Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 14 October 2020

Afternoon (Time: 2 hours) | Paper Reference **9MA0/02**

Mathematics

Advanced

Paper 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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A Level Products

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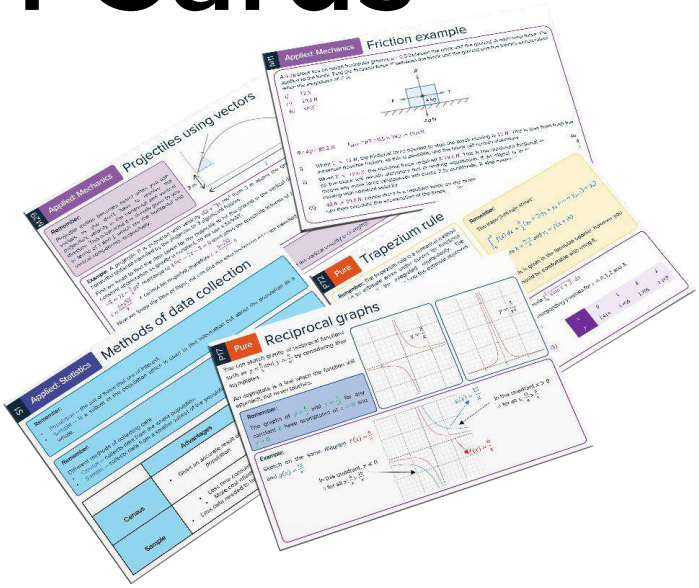

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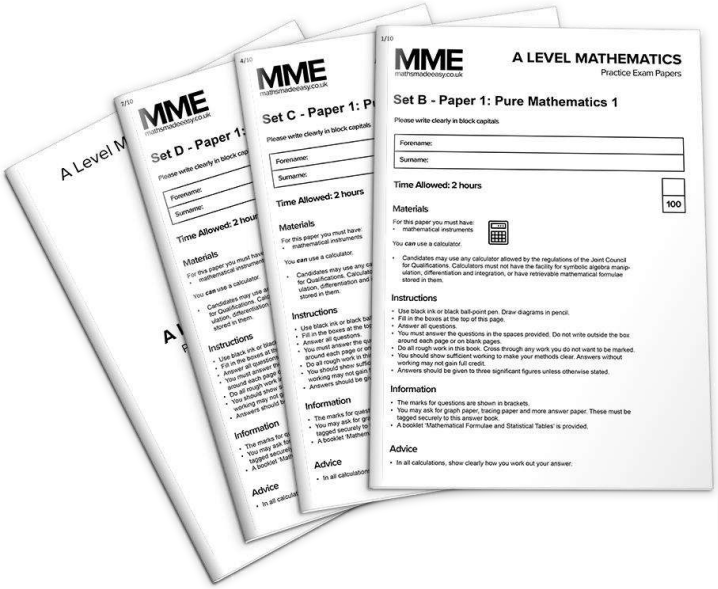

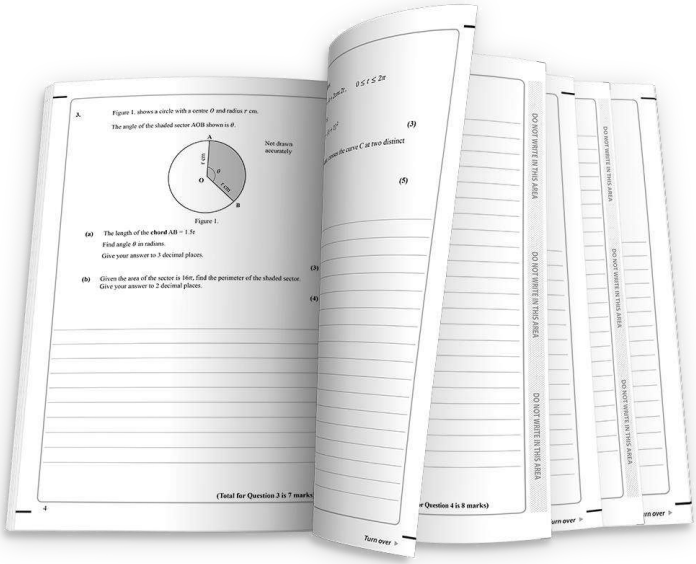

Available to buy separately or as a bundle

1 The table below shows corresponding values of x and y for $y = \sqrt{\dfrac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\displaystyle\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

a) $\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,dx \approx \frac{0.5}{2}\left(0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\right)$

$= 1.50475 \approx 1.50.$

b) $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\,dx =$

$\sqrt{9}\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,dx =$

$3\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\,dx =$

$3(1.50475) =$

$4.51425 =$

$4.51.$

c) The answer is accurate to 2sf.

