Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mathematics

Advanced
Paper 2: Pure Mathematics 2

Wednesday 13 June 2018 – Morning
Time: 2 hours

Paper Reference **9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).
(2)

(b) State the range of g.
(1)

(c) Find $g^{-1}(x)$, stating its domain.
(3)

a) $g(5) = \frac{15}{2}$

$gg(5) = \frac{40}{9}$

b) as $x \to \infty$ $g(x) \to 2$

range : $y > 2$ and $y \leqslant \frac{15}{2}$

$= 2 < y \leqslant 15/2$

c) let $y = \frac{2x+5}{x-3}$

make x the subject : $y(x-3) = 2x+5$

$\Rightarrow yx - 3y = 2x + 5$

$\Rightarrow yx - 2x = 3y + 5$

$\Rightarrow x(y-2) = 3y + 5$

$\Rightarrow x = \frac{3y+5}{y-2}$

so $g^{-1}(x) = \frac{3x+5}{y-2}$, domain: $2 < x \leqslant 15/2$.

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2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

(a) Find the position vector of D.

(2)

Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)

a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= (4\underline{i} - 2\underline{j} + 3\underline{k}) - (2\underline{i} + 3\underline{j} - 4\underline{k})$
$= 2\underline{i} - 5\underline{j} + 7\underline{k}$

so $\overrightarrow{OD} - \overrightarrow{OB} = (2\underline{i} - 5\underline{j} + 7\underline{k})$

$\overrightarrow{OD} = (2\underline{i} - 5\underline{j} + 7\underline{k}) + (4i - 2j + 3k)$
$= (6\underline{i} - 7\underline{j} + 10\underline{k})$

b) $\overrightarrow{AC} = ((a-2)\underline{i} + 2\underline{j} + 2\underline{k})$

$|\overrightarrow{AC}|^2 = (a-2)^2 + 2^2 + 2^2 = 16$
$\Rightarrow a^2 - 4a + 4 + 4 + 4 = 16$
$\Rightarrow a^2 - 4a - 4 = 0$
$\Rightarrow (a-2)^2 - 4 - 4 = 0$
$\Rightarrow (a-2)^2 = 8$
$\Rightarrow a = 2 \pm \sqrt{8}$

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3. (a) "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."

Disprove this statement by means of a counter example.

(2)

(b) (i) Sketch the graph of $y = |x| + 3$

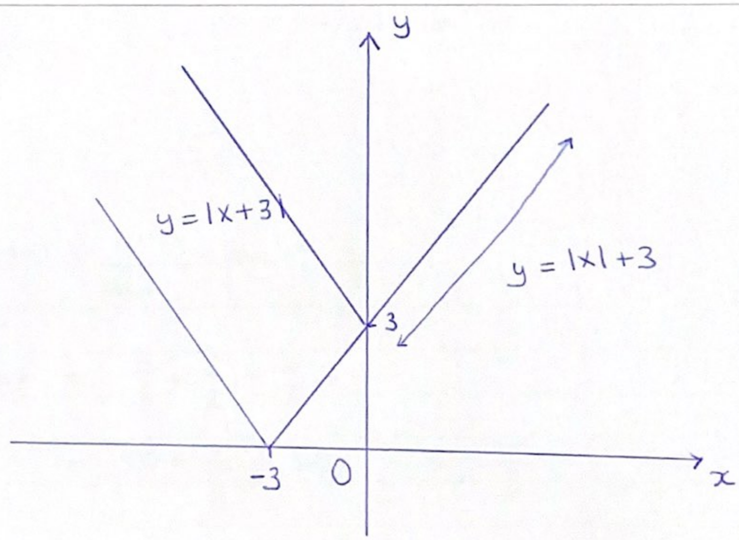
(ii) Explain why $|x| + 3 \geqslant |x + 3|$ for all real values of x.

