 + 3a + 2a^2$ as required.

c) $u_3 = 1 \Rightarrow 1 + 3a + 2a^2 = 1$

$\therefore 2a^2 + 3a = 0$

$\therefore a(2a+3) = 0$

We know $a \neq 0$ hence, $a = -\frac{3}{2}$

d) By computing the first few terms, we can see the terms alternate between values of 1 and $-\frac{1}{2}$

Even terms take the value of $-\frac{1}{2}$

Hence, $u_{4018} = -\frac{1}{2}$

P85

a) $a^2 = b^2 + c^2 - 2bc\cos A$

$AB^2 = AC^2 + BC^2 - 2 \times AC \times BC \times \cos ACB$

$AB^2 = 3.4^2 + 5.6^2 - 2(3.4)(5.6)\cos 103$

$AB^2 = 51.4861 \ldots$

$AB = 7.2\ (1dp)$

b) $\frac{\text{Sin } 103}{7.2} = \frac{\sin x}{5.6}$

$\frac{\text{Sin } 103}{7.2} \times 5.6 = \sin x$

$x = 49.3\ (1d.p)$

If you don't round your answer and use an exact value for the distance of AB you may get $x = 49.50$

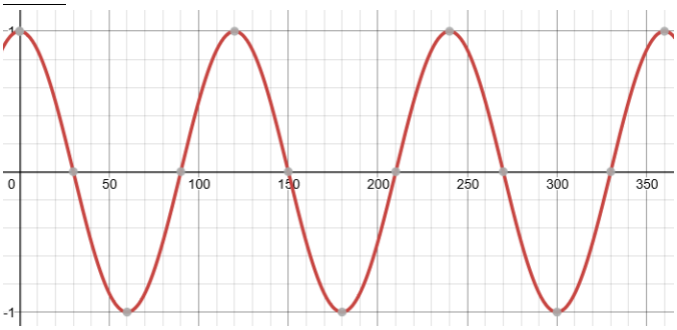
P86

$Area = \frac{1}{2}ab\sin c$

$A = \frac{1}{2}(x+2)(x-4)\sin 30$

$A = \frac{1}{4}(x^2 - 2x - 8)$ or $\frac{1}{4}(x+2)(x-4)$ is sufficient

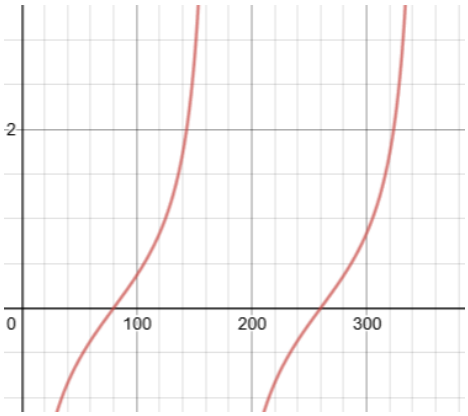
P87



a) Minimum of -1, maximum of 1

Intersection points:

$(0,1), (30,0), (90,0), (150,0), (210,0), (270,0)$ and $(330,0)$



b) There is no maximum or minimum with the tan graphs.

The intersection points are (80, 0) and (260, 0)

P88

$cos\,315 = \cos(360 - 315) = cos(45) = \frac{\sqrt{2}}{2}$

$sin420 = sin60 = \frac{\sqrt{3}}{2}$

$tan\,-120 = tan60 = \sqrt{3}$

$sin\,135 = sin(180 - 135) = sin45 = \frac{\sqrt{2}}{2}$

P89

$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

$LHS = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

Using the identity: $\sin^2 x + \cos^2 x = 1$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