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2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

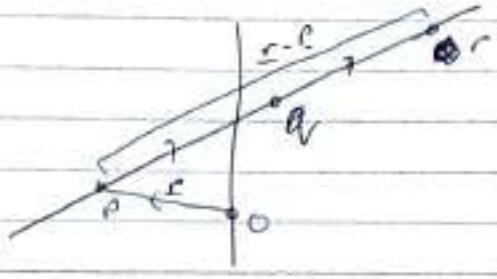
Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)



$\underline{q} = \underline{p} + \frac{1}{3}(\underline{r} - \underline{p})$

$\underline{q} = \underline{p} + \frac{1}{3}\underline{r} - \frac{1}{3}\underline{p}$

$\underline{q} = \frac{1}{3}\underline{r} + \frac{2}{3}\underline{p}$

$\underline{q} = \frac{1}{3}(\underline{r} + 2\underline{p})$.

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3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

$2\log(4-x) = \log(x+8)$

$\log((4-x)^2) = \log(x+8)$

$(4-x)^2 = x+8.$

$x^2 - 8x + 16 = x+8.$

$x^2 - 9x + 8 = 0.$

b) i) $(x-8)(x+1) = 0.$

$x = 8$ $\quad$ $x = -1.$

ii) $x = 8$ is not valid as $\log(4-8) = \log(-4)$ is a log of a negative which is not allowed.

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4. In the binomial expansion of

$$(a+2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

$(a+2x)^7$ has term $\binom{7}{4} a^3 (2x)^4$

$= \frac{7!}{4!\,3!} a^3 2^4 x^4$

Hence $15120 = \frac{7!}{4!\,3!} a^3 2^4$

$15120 = \frac{7\times6\times5}{3\times2\times1} a^3 \times 16$

$15120 = 7\times16\times5a^3$

$15120 = 560a^3$.

$a^3 = \frac{15120}{560}$

$a^3 = 27$

$a = 3$.

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5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

$3 \times 2^x = 15 - 2^{x+1}$

$3 \times 2^x = 15 - 2 \times 2^x$

$5 \times 2^x = 15$

$2^x = 3$

$x = \log_2 3$

$y = 3 \times 2^{\log_2 3}$

$y = 3 \times 3$

$y = 9$

Exact coordinate is $(\log_2 3, 9)$.

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6. (a) Given that

$$\frac{x^2+8x-3}{x+2} \equiv Ax+B+\frac{C}{x+2} \qquad x \in \mathbb{R} \ x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2+8x-3}{x+2}\,dx$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

a) $\frac{x^2+8x-3}{x+2} = Ax+B+\frac{C}{x+2}$

$x^2+8x-3 = Ax(x+2)+B(x+2)+C$

$x^2+8x-3 = Ax^2+2Ax+Bx+2B+C$

$x^2+8x-3 = Ax^2+(2A+B)x+(2B+C)$

$A=1$

$2A+B=8$

$2\times1+B=8$

$2+B=8$

$B=6.$

$2B+C=-3$

$2\times6+C=-3$

$12+C=-3.$

$C=-15.$

b) $I=\int_0^6 \frac{x^2+8x-3}{x+2}$

$I=\int_0^6 x+6-\frac{15}{x+2}\,dx$

$I=\left[\frac{1}{2}x^2+6x-15\ln(x+2)\right]_0^6$

$I=\frac{1}{2}\times6^2+6\times6-15\ln(6+2)-\frac{1}{2}\times0^2-6\times0+15\ln(0+2)$

$I=\frac{1}{2}\times36+36-15\ln(8)-0-0\times15\ln(2)$

$I=18+36+15\ln(2)-15\ln(8)$

$I=54+15\ln(2)-15\ln(2^3)$

$I=54+15\ln(2)-45\ln(2)$

$I=54-30\ln(2).$

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7.