(3)

a) let $m = \sqrt{3}$ and $n = \sqrt{12}$

$mn = \sqrt{36} = 6$

So the statement is true as 6 is rational.



b)(ii) as shown on the graph for all values of x $|x|+3 \geqslant |x+3|$

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4. (i) Show that $\sum_{r=1}^{16}(3+5r+2^r) = 131\,798$ **(4)**

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$ **(3)**

(i) $\sum_{r=1}^{16}(3+5r+2^r) = 131798$

$= \sum_{r=1}^{16}(3+5r) + \sum_{r=1}^{16} 2^r$

$= \cancel{\sum_{r=1}^{16}} \quad \frac{16}{2}(2\times 8 + 15\times 5) + \frac{2(2^{16}-1)}{2-1}$

$= 728 + 131070 = 131798$

(ii) $u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$

$\sum_{r=1}^{100} u_r = \frac{2}{3} + \frac{3}{2} + \frac{2}{3} + \ldots$

$= 50 \times \frac{2}{3} + 50 \times \frac{3}{2} = \frac{100}{3} + \frac{150}{2} = \frac{325}{3}$

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5. The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root.

(a) Show that, for this equation, the Newton-Raphson formula can be written

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

(3)

Using the formula given in part (a) with $x_1 = 1$

(b) find the values of x_2 and x_3

(2)

(c) Explain why, for this question, the Newton-Raphson method cannot be used with $x_1 = 0$

(1)

a) $f(x) = 2x^3 + x^2 - 1$

$f'(x) = 6x^2 + 2x$

$$x_{n+1} = x_n - \frac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$$

$$x_{n+1} = \frac{x_n(6x_n^2 + 2x_n) - 2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$$

$$x_{n+1} = \frac{6x_n^3 + 2x_n^2 - 2x_n^3 - x_n^2 + 1}{6x_n^2 + 2x_n}$$

$$x_{n+1} = \frac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$$

b) $$x_2 = \frac{4 + 1 + 1}{6 + 2} = \frac{3}{4}$$

$$x_3 = \frac{4\left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^2 + 1}{6(3/4)^2 + 2(3/4)} = \frac{2}{3}$$

c) There is a stationary point at $x = 0$.

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6.

$$f(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$$

(a) (i) Calculate f(2)

(ii) Write f(x) as a product of two algebraic factors.

(3)

Using the answer to (a)(ii),

(b) prove that there are exactly two real solutions to the equation

$$-3y^6 + 8y^4 - 9y^2 + 10 = 0$$

(2)

(c) deduce the number of real solutions, for $7\pi \leqslant \theta < 10\pi$, to the equation

$$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0$$

(1)

a) i) $f(2) = -3 \times 2^3 + 8 \times 2^2 - (9 \times 2) + 10$

$= -24 + 32 - 18 + 10 = 0$

ii) $f(x) = (x-2)(-3x^2 + 2x - 5)$

b) $-3y^6 + 8y^4 - 9y^2 + 10 = (y^2 - 2)(-3y^4 + 2y^2 - 5)$

$y^2 - 2 = 0$ has two real solutions $y \pm\sqrt{2}$.

$-3y^4 + 2y^2 - 5 = 0$ has no real solutions

as $b^2 - 4ac = 2^2 - 4 \times (-3) \times (-5) = -56 < 0$

so there are only two real solutions.

c) There are 3 solutions.

7. (i) Solve, for $0 \leqslant x < \frac{\pi}{2}$, the equation

$$4\sin x = \sec x$$

(4)

(ii) Solve, for $0 \leqslant \theta < 360°$, the equation

$$5\sin\theta - 5\cos\theta = 2$$

giving your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

i) $4\sin x = \sec x$

$\Rightarrow 4\sin x = \frac{1}{\cos x}$

$\Rightarrow 4\sin x\cos x = 1$

$\Rightarrow \sin x\cos x = \frac{1}{4}$

$\Rightarrow \frac{1}{2}\sin 2x = \frac{1}{4}$

$\Rightarrow \sin 2x = \frac{1}{2}$

$\Rightarrow 2x = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$\Rightarrow x = \frac{\pi}{12}$ and $\frac{5\pi}{12}$