which equals the RHS

P90

a) $6\,sin\,x = 5\,cos\,x$

$\frac{\sin x}{\cos x} = \frac{5}{6}$

$\tan x = \frac{5}{6}$

b) $\tan^{-1}\left(\frac{5}{6}\right) = 39.8°$ $\quad 0 \le 2\theta \le 720$

$2\theta = 39.8, 219.8, 399.8, 579.8$

$\theta = 19.9, 109.9, 199.9, 289.9$

P91

a) $10\sin^2 x + 6\cos^2 x = 3\cos x$

$10(1 - \cos^2 x) + 6\cos^2 x = 3\cos x$

$10 - 10\cos^2 x + 6\cos^2 x = 3\cos x$

$0 = 4\cos^2 x + 3\cos x - 10$

b) $4\cos^2 x + 3\cos x - 10 = 0$

$(4\cos x - 5)(\cos x + 2) = 0$

$\cos x = \frac{5}{4}\ or\ -2$

$\cos x = \frac{5}{4}$, has no solutions since $\frac{5}{4} > 1$

$\cos x = -2$, has no solutions since $-2 < -1$

P92

a) $200 \times \frac{\pi}{180} = \frac{10\pi}{9}$

b) $10 \times \frac{\pi}{180} = \frac{\pi}{18}$

c) $137 \times \frac{\pi}{180} = \frac{137\pi}{180}$

d) $\frac{6\pi}{7} \times \frac{180}{\pi} = 154.3°$

e) $\frac{3\pi}{14} \times \frac{180}{\pi} = 38.6°$

f) $\frac{20\pi}{21} \times \frac{180}{\pi} = 171.4°$

P93

a) arc length is given by: $r\theta = 10 \times 0.7 = 7cm$

b) $BC^2 = 10^2 + 14^2 - 2 \times 10 \times 14 \times \cos 0.7$

$BC = 9.047$

Perimeter $= 7 + 4 + 9.0467 = 22.0cm\ (3s.f.)$

P94

$3\sin^2 x + 7\cos x = 6 - \cos^2 x$

$3(1 - \cos^2 x) + 7\cos x = 6 - \cos^2 x$

$3 - 3\cos^2 x + 7\cos x = 6 - \cos^2 x$

$0 = 2\cos^2 x - 7\cos x + 3$

$(2\cos x - 1)(\cos x - 3) = 0$

$\cos x = 3\ or\ \frac{1}{2}$

There are no solutions when $\cos x = 3$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}$

$\theta = 1.0, 5.2$ to one decimal point

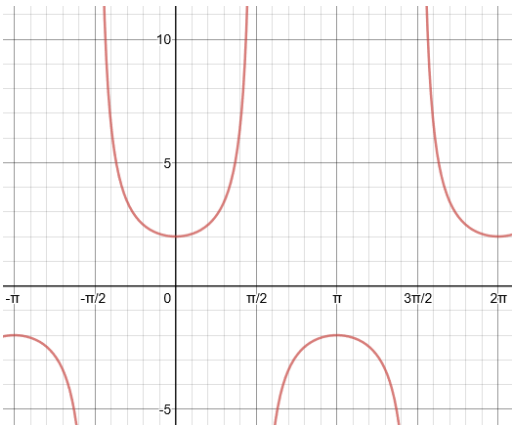
P95

Using the approximations, $\sin x \approx x$, and $\cos x \approx 1 - \frac{x^2}{2}$

$$\frac{2\cos 2\theta + 2\sin\theta}{\sin\theta - 1} \approx \frac{2(1 - \frac{(2\theta)^2}{2}) + 2\theta}{\theta - 1}$$

$$\frac{2 - 4\theta^2 + 2\theta}{\theta - 1} = \frac{(\theta - 1)(2\theta + 1)}{\theta - 1} = -2(2\theta + 1)$$

P96



a) Turning points are at $(-\pi, -2), (0,2), (\pi, -2)$ and $(2\pi, 2)$

b) The coordinates at points at which the gradient is 0 are turning points.
These are $-\pi,\ 0,\ \pi\ and\ 2\pi$

P97

Using the rule of difference of two squares,

$\operatorname{cosec}^4\theta - \cot^4\theta \equiv (\operatorname{cosec}^2\theta + \cot^2\theta)(\operatorname{cosec}^2\theta - \cot^2\theta)$

Using the Identity, $cosec^2 x = 1 + \cot^2 x$

$$\equiv \operatorname{cosec}^2\theta + \cot^2\theta$$

$$\equiv \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1 + \cos^2\theta}{\sin^2\theta}$$

$$\equiv \frac{1 + \cos^2\theta}{1 - \cos^2\theta} = RHS$$

P98

$y = \arccos x$

$x = \cos y$

$x = \sin(\frac{\pi}{2} - y)$

$\arcsin x = \frac{\pi}{2} - y$

P99

$2\cos x \cos 50 - 2\sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$

$2\cos x \cos 50 - \cos x \sin 40 = \sin x \cos 40 + 2\sin x \sin 50$

$\cos x\,(2\cos 50 - \sin 40) = \sin x\,(\cos 40 + 2\sin 50)$

$2\cos 50 - \sin 40 = \tan x\,(\cos 40 + 2\sin 50)$

$\frac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \tan x$

$\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ so

$\frac{2\sin 40 - \sin 40}{\cos 40 + 2\cos 40} = \tan x$

$\frac{1}{3}\tan 40 = \tan x$

P100

a) $\cos 2\theta + \sin\theta = 1$

$1 - 2\sin^2\theta + \sin\theta = 1$

$2\sin^2\theta - \sin\theta = 0$

Therefore $k = 2$

b) $\sin\theta\,(2\sin\theta - 1) = 0$
$\sin\theta = 0\ or\ \frac{1}{2}$
$\theta = 0°, 30°, 150°, 180°$ and $360°$

P101

$R^2 = 6^2 + 8^2$

$\sqrt{6^2 + 8^2} = 10, R = 10$

$\tan\alpha = \frac{8}{6}$

$\alpha = 0.93\ (2d.p)$

$6\cos\theta + 8\sin\theta = 10\cos(\theta - 0.93)$

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P102

$\sec 2A + \tan 2A = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$

$= \frac{1+\sin 2A}{\cos 2A}$

Using double angle formulae

$= \frac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$

Using the identity: $1 = \sin^2 x + \cos^2 x$

$= \frac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$

Factorise the numerator and denominator

$= \frac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$

$= \frac{\cos A + \sin A}{\cos A - \sin A}$

P103

a) Start is when t = 0 and the end is when t = 9

Beginning: $18.2 + 3\sin(-2) = £15.47$

End: $18.2 + 3\sin(0.6(9) - 2) = £17.43$

b) Maximum price is when $\sin(0.6t - 2) = 1$
$18.2 + 3 = 21.2$
maximum price is £21.20