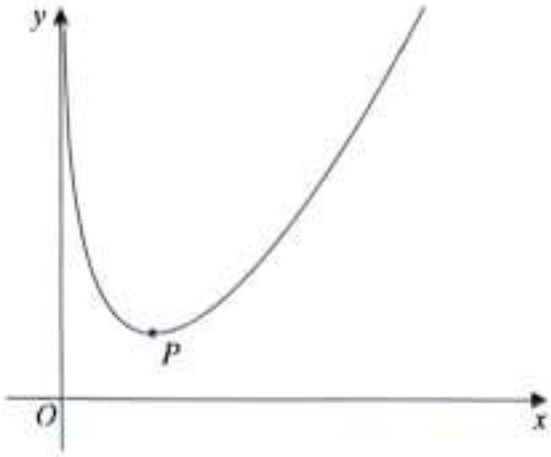

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

a) $y = \frac{4x^2+x}{2\sqrt{x}} - 4\ln(x)$,

$\frac{dy}{dx} = \frac{2\sqrt{x}(8x+1) - (4x^2+x)x^{-\frac{1}{2}}}{(2\sqrt{x})^2} - \frac{4}{x}$.

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Question 7 continued

$$\frac{dy}{dx} = \frac{16x^{3/2} + 2x^{1/2} - 4x^{3/2} + x^{1/2}}{4x} - \frac{4}{x}$$

$$\frac{dy}{dx} = \frac{12x^{3/2} + x^{1/2} - 16}{4x}$$

$$\frac{dy}{dx} = \frac{12x^2 + 1 - 16\sqrt{x}}{4x\sqrt{x}}$$

b) This is where $\frac{dy}{dx} = 0$.

$$12x^2 + 1 - 16\sqrt{x} = 0$$

$$12x^2 + \sqrt{x} - 16 = 0$$

$$x^2 + \frac{\sqrt{x}}{12} - \frac{4}{3} = 0$$

$$x^2 = \frac{4}{3} - \frac{\sqrt{x}}{12}$$

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{2/3}$$

c)i) $x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{2/3}$

$x_1 = 2$

$x_2 = 1.13894$.

ii) $x = 1.15650$

8. A curve C has equation $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $f(x)$

find, in simplest form, $f(x)$

(6)

$f'(x) = 6x^2 + ax - 23$.

$f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x + c$.

y - intercept is -12

$c = -12$.

$f(x) = 2x^3 + \frac{1}{2}ax^2 - 23x - 12$.

$f(-4) = 0$.

$2(-4)^3 + \frac{1}{2}a(-4)^2 - 23(-4) - 12 = 0$.

$2(-64) + \frac{1}{2}a(16) + 92 - 12 = 0$.

$-128 + 8a + 80 = 0$.

$8a - 48 = 0$

$8a = 48$

$a = 6$.

$f(x) = 2x^3 + 3x^2 - 23x - 12$.

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9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ °C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

a) $\theta = A - Be^{-0.07t}$.

$A - B = 18$

$A - Be^{-0.7} = 44$.

$A - B - (A - Be^{-0.7}) = 18 - 44$.

$-B + Be^{-0.7} = -26$.

$B(1 - e^{-0.7}) = 26$.

$B = \frac{26}{1 - e^{-0.7}}$.

$B = 51.6$.

$A - B = 18$

$A - 51.6 = 18$

$A = 18 + 51.6$

$A = 69.6$

$\theta = 69.6 - 51.6e^{-0.07t}$.

b) The maximum temperature of this model is 69.6 °C, which is considerably lower than the boiling point of 78 °C, so the model is not appropriate.