(ii) $5\sin\theta - 5\cos\theta = 2$

LHS: $5\sin\theta - 5\cos\theta = R\sin(\theta - \alpha) = R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

equating coefficients: $5 = R\cos\alpha$

$5 = R\sin\alpha$

$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = 1$

to find R: $25 = R^2\cos^2\alpha$

$\Rightarrow \tan\alpha = 1$

$25 = R^2\sin^2\alpha$

$\Rightarrow \alpha = \tan^{-1}(1) = 45$

$\Rightarrow 50 = R^2(\sin^2\alpha + \cos^2\alpha)$

$\Rightarrow R = \sqrt{50}$

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Question 7 continued

ii cont) $\sqrt{50}\sin(\theta - 45) = 2$

$\Rightarrow \sin(\theta - 45) = \dfrac{2}{\sqrt{50}}$

$\Rightarrow \theta - 45 = \sin^{-1}\left(\dfrac{2}{\sqrt{50}}\right)$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{2}{\sqrt{50}}\right) + 45 = 61.43^\circ$ and 208.6°.

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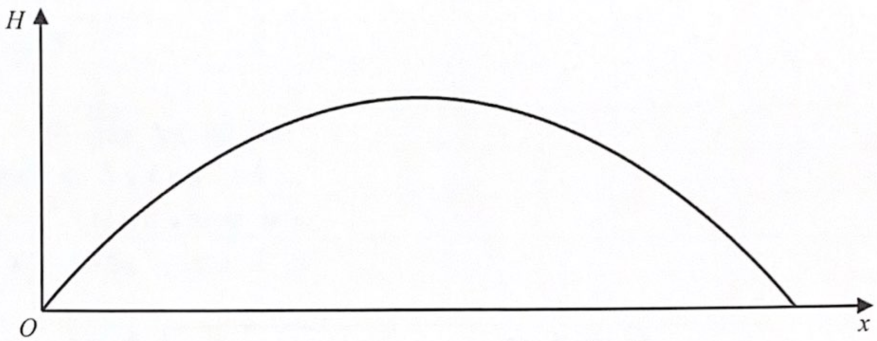

Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.
(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.
(3)

(c) Give one limitation of the model.
(1)

a) $H = Ax(40 - x)$

when $x = 20$, $H = 12$: $12 = 20 \cancel{\times} A \times 20$

$\Rightarrow A = \frac{12}{400} = \frac{3}{100}$

$\Rightarrow H = \frac{3}{100}x(40 - x)$

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Question 8 continued

b) $H = 3$

$\Rightarrow 3 = \frac{3}{100}x(40-x)$

$\Rightarrow 300 = 3x(40-x)$

$\Rightarrow 3x^2 - 120x + 300 = 0$

$\Rightarrow x^2 - 40x + 100 = 0$

$x = \frac{40 \pm \sqrt{(-40)^2 - 4 \times 1 \times 100}}{2} = 20 \pm 10\sqrt{3}$

greatest distance = 37.3m

c) For this model to work there needs to be no wind or air resistence.

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9. Given that θ is measured in radians, prove, from first principles, that

$$\frac{d}{d\theta}(\cos\theta) = -\sin\theta$$

You may assume the formula for $\cos(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

$$\frac{d}{d\theta}(\cos\theta) = \lim_{h\to 0} \frac{\cos(\theta+h) - \cos(\theta)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\theta\cos h - \sin\theta\sin h - \cos\theta}{h}$$

$$= \lim_{h\to 0} \cos\theta\underbrace{\frac{(\cos h - 1)}{h}}_{0} - \sin\theta\underbrace{\frac{\sin h}{h}}_{1}$$

$$= \cos\theta \times 0 - \sin\theta \times 1$$

$$= -\sin\theta$$

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10. A spherical mint of radius 5 mm is placed in the mouth and sucked.
Four minutes later, the radius of the mint is 3 mm.

In a simple model, the rate of decrease of the radius of the mint is inversely proportional to the square of the radius.

Using this model and all the information given,

(a) find an equation linking the radius of the mint and the time.
(You should define the variables that you use.)
(5)

(b) Hence find the total time taken for the mint to completely dissolve. Give your answer in minutes and seconds to the nearest second.
(2)

(c) Suggest a limitation of the model.
(1)

a) $\frac{dr}{dt} = \frac{k}{r^2}$ where r is the radius (mm)
t is the time in mouth (mins)
k is a real number.