P104

a) $a = \binom{p}{q}$ $b = \binom{-2q}{3p}$ $c = \binom{-7}{30}$

Given that 3a + 2b = c, find the values of p and q

$3\binom{p}{q} + 2\binom{-2q}{3p} = \binom{-7}{30}$

$3p - 4q = -7$

$3q + 6p = 30$

Rearrange to get:

$6p = 30 - 3q$

$6p - 8q = -14$

$(30 - 3q) - 8q = -14$

$44 = 11q$

$4 = q$

$12 + 6p = 30$

$p = 3$

b) $\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$

$\overrightarrow{DB} = -t + s = s - t$

c) $\overrightarrow{NC} = \overrightarrow{NB} + \overrightarrow{BC}$

$\overrightarrow{NB} = \frac{1}{4}\overrightarrow{DB} = \frac{1}{4}(s - t)$

$\overrightarrow{NC} = \frac{1}{4}(s - t) + t$

$\overrightarrow{NC} = \frac{1}{4}s + \frac{3}{4}t$

P105

a) Magnitude $= \sqrt{12^2 + 8^2} = 4\sqrt{13}$

Unit vector = $\frac{1}{4\sqrt{13}}(12i + 8j)$ or $\frac{\sqrt{13}}{52}(12i + 8j)$

b) Magnitude $= \sqrt{7^2 + 3^2} = \sqrt{58}$
Unit vector $= \frac{1}{\sqrt{58}}\binom{7}{3}$

P106

Start by finding $\overrightarrow{OX}$ in two different ways.

First consider the route O to B to X:

$\overrightarrow{OX} = b + \lambda\overrightarrow{BA}$

$= b + \lambda$(-b+a)

$= b - \lambda b - \lambda a$

$= \lambda a + b(1 - \lambda)$

Then consider the route via the line OC

$\overrightarrow{OX} = \mu\overrightarrow{OC}$

$= \mu(a + b)$

Therefore

$\mu a + \mu b = \lambda a + b(1 - \lambda)$

Comparing the coefficients of a $\mu = \lambda$

Comparing the coefficients of b $\mu = 1 - \lambda$

$\mu = 1 - \mu$

$2\mu = 1$

$\mu = 1/2$

Therefore $\mu = \lambda = 1/2$

The scalar of ½ means they bisect each other, and x is the midpoint of each of the diagonals

P107

a) 4i - 7j

b) $\sqrt{4^2+7^2} = \sqrt{65}\ km$ or $8.06km\ (2d.p)$

c) $\tan^{-1}\left(\frac{7}{4}\right) + 90 = 150.3\ (1d.p)$

P108

a) $A\ (8,4,4)\quad B\ (5,2,k)$
Distance between =
$\sqrt{(8-5)^2+(4-2)^2+(4-k)^2} = \sqrt{17}$
$\sqrt{3^2+2^2+k^2-8k+16} = \sqrt{17}$
$k^2-8k+29 = 17$
$k^2-8k+12 = 0$
$(k-6)(k-2) = 0$
$k = 6\ or\ 2$

b) $a = 7i + 3j + 4k$
$|a| = \sqrt{7^2+3^2+4^2} = \sqrt{74}$
Therefore $\hat{a} = \frac{1}{\sqrt{74}}\begin{pmatrix}7\\3\\4\end{pmatrix}$

c) $|a| = \sqrt{5^2+-3^2+-2^2} = \sqrt{38}$
$\cos\theta_x = \frac{5}{\sqrt{38}}$
$\theta_x = 35.8°$
$\cos\theta_y = \frac{-3}{\sqrt{38}}$
$\theta_y = 119.1°$
$\cos\theta_z = \frac{-2}{\sqrt{38}}$
$\theta_z = 108.93°$

P109

$AB = 8i + 2k$
$BC = -3i + j + 3k$

$|AB| = \sqrt{8^2+5^2} = 2\sqrt{17}$

$|BC| = \sqrt{3^2+1^2+3^2} = \sqrt{19}$

$AC = AB + BC$

$= 8i + 2k - 3i + j + 3k$

$= 5i + 1j + 5k$

$|AC| = = \sqrt{5^2+1^2+5^2} = \sqrt{51}$

Using: $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos^{-1}\left(\frac{(2\sqrt{17})^2+(\sqrt{51})^2-(\sqrt{19})^2}{2(2\sqrt{17})(\sqrt{51})}\right) = 31.9°$

The area of a triangle is $= \frac{1}{2}ab\sin C$

$Area = \frac{1}{2}\times 2\sqrt{17}\times\sqrt{51}\times\sin(31.9\ldots) = 15.6\ units^2$