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10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

a) $\cos 3A =$

$\cos(2A + A) =$

$\cos(2A)\cos(A) - \sin(2A)\sin(A) =$

$(2\cos^2 A - 1)\cos A - 2\sin(A)\cos(A)\sin(A) =$

$2\cos^3 A - \cos A - 2\sin^2(A)\cos(A) =$

$2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A =$

$2\cos^3 A - \cos A - 2\cos A + 2\cos^2 A =$

$4\cos^3 A - 3\cos A.$

b) $1 - \cos(3x) = \sin^2(x).$

$1 - 4\cos^3(x) + 3\cos(x) = 1 - \cos^2(x).$

$4\cos^3(x) + 3\cos(x) = \cos^2(x)$

$4\cos^3(x) - \cos^2(x) + 3\cos(x) = 0.$

$\cos(x)(4\cos^2(x) - \cos(x) + 3) = 0.$

$\cos(x)(4\cos(x) - 3)(\cos(x) - 1) = 0.$

$\cos(x) = 0$ $\quad$ $\cos(x) = \frac{3}{4}$ $\quad$ $\cos(x) = 1$

$x = -90°, 90°$ $\quad$ $x = 139°.$ $\quad$ $x = 0°$

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11.

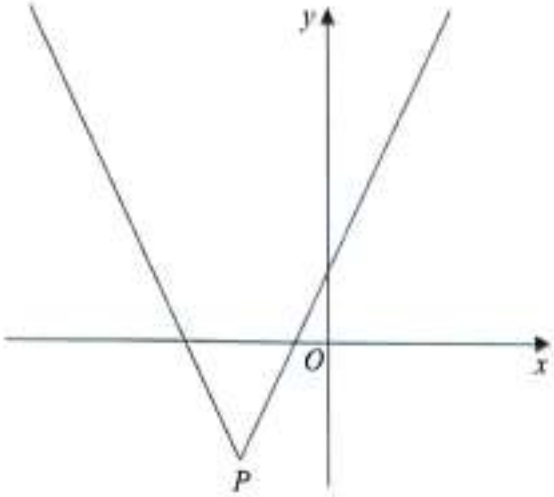

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x+4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x+4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x+4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

a) $y = 2|x+4| - 5$.

$x = -4$

$y = -5$.

b) $3x + 40 = 2|x+4| - 5$.

This only intersects at $-(x+4)$.

$3x + 40 = -2(x+4) - 5$.

$3x + 40 = -2x - 8 - 5$.

$3x + 40 = -2x - 13$.

$5x = -53$.

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Question 11 continued

$x = -10.6$.

c) $a > 2$ to intersect above x-axis.
$a < \frac{5}{4}$ to intersect below x-axis.
$\{a : a < \frac{5}{4}\} \cup \{a : a > 2\}$.

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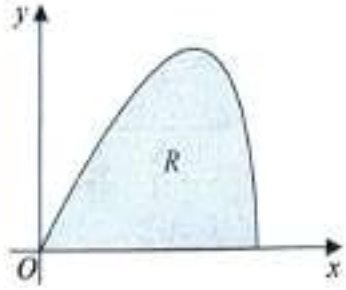

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

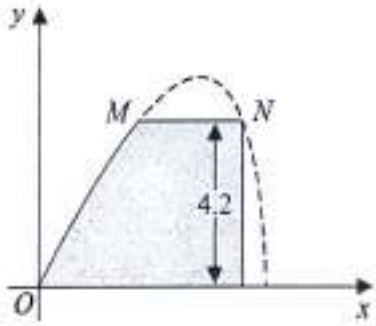

Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

a) i) $\int y \, dx =$

$\int y(t) \frac{dx}{dt} \, dt$

$\frac{dx}{dt} = 6\cos t$.

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Question 12 continued

$R = \int y \, dx$

$R = \int 5\sin 2t \; 6\cos t \, dt$

$R = \int 30 (2\sin t \cos t) \cos t \, dt$

$R = \int 60 \sin t \cos^2 t \, dt.$

ii) $R = \int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt.$

$R = \left[-20\cos^3 t \right]_0^{\frac{\pi}{2}}$

$R = -20\cos^3\left(\frac{\pi}{2}\right) + 20\cos^3(0)$

$R = 0 - (-20) = 20.$

b) $y = 4.2$

$5\sin 2t = 4.2$

$\sin 2t = 0.84$

$t = 0.4986, \; 1.072$

$x = 2.869 \qquad x = 5.269.$

$5.269 - 2.869 = 2.40.$

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k. **(1)**

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g. **(3)**

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

a) $g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2}$.

k is such that $\ln(k) - 2 = 0$

$\ln(k) = 2$

$k = e^2$.

b) $g'(x) = \frac{(\ln(x)-2)\frac{3}{x} - (3\ln(x)-7)\frac{1}{x}}{(\ln(x)-2)^2}$

$g'(x) = \frac{3(\ln(x)-2) - (3\ln(x)-7)}{x(\ln(x)-2)^2}$

$g'(x) = \frac{3\ln(x) - 6 - 3\ln(x) + 7}{x(\ln(x)-2)^2}$

$g'(x) = \frac{1}{x(\ln(x)-2)^2}$

$x > 0$ so $\frac{1}{x} > 0$.