$\int r^2 \, dr = \int k \, dt$

$\Rightarrow \frac{1}{3}r^3 = kt + c$

$t=0, r=5 : \frac{1}{3} \times 5^3 = c, \quad c = \frac{125}{3}$

$t=4, r=3 : \frac{1}{3} \times 3^3 = 4k + \frac{125}{3}$

$\Rightarrow 9 - \frac{125}{3} = 4k$

$\Rightarrow k = \frac{-49}{6}$

equation : $\frac{1}{3}r^3 = \frac{-49}{6}t + \frac{125}{3}$

Question 10 continued

b) $r = 0$, $\quad -\frac{49}{6}t + \frac{125}{3} = 0$

$\Rightarrow \quad t = \frac{125}{3} \times \frac{6}{49} = 5.102$

time = 5 mins 6 seconds.

c) The mint may not retain the same shape as it is being sucked.

11.

$$\frac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A + \frac{B}{(x-3)} + \frac{C}{(1-2x)}$$

(a) Find the values of the constants A, B and C.

(4)

$$f(x) = \frac{1+11x-6x^2}{(x-3)(1-2x)} \qquad x > 3$$

(b) Prove that $f(x)$ is a decreasing function.

(3)

a) $1 + 11x - 6x^2 = A(x-3)(1-2x) + B(1-2x) + C(x-3)$

$= A(-2x^2 + 7x - 3) + B(1-2x) + C(x-3)$

$-2A = -6 \Rightarrow \boxed{A = 3}$

let $x = 3$: $1 + 33 - 54 = \cancel{3(-2\times 9 + 15 - 18)} - 5B$

$-20 = \cancel{-63} - 5B$

$\boxed{B = 4}$

$1 \equiv -9 + 4 - 3C$ (equating x^0 terms)

$6 = -3C$

$\boxed{C = -2}$

$$\frac{1 + 11x - 6x^2}{(x-3)(1-2x)} \equiv 3 + \frac{4}{(x-3)} - \frac{2}{(1-2x)}$$

b) $f'(x) = -4(x-3)^{-2} - 4(1-2x)^{-2}$

$(x-3)^{-2} > 0$ and $(1-2x)^{-2} > 0$

then $f'(x) = -(+ve) - (+ve) < 0$, so $f(x)$ is a decreasing solution.

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12. (a) Prove that

$$1 - \cos 2\theta \equiv \tan\theta \sin 2\theta, \quad \theta \neq \frac{(2n+1)\pi}{2}, \quad n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the equation

$$(\sec^2 x - 5)(1 - \cos 2x) = 3\tan^2 x \sin 2x$$

Give any non-exact answer to 3 decimal places where appropriate.

(6)

a) $1 - \cos 2\theta \equiv \tan\theta \sin 2\theta$

$\tan\theta \sin 2\theta = \tan\theta(2\sin\theta\cos\theta)$

$= \frac{\sin\theta}{\cancel{\cos\theta}}(2\sin\theta\cancel{\cos\theta})$

$= 2\sin^2\theta$

$= 2\left[\frac{1}{2}(1-\cos 2\theta)\right]$

$= 1 - \cos 2\theta.$ $\square$

b) $(\sec^2 x - 5)(1-\cos 2x) = 3\tan^2 x \sin 2x \quad = 3\tan x(1-\cos 2x)$

$\Rightarrow (\sec^2 x - 5)(\tan x \sin 2x) = 3\tan^2 x \sin 2x$

$\Rightarrow \sec^2 x - 5 = 3\tan x$

$\Rightarrow (\tan^2 x + 1) - 5 = 3\tan x$

$\Rightarrow \tan^2 x - 3\tan x - 4 = 0$ so $x = 0$ is

$\Rightarrow (\tan x - 4)(\tan x + 1) = 0$ also a solution.

$x = \tan^{-1}(4)$ and $\tan^{-1}(-1)$

$x = -\frac{\pi}{4}$ and 1.326

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13.