P110

a) If the particle is in equilibrium then

$$F_1 + F_2 + F_3 = 0$$

So

$$\begin{pmatrix}-3\\2\\a\end{pmatrix}+\begin{pmatrix}b\\3\\2\end{pmatrix}+\begin{pmatrix}-9\\-5\\7\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

So

$$-3+b-9=0, b=12$$

$$a+2+7=0, a=-9$$

b) Acceleration is given by $F = ma, a = \frac{F}{m}$

$$F = F_1 + F_2 = \begin{pmatrix}-3\\2\\-9\end{pmatrix}+\begin{pmatrix}-9\\-5\\7\end{pmatrix}=\begin{pmatrix}-12\\-3\\-2\end{pmatrix}$$

$$a = \frac{1}{3}\begin{pmatrix}-12\\-3\\-2\end{pmatrix}=\begin{pmatrix}-4\\-1\\-\frac{2}{3}\end{pmatrix}$$

c) Magnitude of acceleration is given by
$\sqrt{(-4)^2+(-1)^2+\left(-\frac{2}{3}\right)^2} = \sqrt{\frac{157}{9}} = 17.44$

d) find the distance the acceleration vector is from $\hat{k}$, which is found by $\cos\theta_k = \frac{k}{|a|}$
$|a| = 17.44$
then $cos\theta = \frac{\frac{2}{3}}{\sqrt{\frac{157}{9}}}$ (ignore negatives as it is distance from $\hat{k}$ and therefore we can take the magnitude of the direction)
$\theta = 80.82$

Mechanics

M5

1) Distance
2) Speed
3) Speed = $\sqrt{3^2+4^2} = 5ms^{-1}$
Direction = $\tan^{-1}\frac{4}{3} = 53.1°$
4) i. net displacement is given by: $(2\mathbf{i}+3\mathbf{j})+(4\mathbf{i}+\mathbf{j}) = (6\mathbf{i}+4\mathbf{j})$ miles
as a overall distance this is: $\sqrt{6^2+4^2} = 7.21$ miles
ii. total distance travelled given by: $\sqrt{2^2+3^2}+\sqrt{4^2+1^2} = 7.73$ miles

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M6

i. $s = \int v\,dt = \int 24t^3\mathbf{i} + 6\mathbf{j}\,dt = 6t^4\mathbf{i} + 6t\mathbf{j} + c$

When $t = 0, s = 2\mathbf{i} + 3\mathbf{j}$ Substitute $t = 0$ into the above equation for s and equate to and you will find that $s = 2\mathbf{i} + 3\mathbf{j}$ then you will find that $c = 2\mathbf{i} + 3\mathbf{j}$

Therefore, $s = (6t^4 + 2)\mathbf{i} + (6t + 3)\mathbf{j}$

ii. $a = \frac{dv}{dt} = \frac{d}{dt}(24t^3\mathbf{i} + 6\mathbf{j}) = 72t^2\mathbf{i}$

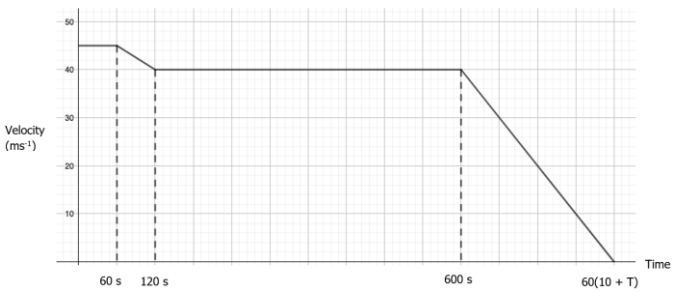
M7

i. 1 hour = 3600 s therefore 3 hours = 10800 s and 2 hours = 7200 s
Therefore displacement, $s = 2(10800) + 1.5(7200) = 32400 \text{ m} = 32.4 \text{ km}$

ii. Average velocity $= \frac{32400}{5.5\times3600} = \frac{18}{11} = 1.64 \text{ ms}^{-1}$

M8

First, draw a velocity-time graphs with the time scale in minutes.



Convert minutes into seconds and write them under the corresponding minutes on the graph.

Find area under the graph in terms of T, using seconds not minutes. This is to find the distance in metres that the train has travelled. Use dotted lines to split the area into shapes you know how to find the area of. (there are other possible ways to split the area and so long as you get the same answer it doesn't matter how you split it up)

Equal this to 29250 m and solve for T:

$2700 + 2550 + 19200 + (40)(0.5)(60)(\mathrm{T}) = 29250$

$1200\mathrm{T} = 4800$

$\mathrm{T} = 4$

M9

S ?
U 100 ms^{-1}
V 120 ms^{-1}
A 8 ms^{-2}
T /

No t, so use: $v^2 = u^2 + 2as$.