$(\ln(x) - 2)^2 > 0$ (square).

$\Rightarrow g'(x) > 0 \ \forall \ x > 0$.

c) $g(a) > 0$

$\frac{3\ln(a) - 7}{\ln(a) - 2} > 0$.

$3\ln(a) - 7 > 0$ and $\ln(a) - 2 > 0$ OR $3\ln(a) - 7 < 0$ and $\ln(a) - 2 < 0$.

$\ln(a) > 2$ $\qquad$ $\ln(a) < 2$

$a > e^2$ $\qquad$ $a < e^2$.

$3\ln(a) > 7$

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Question 13 continued

$\ln(a) > \frac{7}{3}$

$a > e^{7/3}$.

$a > e^{7/3}$

$a < e^2$

Hence: $0 < a < e^2$; $a > e^{7/3}$.

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

a) ~~$5x^2 + (2r-48)x + (r^2-24$~~

Circle is $(x-r)^2 + (y-r)^2 = r^2$.

$2x + y = 12$.

$y = 12 - 2x$.

$(x-r)^2 + (12 - 2x - r)^2 = r^2$

$(x-r)^2 + (2x + r - 12)^2 = r^2$.

$x^2 - 2rx + r^2 + 4x^2 - 4(r-12)x + (r-12)^2 = r^2$

$x^2 - 2rx + r^2 + 4x^2 - 4rx + 48x + r^2 - 24r + 144 = r^2$.

$5x^2 + (2r-48)x + r^2 - 24r + 144 = 0$.

b) $b^2 - 4ac = 0$.

$(2r-48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 0$.

$(r-24)^2 - 5 \times (r^2 - 24r + 144) = 0$

$r^2 - 48r + 576 - 5r^2 + 120r - 720 = 0$.

$-4r^2 + 72r - 144 = 0$.

$4r^2 - 72r + 144 = 0$.

$r^2 - 18r + 36 = 0$.

$r = \frac{18 \pm \sqrt{18^2 - 4 \times 36}}{2}$

$r = 9 \pm \sqrt{9^2 - 36}$

$r = 9 \pm \sqrt{81 - 36}$

$r = 9 \pm \sqrt{45}$

$r = 9 \pm 3\sqrt{5}$.

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15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

a) $S_n = a + ar + ar^2 + \dots + ar^{n-1}$

$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$

$(1-r)S_n = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$.

b) $S_{10} = 4S_5$

$\frac{a(1-r^{10})}{1-r} = \frac{4a(1-r^5)}{1-r}$

$1-r^{10} = 4(1-r^5)$

$1-r^{10} = 4-4r^5$

$r^{10} - 4r^5 + 3 = 0$.

$(r^5-1)(r^5-3) = 0$.

$r^5 = 1$ $\quad$ $r^5 = 3$.

$r = 1$ $\quad$ $r = \sqrt[5]{3}$.

not a sequence.

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16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

All natural numbers are $3n$, $3n+1$, $3n+2$.

$3n$	$3n+1$	$3n+2$
$(3n)^2 =$	$(3n+1)^2 =$	$(3n+2)^2 =$
$\underbrace{9n^2}_{\text{multiple of 3}}$	$\underbrace{9n^2+6n}_{\text{multiple of 3}}+1$	$9n^2+12n+4 =$
		$\underbrace{9n^2+12n+3}_{\text{multiple of 3}}+1$
Hence, $(3n)^2$ is a multiple of 3	$(3n+1)^2$ is 1 more than a multiple of 3.	$(3n+2)^2$ is 1 more than a multiple of 3.

Hence, all square numbers are a multiple of 3 or one more than a multiple of 3.

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