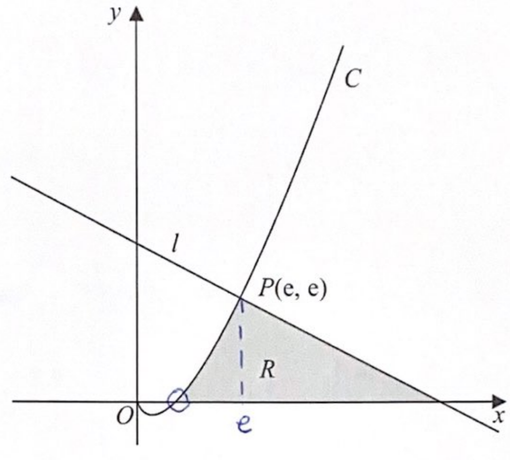

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

$y = x\ln x$, $\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1$

gradient at $P(e,e) = \ln e + 1 = 2$

so gradient of line l (normal) $= -1/2$

Equation of line l : $y - e = -1/2(x - e)$

$\Rightarrow y = -\frac{1}{2}x + \frac{3}{2}e$

line l

Meets x-axis : $(y = 0)$ $0 = -1/2x + 3/2e$

$\Rightarrow 1/2x = 3/2e$

$\Rightarrow x = 3e$

Curve C meets x-axis at $(0,0)$ and $(1,0)$

as $x\ln x = 0 \Rightarrow x = 0$ ~~and~~ or 1.

Question 13 continued

Area of Δ $(e,0)$, $(3e,0)$ and (e,e)

$= \frac{1}{2} \times 2e \times e = e^2$ (Area 1)

Area ~~op~~ shaded region between $(1,0)$ and $(e,0)$.

$\int_1^e x \ln x \, dx$ $\qquad u = \ln x, \ u' = \frac{1}{x}$

$v' = x, \ v = \frac{1}{2}x^2$

$= \left[\frac{1}{2}x^2 \ln x\right]_1^e - \int_1^e \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$

$= \left[\left(\frac{1}{2}\right)x^2 \ln x\right]_1^e - \int_1^e \frac{x}{2} \, dx$

$= \left[\frac{1}{4}x^2(2\ln x - 1)\right]_1^e$

$= \frac{1}{4}e^2 + \frac{1}{4}$ (Area 2)

Area 1 + Area 2 $= \frac{1}{4}e^2 + \frac{1}{4} + e^2$

$= \frac{5}{4}e^2 + \frac{1}{4}$

14. A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

a) $t = 0$, $N = \frac{900}{3+7e^{0}} = \frac{900}{10} = 90$ mice at the start.

b) $N = 900(3 + 7e^{-0.25t})^{-1}$

$$\frac{dN}{dt} = -2 \times 900 \times -0.25 \cdot 7e^{-0.25t} \times (3 + 7e^{-0.25t})^{-2}$$

$$= \frac{900(0.25)(7)e^{-0.25t}}{(3 + 7e^{-0.25t})^2}$$

$$= \frac{900(0.25)\left(\frac{900}{N} - 3\right)}{\left(\frac{900}{N}\right)^2}$$

$$= \left(\frac{900^2}{4N} - 2700\right) \times \left(\frac{N^2}{900^2}\right) = \frac{N}{4} - \frac{3N^2}{900} = \frac{300N - 4N^2}{1200}$$

$$= \frac{N(300 - N)}{1200}$$

Question 14 continued

c) When $N = 150$, $\frac{dN}{dt}$ is at its maximum.

Sub $N = 150$ into $N = \frac{900}{3 + 7e^{-0.25t}}$

$\Rightarrow 150 = \frac{900}{3+7e^{-0.25t}}$

$\Rightarrow 150(3 + 7e^{-0.25t}) = 900$

$\Rightarrow 7e^{-0.25t} = 3$

$\Rightarrow e^{-0.25t} = 3/7$

$\Rightarrow -0.25t = \ln 3/7$

$\Rightarrow t = -4 \ln 3/7 = 3.4$ months

d) Max number of mice, $P = 300$.

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