Rearrange this to get: $\frac{v^2-u^2}{2a} = s$

Sub in values: $\frac{120^2-100^2}{2\times8} = s$

$s = 275\,m$

M10

Normal reaction force $R = g\cos 20$ N

$F_{\max} = \mu R = \mu(g\cos 20)$

Resolve parallel to slope: $g\sin 20 - \mu(g\cos 20) = 1.5$

$g\sin 20 - 1.5 = \mu(g\cos 20)$

$\mu = \frac{g\sin 20 - 1.5}{(g\cos 20)} = 0.2\ (2\text{ s.f.})$

M11

Friction $= \mu R = 0.1(g + 20\sin 30) = 1.98$

Therefore, resolving the forces on the block horizontally and using $F = ma$:

$20\cos 30 - 1.98 = (1)a$

$a = 15.3 \text{ ms}^{-2}$

M12

$(30 - 10p + 3q)\mathbf{i} + (50 + 2q - 12.5p)\mathbf{j} = 25\mathbf{i} + 35\mathbf{j}$

Equate the coefficients of **i**:

$30 - 10p + 3q = 25$ therefore $5 = 10p - 3q$

Equate the coefficients of **j:**

$50 + 2q - 12.5p = 35$ therefore $15 = 12.5p - 2q$

Solve the simultaneous equations to find that $p = 2$ and $q = 5$.

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M13

Normal reaction force between box and slope is: $\mathrm{R} = (g\cos 10 + 15\sin 10)\ \mathrm{N}$

Resolving parallel to slope and using $F = ma$:

$15\cos 10 - g\sin 10 - 0.3(g\cos 10 + 15\sin 10) = (1)a$

$$a = 9.39\ \mathrm{ms}^{-1}$$

M14

i. Resultant force parallel to the slope = $22120 - 1800g\sin 30 - 1000g\sin 30 = 8400\ \mathrm{N}$

Mass of whole system = 2800 kg

$a = \frac{F}{m} = \frac{8400}{2800} = 3\mathrm{ms}^{-2}$

ii. Taking the trailer:

$F = ma$

$\mathrm{T} - 1000g\sin 30 = 1000(3)$

$\mathrm{T} = 7900\ \mathrm{N}$

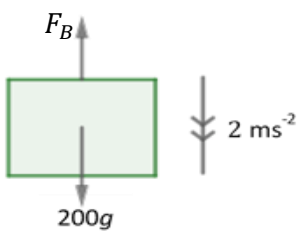
You could have used the forces acting on the car to obtain the same value for tension.

M15

i. Total mass $= 1500 + 350 + 200 = 2050$ kg

$2050g - \mathrm{T} = 2050(2)$

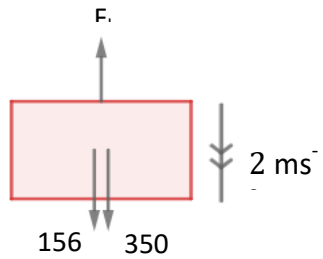
$\mathrm{T} = 15990\ \mathrm{N} = 16000\ \mathrm{N}\ (3\ \mathrm{s.f.})$



ii. Taking F_B as the force exerted by box B by box A.

$$200g - \mathrm{F}_B = 400$$

$$\mathrm{F}_B = 1560\ \mathrm{N}$$



iii. Taking F_L as the force exerted by box B on the lift.

$350g + 1560 - \mathrm{F}_L = 350(2)$

$\mathrm{F}_L = 4290\ \mathrm{N}$

M16

i. Taking A: $\mathrm{T} - 2g\sin 25 = 2a$

Taking B: $2g - \mathrm{T} = 2a$

Add the simultaneous equations together: $2g(1 - \sin 25) = 4a$

Therefore: $a = \frac{g}{2}(1 - \sin 25) = 2.83\ \mathrm{ms}^{-1}$

ii. Substitute a into either equation to get T: $2g - g(1 - \sin 25) = \mathrm{T} = 13.9\ \mathrm{N}$

M17

The particle is at instantaneous rest when $v = 0$

$0 = 3t^2 + 4t - 12$

$t = \frac{-4 \pm \sqrt{160}}{6} = \frac{2}{3}(-1 \pm \sqrt{10})$

$t \geq 0\ so\ t = \frac{2}{3}(-1 + \sqrt{10})$

M18

When the particle is at instantaneous rest, $v = 0. v = \frac{d}{dt}x$

$\frac{d}{dt}x = \frac{d}{dt}(4t^3 - 3t + 5) = 12t^2 - 3 = 0$

$t = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}, t \geq 0\ so\ t = \frac{1}{2}$

Displacement between $t = 0\ and\ t = \frac{1}{2}$

$x_0 = 5, x_{0.5} = 4$ therefore, displacement is 1 units.

M19

Displacement, $x = \int v dt$

$x = \int 5t^2 - 22t + 10\, dt = \frac{5}{3}t^3 - 11t^2 + 10t + C$

Given at $t = 0, x = 0\,, C = 0$

$x = \frac{5}{3}t^3 - 11t^2 + 10t = t\left(\frac{5}{3}t^2 - 11t + 10\right)$

$t = \frac{11 \pm \sqrt{11^2 - 4 \times \frac{5}{3} \times 10}}{2 \times \frac{5}{3}} = \frac{11 \pm \sqrt{\frac{163}{3}}}{\frac{10}{3}} = \frac{33 \pm 3\sqrt{163}}{10}$

Numerical answers for t are also correct however, it is good practise to leave your answers in the simplest form

M20

$20 = F15 \sin 30$

$F = \frac{20}{15 \sin 30} = \frac{4}{3\frac{1}{2}} = \frac{8}{3}$

M21

Set the length of the rod to be x

Clockwise moments: $2 \times 15 + x \times 15 = 30 + 15x$

Anticlockwise moments: $(x - 2) \times 20 = 20x - 40$

$30 + 15x - (20x - 40) = 15$

$70 - 5x = 15$

$5x = 55$

$x = 11$

M22

Total down force on plank is $15g + 45g = 60g$ therefore the reaction at A and at C are both equal to $30g$

Set the distance $AD = x$ and take moments about A

Clockwise moments: $4 \times 15g + x \times 45g = 60g + 45xg$

Anticlockwise moments: $5 \times 30g = 150g$

$150g = 60g + 45xg$

$10 = 4 + 3x$

$x = 2$

M23

Tension in A= 2 ×Tension in B

Tension in $A = \frac{40g}{3}$ so Tension in $b = \frac{20g}{3}$

Set x to be the distance from the left most point of the rod to the centre of mass

Taking moments about the left most point of the rod,

Clockwise moments: $1 \times \frac{40g}{3} + 4 \times \frac{20g}{3} = \frac{120g}{3} = 40g$

Anticlockwise moments: $x \times 20g$

$40g = x20g$

$x = 2$

The centre of mass is 2m from the far-left point of the rod

M24

Force at C is $110gN$

Set x to be the distance from A the mass can be placed

Take moments about A

Clockwise moments: $6 \times 50g + x \times 60g = 300g + 60xg$

Anticlockwise moments: $8 \times 110g = 880g$

$880g = 60xg + 300g$

$580 = 60x$

$x = \frac{580}{60} = \frac{29}{3}$

The mass can be placed $\frac{29}{3}m$ from the point A

M27

Find how long it takes for P to reach 40 m.

S 40 m
U 35 ms^{-1}
V /
A -9.8 ms^{-2}
T t

$s = ut + \frac{1}{2}at^2$

$40 = 35t - 4.9t^2$

$4.9t^2 - 35t + 40 = 0$

Solve for t: $t = \frac{40}{7}$ or $\frac{10}{7}$

The quadratic gives you two values for t. The smaller value of t is the time P passes 40 m for the first time travelling upwards. The larger value of t is the time P passes 40 m for a second time when travelling downwards. Therefore, the length of time between the two values of t is the total time P is above 40 m.

$\frac{40}{7} - \frac{10}{7} = \frac{30}{7} = 4.29$ s

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M28

i) Taking upwards as the positive direction

S ?
U $(45\sin 20°)$ ms^{-1}
V 0
A -9.8 ms^{-2}
T /

$v^2 = u^2 + 2as$

$0^2 = (45\sin 20°)^2 - 19.6s$

$s = 12.1$ m

12.1 m above release position, so greatest height above the ground is

$12.1 + 1.5 = 13.6$ m

ii) Taking upwards as the positive direction

S -1.5 m
U $(45\sin 20°)$ ms^{-1}
V ?
A -9.8 ms^{-2}
T /

$v^2 = u^2 + 2as$

$v^2 = (45\sin 20°)^2 + (19.6)(1.5)$

$v = 33.184936\ldots$

This is the vertical velocity when the arrow hits the ground. The horizontal velocity is $(45\cos 20°)$ ms^{-1}. Therefore,

$$\text{speed} = \sqrt{v^2 + (45\cos 20°)^2} = 45.2 \text{ ms}^{-1}$$

iii) $\tan^{-1}\frac{v}{(45\cos 20°)} = 40.8°$

Solutions may vary slightly due to rounding.

M29

Find the time taken to reach 18m. However, the displacement will be 12 m as the rock starts from 6 m above the ground.

Taking upwards as positive:

S 12 m
U 20 ms^{-1}
V ?
A -9.8 ms^{-2}
T /

$$s = ut + \frac{1}{2}at^2$$

$$12 = 20t - 4.9t^2$$

$$4.9t^2 - 20t + 12 = 0$$

Solve the quadratic to find the values of t: $t = 3.35$ or 0.73

The smaller value of t is the time the rock passes 18 m for the first time travelling upwards. The larger value of t is the time the rock passes 18 m for a second time when travelling downwards. Therefore, the length of time between the two values of t is the total time the rock is above 18 m.

Therefore, the rock is above 18 m for $3.35 - 0.73$

Statistics

S1

Possible advantages for a census are:

- They give an accurate result of the population

Possible disadvantages for a census are:

- They are time consuming
- More expensive
- Could damage the item
- Difficult to process large data sets.

Possible advantages for a sample are:

- Less time consuming
- More cost effective
- Less data needed

Possible disadvantages for a sample are:

- Only estimates the population
- Isn't large enough to represent the whole population.

Pick one from each to get full marks.

S2

Allocate a different number between 1 and 50 to each person. Using a calculator randomly select 15 numbers between 1 and 50 and then use those 15 numbers to select the people for the questionnaire.

S3

There are 21 values so take every $5th$ value from the set since $\frac{21}{5} = 4.25$ and we round up in this case

This gives $1.3, 1.9, 2.5, 2.9, 3.4$

S4

Stratified sampling should be used.

Year 1: $\frac{61}{200} \times 60 = 18.3$

Year 2: $\frac{49}{200} \times 60 = 14.7$

Year 3: $\frac{54}{200} \times 60 = 16.2$

Year 4: $\frac{36}{200} \times 60 = 10.8$

This gives 18 from year 1, 15 from year 2, 16 from year 3 and 11 from year 4. To double check these are the correct answers add them all up to check they add to make 60.

$$18 + 15 + 16 + 11 = 60$$

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S5

Quota sampling was used

It shows that on average female squirrels live longer than males, you can see this if you take the mean average of the data.

S6

Opportunity sampling was used

They should survey people at different times in the day to get a wider range of people and they should survey from different areas. Asking about food outside a food shop is a bias collection.

S7

Weight is numerical, it can take on any value in a given range

Cars in a car park is a numerical value that takes on integer values not a range.

S8

The total number of days is 60 , $(31 + 29 = 60)$

The mean is therefore $\frac{258+174}{60} = 7.2$

The mean is $7.2cm$ of rainfall.

S9

$40\%\ of\ 72\ = 28.8$ so we take the $29th$ value

$70\%\ of\ 72 = 50.4$ so we take the $50th$ value

Here we take

P_4 lies in the range $50 \leq m < 55$, it is the $3rd$ value of 12. So is $\frac{1}{4}$ of the way through the range.

$$P_4 = 50 + \frac{55 - 50}{4} = 51.25$$

P_7 lies in the range $55 \leq m < 60$, it is the $14th$ value of 16. So is $\frac{7}{8}$ of the way through the range.

$$P_4 = 55 + \frac{7(60 - 55)}{8} = 59.375$$

The inter-percentile range is $59.375 - 51.25 = 8.125$

The number of data values in the range is given by:

$50 - 29 = 21$

21 data values in the range.

S10

The mean is given by $\frac{\sum x}{n} = \frac{250}{40} = 6.25$

The standard deviation is $\sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} =$

$\sqrt{\left(\frac{4521}{40}\right) - \left(\frac{250}{40}\right)^2} = 8.60$

S11

$4.4 = \frac{\bar{h}-5}{15}$

$66 = \bar{h} - 5$

$\bar{h} = 71$

The coded standard deviation $= \sqrt{\frac{89}{80}}$

Un-coded this becomes $15\sqrt{\frac{89}{80}} = 15.82$

S12

The lower quartile is at the position: $\frac{20}{4} = 5$

The upper quartile is at position: $3 \times \frac{20}{4} = 15$

So we have the 5th and 15th value

Order the values from lowest to highest and count along the list to find the 5th and 12th number.

$3, 3, 4, 4, 4, 5, 6, 7, 7, 8, 8, 9, 9, 9, 9, 9, 10, 10, 15, 17,$

$5th\ = 4$

$15th = 9$

$9 - 4 = 5$

$5 \times 2.3 = 11.5$

$9 + 11.5 = 20.5$

$17 \succ 20.5$ and therefore 17 is not an outlier

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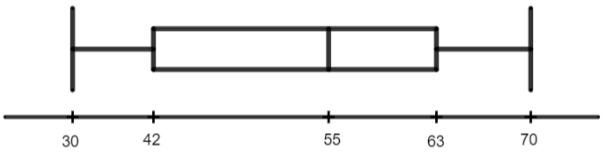
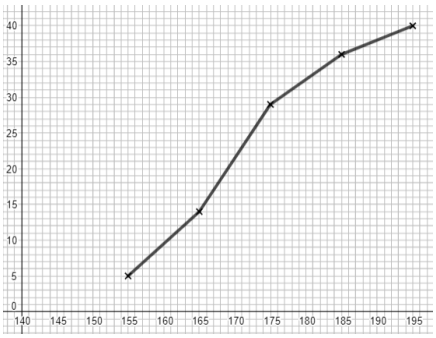
S13



S14

Length, L (cm)	Frequency	Cumulative frequency
$150 \leq h < 160$	5	5
$160 \leq h < 170$	9	14
$170 \leq h < 180$	15	29
$180 \leq h < 190$	7	36
$190 \leq h < 200$	4	40



S15

The area of the bar is $1.5 \times 3.0 = 4.5$

$\frac{4.5}{5} = 0.9$ so the area representing 1 person is $0.9cm^2$

$\frac{108}{0.9} = 120$ so 120 people completed the crossword puzzle in total.

S16

Positive correlation

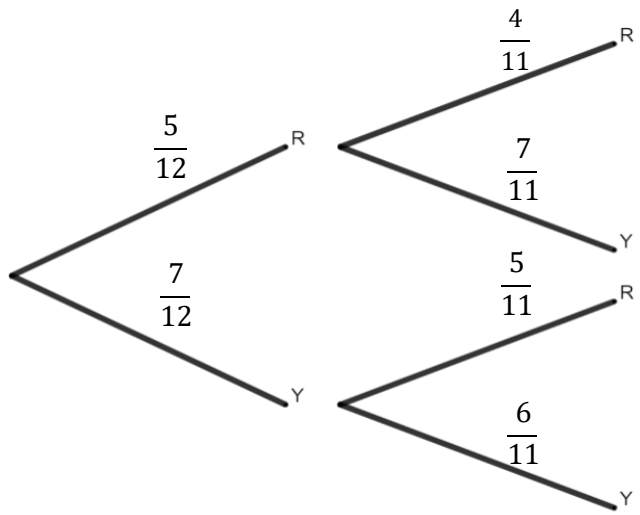
S17

The sample space is: {HH HT TH TT}

$\frac{1}{4}$ probability you get 2 heads (HH)

$\frac{1}{2}$ probability you get a head and tail (TH,HT)

S18



Because the ball isn't replaced then the total number of balls for the second branch decreases by $1\frac{7}{12} \times \frac{5}{11} = 0.265$

S19

Taking $P(M)$ as the probability that an individual chose Maths, and $P(P)$ as the probability that an individual chose Physics.

$P(M) + P(P) - P(M \cap P) = P(M \cup P)$

We know that $1 - P(M \cup P) = 0.15$ as the probability that an individual chose neither Maths or physics is 0.15.

Therefore, we know that

$0.6 + 0.55 - P(M \cap P) = 0.85$

Therefore $P(M \cap P) = 0.3$

For independent events, $P(M \cap P) = P(M) \times P(P) = 0.6 \times 0.55 = 0.33$

$0.3 \neq 0.33$ therefore they aren't independent

S20

$\frac{h}{1} + \frac{h}{2} + \frac{h}{3} + \frac{h}{4} + \frac{h}{5} + \frac{h}{6} = 1$

$h(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}) = 1$

$h(49/20) = 1$

$h = 20/49$

x	1	2	3	4	5	6
P(X=x)	20/49	10/49	20/147	5/49	4/49	10/147

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S21

B(5,0.2)

$P(X \geq 2) = 1 - P(X \leq 1)$

Using the tables we get $1 - 0.7373 = 0.2627$

$P(x \geq 2) = 0.26$

S22

$Ho: p = 0.5$

$H1: p > 0.5$S23

Assume the null hypothesis is true

Using the probability distribution tables we can see that the critical values are 3 and 11

The actual significance of these values is 0.03486

0.035

S23

$\alpha = 0.05$, so $\frac{\alpha}{2} = 0.025$

H_0 : p = 0.6 : Assume H_0 is true, $X \sim B(12, p)$

H_1 : p ≠ 0.6 : Reject H_0 if $P(X \leq x_L) \leq 0.025$

Reject H_0 if $P(X \geq x_u) \leq 0.025$

For x_L: from tables $x_L = 3$

For x_u: $1 - P(X \leq x_u - 1) \leq 0.025$

$0.975 \leq P(X \leq x_u - 1)$

$P(X \leq x_u - 1) \geq 0.975$

From tables, $x_u - 1 = 10$

$x_u = 11$

Therefore, the rejection region is $x \leq 3$ or $x \geq 11$

Actual significance level $= P(X \leq 3) + P(X \geq 11) = 0.015 + 0.02 = 0.035 = 3.5\%$

S24

$\log h = 0.452 + 0.3 \log g = log 10^{0.452} \times \log g^{0.3}$

$h = 10^{0.452} g^n$

$n = 0.3$

$a = 10^{0.452} = 2.831$

S25

Yes, it can be accepted weight is related to age

Since $0.65 > 0.3961$

S26

$P(X \cap Y) = P(Y) \times P(X|Y) = 0.175$

$P(Y|X) = \frac{P(Y \cap X)}{P(X)} = 0.583$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 0.625$

S27

$P(Y < 179) = 0.5$

$P(164 < Y > 194) = P(Y < 194) - P(Y < 164) = 0.68$

They are known results as they are a standard deviation from the mean

S28

$Z = \frac{d-u}{\sigma^2}$

0.1 from the table, $Z = -1.2816$

$-1.2816 = \frac{d-2.3}{0.6^2}$

$d = 1.8386$

$d = 31.84$

S29

$P(X > 40|p = 0.3) > 0.1$

$S \sim N(30, 4.58)$

$Z = 0.109$

$1 - 0.5438 = 0.4562$

The number of people eating packed lunches has significantly increased

S30

$X \sim N(25, 2.4^2)$

Sample mean= $X \sim N\left(\frac{25.5, 2.4^2}{30}\right)$

$H_o: \bar{x} = 25$

$H_a: \bar{x} \neq 25$

$Z > \frac{x-25}{\frac{24}{\sqrt{30}}}$

$$\frac{x-25}{\frac{24}{\sqrt{30}}} = 1.14$$

5% significance $= 0.05$

Can't reject Ho since $1.14 < 1.645$